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The undersigned certify that they have read and recommended to the Midlands State University for acceptance: a dissertation entitled "**Credit Risk Modelling by Commercial Banks in Southern Africa in the Presence of Market Friction (1997 - 2020)"** submitted by Matanda Ephraim in Partial Fulfilment of the Requirements for the Doctor of Philosophy (DPhil) in Finance and Digital Banking.

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DEDICATION

This thesis is dedicated to my beloved wife, Vharawei and kids, Fortune Ephraim, Emmanuel Tatenda, Nyasha Elisha, Elizabeth Paidamoyo and Tabeth Malon, who encouraged and gave me moral and financial to take this project to finality.

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ABSTRACT

The research proposes and examines new structural equity, risk of default, expected loss, and profitability models for banks in frictional and fuzzy financial markets. It is motivated by the need to fill the shortcomings of structural probability-based asset and credit risk models such as Merton (1974) and Black and Scholes (1973) that are characterised by unrealistic assumptions such as crisply precise and constant risk-free rates of return and asset volatilities. The problem investigated here specifically proposes new Kealhofer-Merton-Vasicek (KMV) and vector auto-regression (VAR) models for the valuation of equities, risks of default, expected losses, and profitability of banks respectively which are extended for both market friction represented by transaction costs and uncertainty modelled by fuzziness. The respective novel valuation models are then validated using cross-sectional financial data of listed banking corporations drawn from several emerging economies in Southern Africa. The results from the proposed equity and credit risk models are fairly stable, reliable, and consistent compared to those from conventional or structural credit risk models currently used for bank valuations in the markets. Therefore these proposed models are relevant in that they fairly capture practical conditions faced by banks in emerging markets that influence their equity, risk metrics, and credit exposures in their quest to improve capitalisation, financial performance, and shareholders' wealth. The study recommends that banks in frictional and fuzzy financial markets, such as those in emerging economies can adopt and implement the proposed models to even out under and overestimation errors caused by unrealistic assumptions underlying the structural models currently used worldwide.

Keywords: frictional and fuzzy financial markets, credit risk models, vector auto-regression, transaction costs, fuzziness, and emerging economies

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CHAPTER I

INTRODUCTION

1.1 Introduction

Over the years, the sub-discipline of credit risk modelling (CRM) has gained increasing importance in academic circles and the financial sectors of countries in particular. This is so because it is universally agreed that effective credit risk management is a prerequisite for institutional stability and global economic growth and development. Consequently, financial institutions, particularly commercial banks, have consistently invested substantial resources in the design and implementation of sound credit risk management models over the years. Credit risk models, for instance, structural and reduced-form models used in the valuation of firms and risk metrics of banks in particular are founded on the assumption that financial markets are frictionless as is the case for Black and Scholes (1973) and Merton (1974). It is however a fact that the history of the entire human society is characterised by market friction, for example, the exposure to risks of all kinds and human efforts undergone to deal with the same risks.

Shen (2014) defines market friction as a set of financial costs that comprise transaction costs and taxes on capital gains that impact on the performance of banks. He further argues that market friction is not purely a monetary cost as it can include incentives and commissions given to financial agents and fees for brokers. Therefore, market friction encompasses costs that banks incur in their business activities such as capital and concentration charges, as well as time-to-recovery and insider trading costs. From ancient times, at the emergence of species, people practised risk management to survive (Asset and Kazakhstan, 2015). Contemporary CRMs are founded based on precise classical probability theory (Elizalde, 2005) but are not adjusted for market friction and uncertainty which are real formidable costs faced by banks in Southern Africa. Accurate prediction of credit risk in banks could be transformed into a more efficient use of economic capital in their business operations. Hence to effectively assess credit risk in banks, the current structural models must be extended for market friction and fuzzy logic prediction, the so-called expert system in bank credit risk financial modelling.

According to Cao and Chen (1983) and Blockley (1980) reality in financial markets creates uncertainty that is vagueness stochastic uncertainty in contrast to vagueness from the semantic meaning of events called fuzziness, found in many areas of daily life. In other words, fuzzy sets or environments are designed to be used in handling particular kinds of uncertainty, which is the degree of vagueness for an exposure that can be possessed by objects to varying degrees, for example, the volatility of stock returns in financial markets. Moller et al (2002) researched fuzzy randomness and uncertainty modelling and concluded that these variables were a true reflection of the behaviours of investors in the construction of portfolios in financial markets. Hence the research on CRM is motivated by the need to investigate the impact of market friction, for instance, commissions, taxes, costs, and fees on banks' risk metrics such as the probability of default (PD), exposure at default (EAD) and loss given default (LGD) in the measurement of expected loss (EL) in fuzzy financial markets.

Zadeh (1980) argues that the concept of fuzziness is intimately related to expert value judgment, vagueness, generality, and ambiguity concerning investors' investment decisions in financial markets. Fuzziness does not have a well-defined set of bounds and is not resolvable with specific reference to context as opposed to the other related terms (Qin and Li, 2008). The other terms above can be contextually eliminated and conclusions that are closely linked to investors' language judgments can be made. It is a fact that integral applications that combine semantics, linguistic variables, and pragmatism are more powerful and beneficial to individual investors and firms in a given financial system compared to theories and models based on unrealistic assumptions such as constant risk-free rate of return and asset volatilities used in Merton (1974) and Black-Scholes models. It is against the above background that this research seeks to propose a credit risk model for banks in emerging markets such as Southern Africa characterised by frictional markets and uncertain (fuzzy) returns and risks. Such a model will go a long way in improving the rigour, accuracy, and prediction ability of asset values and risk metrics of banks in their quest to grow and develop.

This research's introduction chapter is organised in such a way that next is the background to the study, followed by the problem statement, aim and objectives, justification, significance, and contribution and ends with the research methodology.

1.2 Background to the Study

The real beauty of the Merton (1974) model lies in its capacity to treat a company's equity as a call option on its assets, paving way for applications of Black-Scholes (1973) option pricing methods if corresponding modeling assumptions are made. As credit risk has become an increasing concern in recent years in economics and finance, various advanced valuation methods have been employed widely to measure credit risk exposures. Two sets of credit risk models have emerged namely structural and reduced-form models as primary classes of credit risk modelling approaches. Wang (2009) stresses that structural approaches aim to provide an explicit relationship between default credit risk and capital structure, while reduced-form approach models on the other hand treat credit defaults as exogenous events driven by a stochastic or random process (such as a Poisson jump process). Structural models, pioneered by Black, Scholes (1973), and Merton (1974), employ modern option pricing theory in the valuation of corporate debt. However, the Merton model was the first structural model for the valuation of debt and has served as the cornerstone for all other structural valuation models. However, both Black-Scholes and Merton models are criticised for being founded on unrealistic assumptions, for instance, frictionless markets yet in reality financial conditions in most economies such as those in Africa are seriously characterised by frictional markets as outlined above.

Because of the unrealistic nature of the theoretical assumptions on which structural models are based, attempts have been made to extend the Merton model along this direction pioneered by Black and Cox (2000). This group of structured models is often referred to as the First Passage Time model. These models acknowledge that the constant interest rate assumption is not reliable, and hence a stochastic interest rate model can be incorporated into the Merton model or its extended versions. In this case, correlations between asset and interest rate processes can also be introduced as and when the need arises. It is further argued by Black and Cox (2000) that mapping all debts of a firm into a single zero-coupon bond is not always feasible. Research has shown that multiple debts with different characteristics can also be modelled using a structural approach. The Geske Compound Option model developed by Robert Geske was the first structural model of this nature.

Several more sophisticated structural models involving stochastic interest rates, volatility, jump-diffusion, and even regime-switching methods have also been proposed in the desire to move away from models based on frictionless to frictional market models. These new model applications can help explain market observations with a higher degree of accuracy, but they often involve a high level of analytical complexities.

Structural approaches, led by Merton's (1974) model, have the highly appealing feature of connecting credit risk to the underlying structural variables. Merton's model provides both an intuitive economic interpretation and endogenous explanation of credit defaults and allows for applications of option pricing methods to the valuation of firms. As a result, structural models not only facilitate asset valuation but also address the choice of financial structure for the firm. Wang (2009) points out that the main disadvantage of structural models lies in the difficulty of implementation. For instance, the continuous traceability assumption for corporate assets in structural models is unrealistic, and calibrating stochastic asset processes using publicly available information may be sometimes more difficult than anticipated. Furthermore, while improved structural models have addressed several shortcomings of earlier models, they tend to be analytically complex and computationally intensive.

Most of the extended structural models represent important improvements to Merton's original framework. New-look structural models are more realistic and able to better align with market data for example credit default swap (CDS) spreads for the betterment of the model. In Merton's framework, a company could only default at its debt maturity date hence model can be modified to allow for early defaults by specifying a threshold level such that a default event occurs when asset value, At falls below a such critical level.

1.2.1 Financial theory and structural credit risk models

The financial theory holds that contemporary structural credit risk models suit very well the circumstances of banks in developed countries with frictionless financial markets (Bluhm et al, 2013). Frictionless financial markets are markets in which investors face no costs in transacting their business transactions or investments. In all financial markets with no friction, investors can achieve risk-return trade-offs by holding market portfolios and possibly combining with short positions in riskless assets. These investors can hold maximally diversified portfolios and achieve their preferred risk levels by adjusting their holdings of riskless assets. Such allocations will dominate portfolios of only risky assets in all cases except the point of tangency between the efficient frontier of risky assets and the capital market line (CML). However, in frictional financial markets such as those in countries in Southern Africa, investors cannot adjust without the costs of their holdings (Federal Reserve Bank of Atlanta, FRBA, 2007). An investor holding for instance a sub-optimal portfolio drawn from inheritance funds, or a change in employment or marital status could lower their own risk without sacrificing expected return by rebalancing the investment portfolio or improving the portfolio's expected return without accepting any more risk. The rebalancing of investable funds between risk-free and risky securities is costly or impossible in frictional financial markets.

1.2.2 Why Care about financial market friction

Banks must extend contemporary credit risk models for market friction for several central reasons. Firstly financial market friction can generate real costs for investors in the construction of investment portfolios. Recognising these investment costs helps in understanding the total costs of transactions faced by investors. They can then decide where to place, hedge, and even whether to make them at all in the first place for example capital gains tax (FRBA, 2007). Constantinides (1984) in FRBA (2007) shows that the option to assume or defer capital losses or gains has substantial value in the eyes of investors. The option's exact value in derivative markets and the corresponding optimal trading strategy normally depend on factors such as transaction costs, capital gains tax rates, and asset volatilities. Financial market frictions can also generate business opportunities such as investment in mutual funds, which relax wealth constraints and asset indivisibilities.(DeGennaro and Kim 1986). Financial market friction is not exact and hence can change over time.

The degree of existing market friction in markets varies, such that new types can appear while existing ones disappear. Bank analysts currently face the daunting task of analyzing far larger and more sophisticated institutions than existed twenty years or more ago. This challenge however can be offset in part by a vast increase in information and computing power that are readily available to them. Kane (2000) shows that regulators or central banks of financial systems face similar challenges of complexity and difficulty of resolving undercapitalized institutions, shifting the political calculus of a resolution and all financial market frictions to do with shifting from the use of qualitative to quantitative information.

1.2.3 Fuzzy logic with application to financial markets

Although contemporary credit risk models such as Merton (1974) used by banks are based on assumptions such as frictionless markets and constant returns and asset volatilities, in reality, financial markets are mainly characterised by friction and uncertainty with specific reference to fuzziness (Duffie, 2003 and Zadeh, 1965). Fuzziness is defined by Zadeh (1980) and Zimmermann (2001) and Zadeh (1980 and 1965) as a market condition in which returns to financial market investments are not precisely defined as is expressed in probability theory but in linguistic terms such as high, average or low. The history of the entire human society is characterised by the exposure to risks of all kinds and human efforts undergone to deal with the same risks. From ancient times, at the emergence of species, people practised risk management to survive (Asset and Kazakhstan, 2015).

The practice of survival instincts by humans leads to the avoidance of risks threatening to extinct the human species. However, the existence of humankind today is enough testimony of the successful application of risk management strategies by our ancestors. Hence the proposed research model is motivated by the need to examine the impact of both transaction costs and fuzziness on CRM and the valuation of banks in emerging markets. According to classical probability, an investment can have a return of 40%, which can be described as a high return in fuzzy theory and logic. Based on fuzzy mathematics this return of 40% to an investment will then be considered to be a number, X which is a continuous variable such that it will be lying in the range of 35% to 45%.

According to Zadeh (1965), fuzzy logic is an approach used in variable processing that allows multiple possible truth values to be processed through the same quantity to achieve a family of accurate conclusions. In other words, fuzzy logic refers to generalisations from the standard logic value in which all statements have a truth value of 1 or 0 that is statements can have partial truth values such as 0.95 or 0.50. Theoretically, this valuation approach gives more opportunity to mimic real-life circumstances where statements of the absolute truth of falsehood are very rare (Zimmermann, 1980). Fuzzy logic can be used by quantitative analysts to improve the execution of their algorithms in various economic and commercial disciplines. Because of similarities with ordinary language, fuzzy algorithms are comparatively simple to code, although they may require thorough verification and testing processes.

The following terms are defined to assist in the conceptualisation, understanding, and internalisation of fuzzy logic and set theory.

1.2.3.1 Uncertainty

The concept of uncertainty is defined from two perspectives namely the state of being unsettled, in doubt or dependent on chance and that of being unsure of an event or something happening. The following are some of the words often used in combination with *uncertainty* (Cambridge English Corpus).

.Conditional uncertainty

This means that the rules used must account for every condition of uncertainty.

.Considerable uncertainty

This refers to considerable uncertainty about the future of informal care given to an event or situation.

.Degree of uncertainty

As is usually expected, the values drawn from events are subject to high degrees of uncertainty.

1.2.3.2 Vagueness

A word, phrase, or sentence is said to be vague when it refers to unclear or imprecise circumstances. Vague statements often call for follow-up questions. For example, if we say, "The son did not live up to our expectations" this is a vague statement that may call for another question such as, "What were our expectations?" Readers or listeners should not be put in the position of assuming our intentions by asking vague questions to them.

1.2.3.3 Fuzzy set

A fuzzy set also called an uncertain set is a set whose elements have degrees of belongingness or membership (Zadeh, 1965). Fuzzy sets are perceived as extensions of the classical notion of a precise set that we all know.

Membership function

We first assume we have a function R and E_1,……………,E_n be fuzzy variables on some credibility space ($\theta; P; Cr$). By credibility theory or space we mean a form of statistical inference developed by Thomas Bayes used to forecast an uncertain future event. Credibility space is used to combine multiple estimates into a single summary estimate that puts into consideration information on the accuracy of the initial variable estimates. The concept of credibility space is commonly used by Insurance Companies when determining premium values. Therefore if the function:

$$U = \{\theta; P; Cr\} \text{ is a credibility space,} \tag{1.1}$$

then its membership function is defined from equation 1.1 by:

$$u(x) = 2Cr\ (E = x) - 1,\ x \in R. \tag{1.2}$$

Therefore the membership function of a fuzzy set is a generalisation of the indicator function of classical sets. In fuzzy logic a membership function represents the degree of truth as an extension of valuation. Degrees of truth are often confused with probabilities although they are conceptually distinct or different because fuzzy truth represents membership in vaguely defined sets, not likelihood of some condition or event. Zadeh, (1965) demonstrated that since most fuzzy sets haave a universe of discourse X, consisting of the real line, R, it is impractical to list all pairs of values defining a membership function. Therefore membership functions characterise fuzziness that is all the information in a fuzzy set whether the elements in the set are discrete or continuous. The membership function μA assumes values in the range 0 to 1 given as [0; 1].

$\alpha - \text{cut}$

Given a fuzzy set A defined on variable X and any number $\alpha \epsilon [0; 1]$, the $\alpha - cut$ (αA is the crisp set that contains all the elements of a universal set, X whose membership grades in A are $\geq$ $specific\ values\ of\ \alpha,$ that is $\alpha A = \{x |\ A(x) \geq \alpha\}$ where A is a fuzzy set.

Fuzzy number

A fuzzy number is a generalisation of a regular, real number in the sense that it does not refer to a single value but rather a connected set of possible values each with a weight between 0 and 1 (Zimmermann, 1980 and Zadeh, 1965). This weight is called the membership function (definition 1.4 above). The membership function of a set A is given by μ_A (x) = $(A_1; A_2; A_3)$. For example we may have set A = $(A_1; A_2; A_3)$ where $(A_1; A_2; A_3) \in R$ $(Real\ numbers)$ and $A_1 \leq A_2 \leq A_3$ such that μ_A (x)= $\propto$, a fuzzy number.

There are three common variables found in fuzzy mathematics as illustrated below.

Equi-possible fuzzy variable

It refers to a fuzzy variable, X that is determined by a pair of values (a; b) of crisp numbers such that a$< b$ with M membership function:

$$\mu(\text{x}) = \begin{cases} 1, if\ a \leq x \leq b \\ 0, Otherwise \end{cases} \tag{1.3}$$

This type of fuzzy variable is as demonstrated graphically by the succeeding figure 1.1 below.

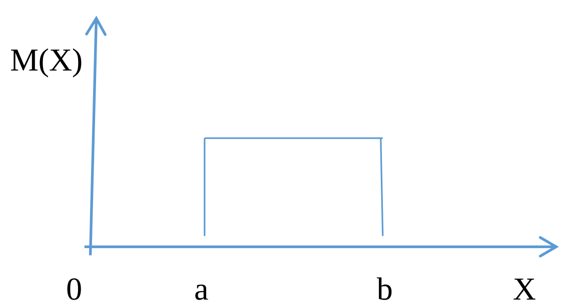


Fig. 1.1 Showing an equi-possible fuzzy variable, X

Triangular fuzzy variable

This is a fuzzy variable, X that is determined by a triplet of values (a; b; c) of crisp numbers such that a$< b < c$ with M membership function given by:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & for\ a \leq x \leq b \\ \frac{x-b}{c-b} & for\ b \leq x \leq c \\ 0, Otherwise \end{cases} \quad (1.4$$

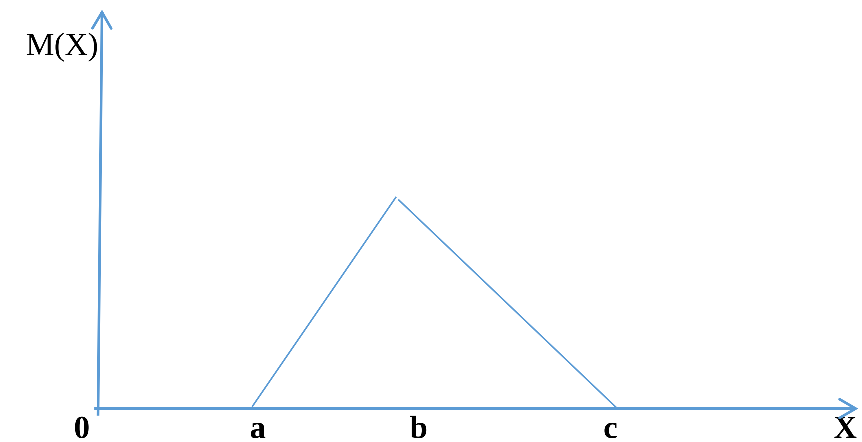


Fig. 1.2 Showing a triangular fuzzy variable, X

Trapezoidal variable

This is a fuzzy variable, X that is determined by a set of quadruplet values (a; b; c; d) of crisp numbers such that:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & for\ a \leq x \leq b \\ \frac{x-b}{c-b} & for\ b \leq x \leq c \\ \frac{x-c}{d-c} & for\ \ c \leq x \leq d, \\ 0, Otherwise \end{cases} \quad (1.5)$$

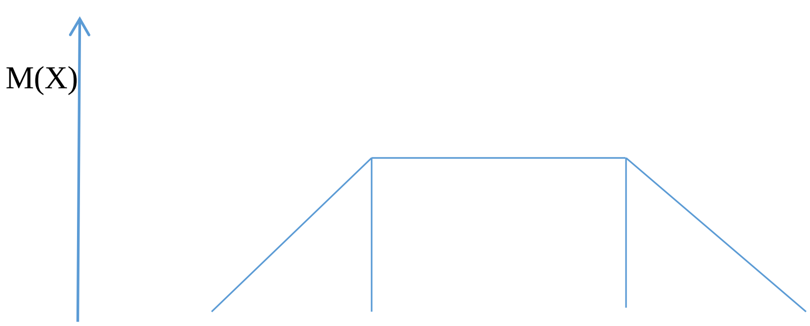


Fig. 1.3 Showing a trapezoidal fuzzy variable, X

1.2.4 Classification of financial market friction

The universe of types of financial market friction can be partitioned in many different ways. The FRBA (2007) builds the structure for the classification of market frictions on the economic forces underlying financial market frictions. This structure used takes a step toward the identification process of those entities best able to reduce their costs of market frictions. There are five primary categories into which market frictions are divided and these are transaction costs, taxes and regulations, asset indivisibility, nontraded assets, and agency and information problems. While all these forms of market frictions are known for impacting negatively the performance of markets in emerging economies, only transaction costs will be drawn into the research at hand because they are the greatest constraint to the growth and development of banks.

Transaction costs are partitioned into two categories which are the costs of trade and the opportunity costs of time. The costs of trade in financial markets will include expenses such as postage, telephone charges, computer power, and similar real expenditures of resources used in business by investors. These are costs that have been declining with technological improvements or innovation for instance costs of communication and data analysis have fallen over time. Financial trading requires time and will include both search costs and the time to gather information, find a trading partner, and the time to make the trade itself (Fuchs and Uy, 20100. The process of minimizing these types of costs represents a profit opportunity for instance automation of the process by means such as automatic electronic payments. Other reductions in the time required to trade in financial assets are sure to follow, both because technology continues to advance and the opportunity cost of time tends to rise over time. Some of the hazards facing most emerging economies are government-imposed regulations that arise from repressed or administered financial systems that lack autonomy and independence as far as their operations are concerned.

The World Bank (2013) argues that technology can be a game changer in the economics of retail banking particularly in Africa, where financial systems face huge barriers to further outreach, including high transaction costs. Hence fostering competition and the adoption of technology-driven financial innovation will be even more important in agriculture than in other economic sectors. The World Bank provides several examples of successful innovations in the African region that involved new products, providers, delivery channels, and connections among different segments of the financial systems of countries and the agricultural sectors. However, improvements in financial and agricultural sectors alone are not sufficient, and hence the need to manage other major challenges in policy formulation and implementation, business environments, and supporting infrastructure for the economies to be able to grow and develop.

According to Wang (2009) the two years, 2007 and 2008 were characterised by global financial markets experiencing an unprecedented crisis. Although the causes of the 2007-08 global crisis are sophisticated, it is agreeable that credit risks contributed the most to the challenges faced by banks but the African continent was spared from the crisis. In other words, the African continent if had been hit by the global financial crisis could have sunk into oblivion given that it has its own family of perennial challenges. The World Bank (2013) goes further to argue that emerging economies of Africa have serious challenges in accessing long-term finance and these were as many as those involved in expanding the outreach of their financial systems.

Africa faces many long-term investment needs compared to other continents but lacks the necessary capital resources, markets, and products to satisfy these needs. There should be a paradigm shift where market-replacing activist policies are employed through the abandonment of government intervention policies and strategies in favor of modernist strategies geared towards a pro-market activist role of a democratic government. However, according to Witte (2015), politics is a key determinant of the effectiveness of government interventions in the African continent. The research stresses critical differences between East Asia and Africa for instance that can explain why the government was more successful in its interventions in the Pacific Basin Model of the late 20th century than in the latter.

1.2.5 Market structures in Southern Africa

Financial market frictions, especially transaction costs are to a greater extent influenced by the structure of the market. Market structure, in turn, depends on both the risk of the traded security and trading volumes. Southern Africa is characterised by shallow and non-broadened markets for risky assets that are also very unmarketable and illiquid coupled with market participants searching for counterparties directly. This is because transaction costs such as fixed costs of capital investments by banks (including communication) are too huge to be offset by the lower marginal costs of each transaction if transactions turn out to be few (Fuchs and Uy, 2010). It is based on the above challenges including the accumulation of NPLs faced by banks in Southern Africa that this research seeks to extend structural credit risk models applied to frictionless markets to include market friction.

A model of the above nature can be used to accurately measuring the leverage of banks in Southern Africa, their investment values, and risk metrics in their quest to grow and develop. As trading volumes of banks increase, markets evolve from direct searches through brokered, dealers and continuous auction markets. This evolution is a simultaneous process such that as volumes increase, the structure evolves, and leads to an increase in trading volumes. It can also be argued that the potential size of the financial market determines the equilibrium structure. Hence as bank trading volumes increase, it starts to make sense to invest in capital markets and acquire specialized knowledge about potential buyers and sellers of risky securities to facilitate trading for example the case for stockbrokers.

1.2.6 Opportunities and challenges faced by Sub –Saharan Africa (SSA) Banks

Banking systems in some parts of Sub-Saharan Africa (SSA) have grown notably over the past decades due to stable macroeconomic, regulatory, and financial trends. Nonetheless, downside financial risks remain elevated by structural issues, commodity price fluctuations, reversal of capital flows, and spill-over effects from external shocks like that witnessed in Central East and South East European (CESEE) countries. In the light of the 2007-2008 global financial crisis, great attention was given to understanding the causes of banking instability with most of the research centred on advanced economies and large emerging markets while little attention was paid to the bank-based financial sectors of Sub-Saharan Africa. To worsen the situation, there is a scarcity of studies aiming at knowledge-sharing among different emerging economies of the world.

The World Bank (2017) outlines the determinants of bank credit risk by focusing on five SSA countries namely Kenya, Namibia, South Africa, Zambia, and Uganda. The study used the ARDL approach to co-integration and discovered that increased money supply conditions had a decreasing effect on non-performing loans (NPLs) in all counties. It was also discovered that banking industry-specific variables played a significant role in the case of South Africa and Uganda while NPLs were driven by country-specific variables in the case of Zambia, Kenya, and South Africa. The effects of the global financial crisis in these selected SSA countries are evidenced indirectly. Drawing on evidence from CESEE countries with long experience in banking crises, reforms, and financial deepening processes, the study provides lessons for SSA countries and offers policy recommendations in the direction of strengthening banks' statements of financial position (SFP) to ensure financial stability, growth, and development of these emerging countries.

1.2.7 Overview of the SSA economies and banking systems

Over the last decade, growth in SSA countries has been characterised by economic diversification, greater trade integration, and improved political and macroeconomic stability but not in all countries. However, the outlook in SSA countries can be obscured by downside risks such as political instability and economic recessions, the Covid-19 pandemic, violent insurgencies, lower commodity prices, and volatile global financial conditions. Below is a comparison of the economies and the banking sectors of the SSA countries with respective sectors of the Central Eastern and South-Eastern Europe (CESEE) countries.

Overview of SSA economies

Broadly, the CESEE economies were found to have gone through a remarkable transformation since 1989. This was considered the seal of transition from centrally planned to market economies. During the 1990s for instance, several countries in the region suffered high inflationary periods and episodes of sovereign crises due to macroeconomic imbalances and policy uncertainty and inconsistencies. However, over the period 2000–2007 the economic performance of most CESEE countries impressed with real GDP growth reaching an average of about 5.90% (World Bank, 2017). In the pre-crisis years, growth in the region was driven by the booming property and financial sectors magnified by ample credit. In 2007-08, several countries were in a state of overheating characterised by a credit boom and bust cycle which ensued as private debt stocks across CESEE increased much faster than GDP, opening up credit gaps in the process.

The current account deficits of the countries were financed by large FDI inflows and many countries were running budget deficits. Thus, despite the remarkable economic performance of CESEE countries overall, macroeconomic imbalances remained prevalent. In the fall of 2008, several CESEE countries were severely hit by the Global Financial Crisis. Hence many lessons can be drawn from the CESEE's diverse economies and reforms, as the crisis made the unthinkable possible (Aslund, 2012). It was noted that their output plunged and unemployment rates soared as several countries slipped into recessions. Despite many economic differences, in most CESEE countries private investment and growth remained below pre-crisis levels reflecting uncertainty about economic recovery and private sector statements of financial position (SFP) weaknesses reflected in the stock of NPLs.

Since the 1990s by comparison, most SSA economies have been among the world's fastest-growing regions. The majority of SSA countries experienced accelerated growth in the post-2000 era, led by mining and hydrocarbon exports. The acceleration in growth has been accompanied, and facilitated, by a sharp reduction in inflation rates. In most SSA economies inflation rates are typically in the single-digit range, despite persistent vulnerabilities to food and fuel price shocks (Mecagni et al., 2015). Some of the key factors contributing to this turnaround in most SSA economic fortunes were improved macroeconomic policies.

Table 1.1 below provides a concise overview of the GDP and the GDP per capita along with the average growth rates and volatilities for each of the focal SSA countries as well as the averages for SSA and CESEE countries.

Table 1.1. GDP, GDP per capita in 2014, Growth Rate and Volatility Over the Period 2000–14

Country	GDP per capita (in 2005 USD)	GDP per Capita Growth Rate (%)	GDP (in 2005, USD Billion)	GDP Average Growth Rate	World Bank Classification(per Income)

Kenya	658.7	1.7% (2.4%)	29.6	4.4% (2.5%)	Lower-middle
Namibia	4 674.6	3.1% (2.9%)	11.2	4.9% (2.9%)	Upper-middle
South Africa	6 086.4	1.6% (1.8%)	328.7	3.2% (1.8%)	Upper-middle
Uganda	435.5	3.1% (2.2%)	16.5	6.6% (2.3%)	Low
Zambia	1 032.8	4.0% (1.6%)	16.2	7.0% (1.7%)	Lower-middle
SSA Countries	1034.1	1.9% (1.6%)	29.6	4.4% (2.5%)	All income levels
CESEE Countries	7966.2	4.0% (1.3%)	11.2	4.9% (2.9%)	All income levels

Note: In parentheses, the volatility of GDP growth in the period 2000–2014.

Source: World Bank and Authors' Calculations

Overview of the banking systems

It was noted that developing modern and market-oriented banking sectors was a challenge for the transition economies of CESEE following a long period of mono-bank systems, where credit evaluation and risk management were irrelevant (IMF, 2013). The early phases of the transition were marred by regular banking crises followed by privatization efforts and foreign control of a large share of CESEE banking systems. From the early 2000s, the incidence of crises in CESEE declined sharply because of foreign ownership which brought greater financial stability but also generated boom-bust cycles and transmitted international shocks. Foreign ownership has been a strong factor in CESEE banks because foreign-parent banks supported their subsidiaries before and through the financial crisis of 2008. The process of financial broadening and deepening in the economies was facilitated by capital inflows which funded rapid growth in credit, consumption, and external debts. The 2007-08 Global Financial Crisis triggered a sudden stop in capital inflows, credit growth came to a point of stagnation and investment collapsed across the region with only exceptions (IMF, 2013). External demand for products of the countries slumped and a negative feedback loop pushed most of the CESEE countries into recessions. Countries' banks were unable to absorb losses from non-performing loans to facilitate a fresh reallocation of credit to the economies.

Many CESEE banking systems were faced with sizeable non-performing loans and the share of these loans rose above 10% in many countries and even exceeded 20% in some cases, undermining banks' profitability. Given the systemic importance of the Euro area banks' subsidiaries in CESEE along with funding shocks facing the parent banks, contagion from the Euro area became a major risk for CESEE banks (Mecagni et al., 2015). Broadly, the boom–bust cycle left a legacy of non-performing loans in various CESEE countries. A high credit growth during 2003–2008 gave rise to an unsustainable boom that ended abruptly with the Global Financial Crisis. The deep recession that followed brought to the front several accumulated underlying problems while countries with more pronounced boom–bust cycles worsened considerably during the crisis. Retrospectively, neither the banking sectors of CESEE were sufficiently resilient nor the institutions were fully flexible to deal with the debt stock challenges and design supportive macroeconomic policies to rescue them from the crisis.

The acceleration of economic growth in Sub-Saharan Africa in the post-2000 era has been accompanied by an expansion of access to financial services, mainly through commercial banks, which have the traditional backbone of the financial systems of most SSA countries. Banking in SSA has undergone dramatic changes over the past 20 years (Mecagni et al., 2015). Like CESEE countries, financial liberalization and related reforms, improved regulatory capacity, and expansion of cross-border activities have significantly changed the SSA banking landscape in the 21st century. Once dominated by state-owned institutions, banking systems in SSA, once dominated by state-owned institutions became more stable as evidenced by the dramatic decrease in incidences of banking crises in the past two decades (Mecagni et al., 2015). Since 1990, banking systems in SSA have steadily shifted from majority state-owned to private-owned corporations, with increasing levels of foreign ownership of such entities. However, policy inconsistencies, greed, and corruption, as well as operations of regular budget deficits, have led to most SSA countries being over-borrowed and using repressed financial systems, leading to crowding out and the emergence of parallel or black markets.

The ownership structure and the market share controlled by the largest banks in each of the focal SSA countries are presented in Table 1.2 (IMF, 2013).

Table 1.2. Ownership structure and market share in the focal SSA countries

Variable	**Kenya**	**South Africa**	**Namibia**	**Zambia**	**Uganda**
Number of banks	43	74	4	19	25
Ownership (%)					
Foreign	33	60	75	37	84
Local	67	30	25	63	16
Mutual	–	10	–	–	–
Asset size (%)					
Foreign	34	27	80	67	88
Local	66	73	20	33	12
Market share (%)	52% (top-6)	90% (top-5)	62% (top-2)	58% (top-4)	60% (top-5)

Source: Focal Countries' Central Banks

The IMF (2013) notes that the availability of reliable, consistent, and sufficiently long time-series data is identified as a key limitation in all studies dealing with developing countries such as SSA. Data limitations make conducting analytical work in SSA countries very difficult and restrict economic analysis and interpretations. Furthermore, it is observed that the lack of harmonised definitions of credit risk proxies across SSA countries could result in measurement issues that may obscure the interpretations of the empirical findings. The study aimed to include countries at different phases of economic development and given these considerations, several indicators were collected for each sampled country from publicly available sources such as the World Bank/IMF, Central Banks, and other sources. Monthly data were used in the case of South Africa (June 2008–June 2014), Kenya (December 2004–June 2014), and Zambia (December 2008–December 2013). However quarterly data were used for Uganda (2001Q1 -2014Q2) and Namibia (2001Q4–2014Q2).

1.2.8 Why research on banks in Southern Africa

Therefore the research at hand focuses on banks in Southern Africa, as a subset of SSA countries, a decision that was arrived at based on availability and access to data from publicly available sources for the countries in question. The table below summarises some of the major challenges cited by Fuchs and Uy (2010) that banks in Southern Africa in particular face in their desire to grow toward sustainable development.

Table 1.3. Showing Challenges Faced by Banks in Southern Africa and Comments

Challenge	Comment
Ratio of liquid liabilities to the GDP of countries	Too high to allow for growth of the sectors'
Absolute sizes of financial systems of countries	Too small to allow for growth/development
Access to financial services by households	Very limited access to products/services
Net interest margins attained by banks	Very high costs and risks realized
Non-performing loans (NPLs)	Accumulating over time and irrecoverable
Total overhead costs across countries	Very huge overheads which affect growth
Banks' returns on ordinary equity across countries	Constrained leading to mergers/acquisitions
Ratio of checking account fees to GDP per capita	Exorbitant fees relative to GDP per capita
Documentation requirements across countries	Discourage financial activity/investment
Transaction Costs (Agency and Taxes)	Very high costs that erode incomes of banks
Stock market turnover across countries	Small and illiquid stocks traded on markets
Financial innovation and deepening	Very low deposits/credit and high liabilities
Development of credit registries in Africa	Presence of asymmetric credit information
Ownership of banks in Africa over time	Above 70% foreign owned and government

Source: Fuchs and Uy (2010; 14)

The table above summarises the major constraints that are faced by banking institutions in Southern Africa. Duffie and Singleton (2003) argue that innovations in CRM and transfers were central if banks are to attain financial stability. The above view by Duffie and Singleton (2003) is complemented by Fuchs and Uy (2010) who argue that banks in Southern Africa lack financial innovation and deepening. Fuchs and Uy (2010) divide transaction costs into direct and indirect costs that are associated with the execution of financial transactions. Direct costs are total costs that borrowers incur in the process of applying for loans from banking corporations for instance processing and interest expenses on loans borrowed. These direct costs faced by such borrowers can be extended to include finance and insurance costs that accrue on the loans in the event of borrowers' failure to settle both principal and interest amounts when they fall due. Fuchs and Uy (2010) proceed to define indirect costs as other costs that are outside the scope of those levied on borrowers for applying for credit and will include overheads, and general and administration expenses.

Southern Africa as a subset of SSA is associated with banks that are most vulnerable and underdeveloped compared to those of other continents. Emerging countries of Southern Africa were seriously signified by capital inflow challenges, financial friction, non-performing loans, poor corporate governance, and ethics that need to be redressed to turn their fortunes the other way around (World Bank, 2009). Therefore after realizing that African Central Banks were busy implementing structural credit risk models and Basel Capital Accords that suit frictionless markets (unrealistic assumptions) like those in USA and Europe, the study seeks to propose and validate a credit risk model for use in the valuation of banks in Southern Africa characterised by friction in fuzzy financial markets. While most structural credit risk models have been extended to the case for market friction or transaction costs, it is the uncertainty component which we call fuzziness which is real in emerging markets that this study will add to so that it will go a long way in its contribution to existing credit risk theory. The research has a strong conviction that a homegrown credit risk model for Southern Africa will go a long way in addressing the countries' vulnerabilities to economic shocks because they are in one regional block despite being different in geographical areas and growth capabilities.

1.3 Problem Statement

Recent developments in credit risk modelling acknowledge that Merton's (1974) and Black-Scholes' (1973) structural models are the cornerstones of all valuation models used in economics and finance and other related disciplines. However, these conventional models are criticised for making unrealistic assumptions for example the theoretical assumptions of their applicability in frictionless financial markets and precise decisions by investors. While the models could apply to frictionless markets such as those in developed economies of the West, their estimation abilities could be questionable in most emerging markets such as those in Southern Africa whose markets are fuzzy and frictional as signified by high monetary and non-monetary transaction costs. The Merton (1974) model and other structural models assume that returns to investments are precise or exact according to classical probability theory but in reality, financial markets operate based on investments whose returns and risks are uncertain.

Therefore to improve efficiency and effectiveness in banks in emerging markets, current credit risk models need to be extended to include friction or uncertainty to make them more robust and practically oriented. However, trends from most African financial markets indicate that banks are over-borrowed and hence use leverage more conservatively compared to developed economies Moody's (GCR 2013). In contrast, financing, investment, and economic growth are in a state of stagnant in Africa (IMF, 2015), and hence there is a huge investment gap to be closed (Rod et al., 2015). Banks in Southern Africa for instance are heavily characterised by insufficient capital, the decline in liquidity levels, too much cash flow volatilities, increased bankruptcy, and accumulation of non-performing loans (NPLs).

Central Banks of emerging economies must therefore shift from dependence on structural credit risk and asset valuation models to contemporary approaches that are suitable to the challenges that they face which are far from being congruent to those faced by rich countries of the world. Such a paradigm shift will go a long way in assisting Banks in coming up with credit risk models that suit their market conditions, and prudent regulatory and supervisory frameworks in their quest to improve financial performance and contribution to growth and sustainability.

The most prevalent challenges faced by banks in emerging markets are insufficient capitalisation and buffer stocks, over-reliance on borrowing, insider trading, and poor credit ratings that lead to non-performing loans (NPLs), market friction or high transaction costs to poor corporate governance and ethics (Fuchs and Uy, 2010). The above challenges coupled with a lack of autonomy and independence of Central Banks due to massive government interventions and directives render the valuation of banks and their risk metrics through existing structural CRMs not feasible and very inaccurate. Hence it is against the above background that this research is motivated to propose and investigate the impact of transaction costs faced by investors in fuzzy financial markets on the valuation of banks and their risk metrics namely PDs, EADs, LGDs, and ELs in their quest to grow and develop.

1.4 Aim of the Study

The main aim of the research study is to propose and investigate the effects of the inclusion of market friction such as transaction costs, taxes, and commissions on the risk measures or metrics of commercial banks in Southern Africa.

1.5 Objectives of the Study

The research study sought to:

1.5.1 Investigate the Merton asset valuation (AV) model as used in credit risk modelling in modern markets characterized by friction in fuzzy financial markets.

1.5.2 Propose a fuzzy probability of default (PD) model for use in credit risk modelling in banks in emerging markets.

1.5.3 Examine the effects of a market friction-adjusted expected loss (EL) model on the valuation of banking corporations in fuzzy environments.

1.5.4 Assess the impact of market friction on the financial performance of banking corporations in emerging economies such as Southern Africa.

1.6 Justification of the Study

Most countries in emerging economies such as those in Southern Africa are characterised by frictional and fuzzy financial markets where returns to investors are very uncertain and influenced by human language such as expert judgments. Corporate loans or credit exposures issued by banks in emerging economies are mostly delinquent and associated with high transaction costs such as agency costs, capital costs, taxes, and commissions. These cost challenges are reflected in banks' failure to meet their working capital needs, minimum capital requirements (MKRs) set by Central Banks, access credit lines, and mobilise foreign direct investment (FDI) as well as deposits from the general public and corporates. It is against the above background that this study proposed Kealhofer, Mc-Quown, and Vasicek (KMV) equity, risk of default, fuzzy expected loss, and financial performance models extended for market friction fuzziness for use by banks in emerging economies such as those in Southern Africa. Valuation models that capture market friction and fuzziness components are likely to go a long way in being precise in the estimation of equity values, risk metrics, and financial performance of banks in their quest to grow towards sustainable development.

1.7 Significance of the Study

Upon completion, the study will benefit various stakeholders in many different ways in developing countries of the world including Southern African nations. Firms in Southern Africa and other parts of the world will adopt CRMs that are realistic and employ them to improve their capital bases and manage their credit risks accurately for their growth and development. The study envisages an intimate relationship between market friction and human psychology-adjusted CRMs and well-regulated, supervised, efficient, and effectively managed firms (Palma and Ochoa, 2013). These firms will be able to maximize their profits and in turn, improve human abilities and their conditions service, accumulate assets, grow shareholders' wealth and attract other potential investors. Sufficiently capitalized, regulated and managed firms can improve their financial performance and risk assessment and management techniques. The governments of countries in emerging economies will be able to collect corporate taxes efficiently from well-performing firms and use them for financing their development processes. Furthermore, communities and societies of the region and other similar economies at large will benefit through social responsibility that is plowing-back activities by the firms from time to time.

1.8 Contribution of the Study

The application of existing credit risk models is suitable to circumstances in developed countries where transaction costs in financial markets are very minimal or negligible. The research proposes an extension of asset and CRMs to the inclusion of market friction under fuzzy conditions to make them more realistic and accurate in the estimation of firm values and risk metrics. It is hoped that statistical packages such as E-Views 8, logit, and logistic and vector auto-regression models that are relatively straightforward would be used to validate the proposed CRMs for banks in emerging markets. The need for the research to investigate asset valuation and CRMs extended to the case for market friction and fuzziness for banks is a step towards coming up with home-grown models. Such models are likely to be efficient and effective in addressing the unique impediments to the growth and development of banks which are common in the Southern African region and other nations of the world that are at the same level of development.

1.9 Research Methodology

1.9.1 Population, Sampling Techniques, and Data Sources

This section presents the research methodology used by the study which is divided into several dimensions that is the population of interest, sampling construction, research data variables, and proposed models.

Description of the population

The population of this research comprises all banks listed on all the 14 Southern African Stock Exchanges as of the base year, 1997. Listed banks are selected intuitively because data on financial statements about them are readily and publicly available on several databases, hence the ease of access.

Sample construction

A sample of 16 commercial banks was drawn at random from 6 Southern African countries conveniently selected based on their economic stability and growth potential for the validity and reliability of research findings to be enhanced. The six countries used are South Africa, Botswana, Zambia, Malawi, Tanzania, and Namibia. The banks used are assumed to provide sufficient financial data for the validation of the proposed research models. The research is premised on a large number of observations based on the period extended from 1997 to 2020. Based on this sample size a procedure of data cleansing was conducted as follows: Banks from countries such as Zimbabwe characterised by high inflation rates and weak currencies are dropped because their financial statements may be under or overcast leading to biased study findings. Foreign international banks from developed economies are also excluded as they may follow the credit risk patterns of their mother companies and their behaviours are assumed to be influenced by other unique factors compared to African listed banks, and outliers were discarded as well to get the final sample.

1.9.2 Research data for the proposed models

The study employs unbalanced panel data (after checking and screening for apparent coding errors and missing variables). Panel data have the advantage of reducing the co-linearity among explanatory variables, hence that improves the efficiency of econometric estimates. Data were obtained from the Bloomberg and World Development Index financial databases. Bloomberg data give us the leverage to convert financial statements in different currencies into the same currency and accounting standards. Where multiple regression techniques are used, Pearson's product-moment correlations and analysis of variance (ANOVA) analytical techniques are conducted using the STATA package.

1.9.3 Variables of the proposed models

The study proposes four related models namely equity, risk of default, expected loss, and bank financial performance valuation models, all extended for market friction and fuzziness. Assets, liabilities, equities, and credit exposures for the proposed models are maintained at their book values because of their being readily available and their ability to account for what has already taken place (Frank and Goyal, 2013). The main model proposed by the research is one on the impact of transaction costs on bank financial performance. The model specifically examines the impact of firm-specific and market-wide factors on bank performance as measured by return on assets (ROA), return on equity (ROE), and return on investment (ROI).

1.9.4 Other independent (explanatory variables)

Standard variables influence the risk of default, expected loss, and bank financial performance such as policy and non-policy explanations, bank supervision and regulation, market friction also called uncertainty facing financial investments, corporate governance, and ethics which may impact bank growth opportunities are included in this study in line with previous studies.

1.10 Organisation of the Research Study

The first chapter of the research presents the introduction to the study followed by chapters two and three on literature review and research methodology. Chapter four investigates the Merton AVM for bank equity valuation in financial markets characterised by friction and fuzziness. This is followed by chapter five which proposes and validates a fuzzy probability of default (PD) model for use in banking corporations in Southern Africa. The sixth chapter study proposes fuzzy PD, EAD, and LGD models for generating fuzzy input variables for the valuation of ELs of banks. These respective models are extended for market friction and uncertainty and validated using the financial data of banks in emerging financial markets. The research study's chapter seven examines the impact of market friction on bank financial performance in the presence of variables such as corporate governance and ethics. The chapter combines bank-specific and market-wide factors to assess bank performance as represented by key factors such as return on assets, equity, and investments (ROA, ROE, and ROI). The last chapter which is eight wraps up the project by presenting conclusions and recommendations of the study on the impact of transaction costs on credit risk modelling in banks situated in frictional and fuzzy financial environments.

1.11 Summary

The chapter has demonstrated that the research on credit risk modelling in banks in emerging economies is motivated by the need to fill critical gaps in structural and reduced-form models mainly premised on rigid and unrealistic assumptions such as frictionless markets and constant risk-free rates and asset standard deviations. In reality, most financial markets are frictional and fuzzy rendering current structural models for bank valuations and risk metrics not suitable for such markets. The chapter to come discusses the literature related to the study on the impact of transaction costs on credt risk modelling in banks in emerging economies to determine the gaps in knowledge to be filled.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

The first interest in credit risk modelling (CRM) originated from the need to measure the amount of economic capital necessary for supporting a bank's exposures. CRM is the best strategy for lenders to understand the likelihood of a particular loan getting paid. It is a tool for understanding the credit risk of an obligor because the credit risk profile keeps changing with time and circumstances. Risk metrics such as the probability of default (PD), loss-given default (LGD), and exposure at default (EAD) are used to estimate the credit risk capital of banks and similar financial institutions. Banks often estimate the EADs of different loans and then use these figures to calculate their overall default risk. These loans affect banks and non-financial firms in terms of poor performance of their investments due to low-interest rates, demand for loans, and purchasing activities in the economy.

This study anchors on the Merton (1974) model to determine the impact of transaction costs on the performance of banks in fuzzy financial markets such as those in Southern Africa. It is used to understand how capable a bank or company's finances and assets are at delivering its financial obligations including debts. The Merton model is flexible in that it can be adapted to include ordinary dividends in the calculation of ex-dividend values of reference or underlying stocks of companies. The KMV-Merton model specifically is the only CRM developed to provide a probability assessment of banks' or firms' likelihood of default. The following sections emphasise literature on the historical background of CRMs, CRM in general and in banks, CRM with transaction costs, and fuzzy markets before ending by identifying gaps in CRMs to be filled by the study.

2.2 Option Pricing Models (OPMs) and Valuation of Firms

In disciplines such as economics and finance, the asset valuation problem has preoccupied the minds of several researchers and practitioners for time immemorial. For a more comprehensive treatment of this subject of the valuation of assets of a firm, works by Black and Scholes (1973), Merton (1974), Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Wang

(2009), Jovan (2010) and Chen, Zhang, and Gupta (2014) and references therein are critical in motivating conceptualisation of the structural asset valuation models (AVMs). The underlying philosophy behind the study of modern finance is classical probability theory as propounded by Kolmogorov (1933). Kolmogorov's probability theory rests on the classical set theory, where a given object or member either belongs or does not to a crisply or precisely defined set.

The Black and Scholes (1973) popularly known as the Black-Scholes option pricing model (OPM). It was developed specifically for the valuation of European calls and put options under the following strict assumptions:

.Market movements cannot be predicted, random walk assumption,

.Underlying stocks pay no dividends

.Investors incur no transaction costs in buying options,

Returns of underlying stocks are normally distributed,

.Financial markets are frictionless,

. The risk-free rate of return and asset volatility is known and constant (are precise numbers),

European options can only be exercised at expiry that is at T=1 year.

The Black-Scholes (1973) model further assumes that stock prices follow a lognormal distribution based on the principle that asset prices cannot take negative values that is they are bounded by 0 as the lower value. It is based on the Black-Scholes (1973) model that the Merton (1974) OPM was developed under the same major and restrictive predecessor model assumptions. Merton's model is robust in that it includes OPMs in the estimation of probabilities of default (PDs) of banks and companies by providing a framework for extraction of the necessary information about the bankruptcy of market prices. The Merton (1974) model explicitly defines a default event as a firm's inability to honour its debt obligations by modelling its equity value, E = A-D, as a call option.

The transition from the Black-Scholes (1973) to Merton (1974) has become to be known as the Black-Scholes-Merton differential equation widely used to price options contracts. The model requires 5 inputs to be applied which are option strike price, the underlying stock value today, time to expiration, risk-free rate of return, and asset volatility. This is a model for the dynamics of a financial market containing derivative investment securities. It gives a theoretical framework for the estimation of the price of European options and demonstrates that options have unique or precise prices given their risks and expected returns. Investors in options can hedge their exposures by buying and selling the reference or underlying assets uniquely to eliminate the risks (continuously reused delta hedging) (Hull, 2008). This framework is the basis for more complicated hedging strategies such as those employed by investment banks and hedge funds worldwide today.

The Black-Scholes option pricing model (OPM) is used to determine the fair price or theoretical value for a call or a put option. The model is based on six variables which are asset volatility, type of option, underlying stock price, time, strike price, and the risk-free rate of return. The variables used in the model are assumed to be constant but in practice, they change over time (Black-Scholes, 1973, Durbin, 2018). The concept of speculation is more prevalent in the case of stock market derivatives, and hence the use of proper pricing of options to eliminate the opportunity for any arbitrage. There are two essential models for option pricing which are the Binomial Model and Black-Scholes Model. These models are used to determine the price of a European call option. This means that the European option can only be exercised on the expiration date of the contract. The Black-Scholes option pricing model is mainly used by options traders who buy options that are priced under the formula calculated values, and sell options that are priced higher than the Black-Scholes calculated values. The general form of the Black-Scholes model for the pricing of options is given by:

$$V_c = SN(d_1) - Ke^{-rT-}N(d_2) \text{ for all call options} \tag{2.1}$$

$$\text{and } V_p = Ke^{-rT-}N(-d_2) - SN(-d_1) \text{ for put options} \tag{2.2}$$

where;

$$d_1 = \frac{\left[Ln\left(\frac{S}{K}\right) + \left(r + \frac{{\sigma_v}^2}{2}\right)T\right]}{\sigma_v\sqrt{T}} \tag{2.3}$$

and

$$d_2 = d_1 - \sigma_v \sqrt{T} \quad (2.4)$$

where T is the expiry date of the contract, S is the share price of the asset, K is the strike price of the underlying stock, σv is the stock volatility, and r is the interest rate (Hull, 2006). Ferreira and Koekemoer (2020) came up with a structural equation model for South African banks but it was not extended to include market friction and fuzziness. Hence this research proposes a credit risk model for Southern African countries which includes both market friction and fuzziness to improve its estimation ability and contribution to the growth and development of banks.

2.3 Credit Risk Modelling (CRM) in General

CRM is a technique used by lending institutions to determine the level of credit risk associated with extending credit to a borrower, also called an obligor. There are three common types of credit risk firms face in practice namely credit spread, default, and downturn risks. Credit spread risk is the risk caused by the variance between interest and risk-free rates of return (Wang, 2009). Default risk on the other hand is the failure of a borrower to meet loan contractual obligations as and when they fall due. Downturn risk on the other hand refers to risk emanating from the downgrading of issues of credit in an economy mainly due to market factors such as recessions. Credit risk is the risk that a borrower cannot repay the loan, credit card, or any other type of obligation Sometimes customers pay some installments but not full amounts comprising principal borrowed and interest, and hence the part not paid becomes a loss to the bank (Damodaran, 2017).

This risk can be extended to include small, medium, and large corporate loans that lead to non-performing assets or loans (NPAs/NPLs) that borrowers have not been able to pay when due. Therefore banks must have sufficient capital to protect depositors from these credit risks. Poor credit raters of borrowers in banks are responsible for default risk on loans issued.

Hence to compensate for the risks above banks usually charge huge interest rates compared to normal standard rates prevailing in the markets. Banks can sell such risks to investors in the secondary markets, a process called collateralized debt obligations (Casarin, 2005). CRM is mainly attributed to traditional models called structural and reduced-form models. Structural models for PD estimation in banks are based on their asset values and liabilities Banks face credit transfer risk in their credit exposures which is the risk of financial loss and negative business performance related to loans issued to customers, caused by inadequate policies regulating loan disbursements, weak follow-ups, and recovery strategies. Wang (2009) states that CRM approaches fall into two main classes which are structural and reduced–form models. Structural models provide explicit relationships between default risk and the capital structure of a firm, while reduced-form approaches are used in modeling credit defaults as exogenous events driven by stochastic processes such as Poisson diffusion jumps.

Option pricing in economics and finance represents a jump-diffusion model as a form of a mixture model that mixes jump and diffusion processes. Jump-diffusion models are introduced by Merton (1974) as an extension of jump models. Due to their computational tractability, the special case of a basic jump-diffusion process is popular for some credit risk and short-rate models. In other words, a jump-diffusion is a mathematical tool for modelling fat-tail risk. Robert C. Merton is the first to explore this concept in the paper "Option pricing when underlying stock prices are discontinuous", and called it to jump diffusion. Casarin (2005) argues that jump processes are widely used in credit risk modelling to describe both default and rating migration events. Stochastic calculus refers to work to do with a review of some basic definitions and properties of the jump processes intended for a preliminary step before more advanced issues on credit risk modelling are addressed. This type of calculus focuses on the Poisson process and some generalisations, such as the compounded and the double stochastic Poisson processes.

Poisson processes are widely used for describing the time-in homogeneous dynamics either of the default processes or of the credit rating transitions (Bielecki and Rutkowski, 2003). Wang (2009) argues that Merton's (1973) model is beautiful in that it was the first structural approach that treated a firm's equity as a call option on its assets, and allowed the application of the Black-Sholes (1973) option pricing model to be used in the valuation of the market value of equity. Reduced-form models go further to specify recovery rates (RRs) after credit events have happened in banking corporations (Wang, 2009). However, structural models on the other hand do not determine the time to default using the value of the firm but take it to be an exogenous jump process parameter governing the default hazard rate inferred from observable market data. Wang (2009) further argues that structural models provide linkages between the credit quality of a firm and the economic and financial conditions that it faces.

Structural and reduced-form models are criticised for not specifying recovery rates (RRs). However, the models provide values of assets and liabilities of banks and similar firms that are at default to be used in the estimation of the recovery rates. This flexibility in the models will suit very well the varied circumstances facing banks in emerging markets (Damodaran, 2017). The core of the structural models of default events in credit exposures is concerned with firm-specific variables that govern the decision to default by the obligor. The evolution of structural credit-risk models is motivated by firm-specific variables such as transaction costs, tax regimes, and overheads together with several stylized facts on credit spreads, renegotiating debt obligations, time-series, and cross-sectional variations in banks' capital structures. The structural models described above imply that as the time to debt maturity approaches zero, the credit spreads will approach zero as well (Kubo and Sakai, 2011). This is based on the assumption that the option value of default approaches zero as the time to maturity approaches zero. However, the asset values will evolve according to a Geometric Brownian model (GBM).

Merton (1974) derives a simple model of option pricing when the underlying stock prices are governed by a mixed jump-diffusion process. It is simple therefore to combine the above insight with Merton (1974) to develop a structural model of default in which short-term credit spreads will be non-trivial. Rogers (2000) extends the Merton structural model with endogenous default to Levy processes with one-sided downward jumps characterized by both the credit spreads and optimal capital structures of firms. Another approach used in reconciling significant short-term spreads relies on incomplete accounting information as done by Duffie and Singleton (2003). In their approach, the underlying value process is not observable, and the investor must rely on imperfect accounting information to make decisions. However, the research by Duffie and Singleton assumes that the equity of a firm is not traded and thus may be more relevant for private firms.

According to Elizalde (2005), countries of the world use structural and reduced-form models in the estimation of credit risk in banking institutions but notes that reduced-form models do not consider the link between default and firm value in an explicit manner in the modelling of credit risk. Reduced-form models go further to specify recovery rates (RRs) after credit events have happened in banking corporations. However structural models on the other hand do not determine the time to default using the value of the firm but take it to be an exogenous jump process parameter governing the default hazard rate inferred from observable market data. Structural models provide linkages between the credit quality of a firm and the economic and financial conditions it faces. According to Crouhy, Galai, and Mark (2009), structural models do not specify RRs but provide values of assets and liabilities that are at default for use in the estimation of the recovery rates. This flexibility will suit very well the varied circumstances of banks in Southern Africa, to be drawn for investigation by the study. The current credit risk models used by international banks are based on the stipulations of the Basel Committee's Banking Supervision (BCBS, 2009)'s Basel I, II, and III Capital Accords. The internal-based rating (IRB) models postulate that credit risk modelling may indeed result in better risk management systems. The IRB models can also be used in the supervisory oversight of banking corporations including those in developing countries such as those in Southern Africa provided they are adjusted for market friction.

2.4 Credit Risk Modelling in Banks

Banks have been focusing on credit risk for many tears, that is banking is a successful, centuries industry, with an equally long and old history, of adapting to new challenges, innovations, and technologies. However, of late the banking industry is facing a myriad of existential threats from digitalisation and technological advancement. Credit risk is a serious threat to bank valuations, profitability, and credit exposures concerning interest and growth rates. A Committee was established in 1974 by the Central Banks of G10 countries in Geneva Switzerland to come up with Basel Regulations for countering the vulnerability of banks to credit risk. The Basel Regulations were meant to ensure that banks have a minimum but sufficient capital stocks to pay back depositors' funds (BCBS, 1988).

The G10 members meet regularly to review or discuss banking regulatory and supervisory matters at the Bank for International Settlement (BIS) in Basel, Switzerland. The committee was expanded in 2009 to have 27 jurisdictions including South Africa to represent the African continent. The Basel I Accord was the first pact introduced by the G10 in 1988 for bank CRM and capital adequacy ratio (KAR) calculation. This ratio measures the relationship between a bank's capital and its risk-weighted assets (RWA) which is pegged at 8%. A bank's capital (aggregate tiers 1 and 2 capital) must be more than 8% of its RWA. Under Basel I Capital Accord, fixed risk weights are set based on the level of exposure of a bank for instance the weights are 50% for mortgages and

100% for non-mortgage exposures such as credit cards, bank overdrafts, automatic loans, and personal finance.

Basel II Capital was introduced in June 2004 to eliminate the shortcomings of the Basel I Accord. Basel I Accord focused mainly on credit risk whereas the second Accord focused not only on credit risk but also on operational and market risks. Basel II Accord stipulates three ways to the estimation of bank credit risk which are the standard, foundation and advanced internal rating-based (IRB) approaches. The standard approach for companies allows banks to rely on ratings from certified credit rating agencies (CRAs) like Standard and Poor (S and P) and Moody's to quantify regulatory capital for credit risk. Risk weights are 20% for high-rated exposures and the ratio rises to 150% for low-rated exposures.

The rates are also pegged at 35% for mortgages and 75% for non-mortgage exposures ad no ratings by CRAs are needed for retail exposures (Zimmermann, 1980). IRB approaches are based on four credit risk components which are PD, exposure at default (EAD), loss given default (LGD), and effective maturity (M), which is the duration that reflects standard practice used by a bank. Foundation and advanced IRB are an improvement to the standard approach to credit risk modelling. The Foundation IRB approach estimates the PD of a bank internally while LGD and EAD are prescribed by the regulator. Under the Advanced IRB, PD. LGD and EAD can all be estimated internally by the bank. For Foundation and Advanced IRB approaches effective maturities are 2.5 years and more than a year respectively.

Basel III Accord was established in June 2010 and scheduled for implementation in March 2019 but has since been postponed to 1 January 2023 due to the COVID-19 pandemic. The Accord incorporates several risk measures in CRM to counter various issues identified in the 2008 global financial crisis. Basel III Accord emphasises revised capital standards (such as leverage ratios), stress testing, and tangible equity capital, with the greatest loss absorption capacity. The concept that banks must build internal models and external ratings for PD, EAD, and LGD remains the same under Basel II and III Accords. However, some changes have been introduced in the Basel III Accord as tabulated below.

Table 2.1 Showing Comparison Between Basel II and III Capital Accords

Type of Capital	Basel II Accord	Basel III Accord
Tier I	2% of RWA	4.5% of RWA
Tier I Capital Ratio	4% of RWA	6% of RWA
Tier II Capital Ratio	4% of RWA	2% of RWA
Common Equity	---------------------	2.5% of RWA

Source: Authors'

The Basel Committee has also gone on to replace International Accounting Standard (IAS) 39 with the International Financial Reporting Standard (IFRS) 9 which deals with accounting for financial instruments owned by a bank. The IFRS 9 takes over from IAS 39 based on banks' incurred loss model and focuses on the expected loss model, which also covers future bank losses. It identifies three stages of credit risk in banks namely:

Stage I: Credit risk has not increased significantly since initially recognised, thus indicating low credit risk at the reporting date.

Stage II: Credit risk has increased significantly since the initial recognition stage.

Stage III: Permanent decrease in the value of a bank's financial assets at the reporting date.

IFRS 9 differs significantly from Basel III Accord though both call for building PD, LGD, and EAD models. The table below summarises the major key features that separate Basel III Accord from the IFRS 9:

Table 2.2 Showing Basel III Accord and IFRS 9

Parameter	Basel III Accord	IFRS 9
Goal/Objective	Expected and unexpected loss	Expected loss only
Probability of Default, PD	One year PDs	.1 year PD for stage I assets .Life time PDs for stages 2 and 3 assets
Rating Philosophy	Through the Cycle (TTC) Rating	Point in Time (PIT) Rating
Loss given default, LGD	Downturn LGD under both direct and indirect costs	Best LGD estimation based on direct costs only
Exposure at default, EAD	Downturn EAD	Statistical estimation of EAD
Expected credit loss, EL	EL=PD$\times LGD \times EAD$	EL=PD$\times$ $\text{Present Value (PV)of}$ Bank Cash Shortfalls

Source: Authors:

2.5 CRMs Under Conditions of Transaction Costs and Fuzziness

Structural models of credit risk modelling are derived from theory and often include unobservable parameters or variables that help describe behaviour at a deep level. Reduced-form models on the other hand evaluate endogenous variables in terms of observable exogenous variables and serve to identify relationships between or among the variables (Kubo and Sakai, 2011). Scholars have challenged the thinking behind traditional CRM namely the assumptions that markets are frictionless and randomness is the form of uncertainty that drives risk in financial markets. Zamore et al (2018) provide a comprehensive review of research on bank credit risk modelling and measurement. The research findings suggest that credit risk research is multifaceted because it can be classified into six main dimensions which are defaultable security pricing, default intensity modelling, comparative analysis of credit models, comparative analysis of credit markets, credit default swap pricing, and loan loss provisions. It is based on findings by Zamore et al (2018) that structural CRMs need to be extended for market friction and uncertain variables such as vagueness and fuzziness to make them more relevant, rigorous, and realistic.

Structural approaches, led by Merton's (1974) model, have the highly appealing feature of connecting credit risk to the underlying structural variables. Merton's model for instance provides both an intuitive economic interpretation and endogenous explanation of credit defaults and allows for applications of option pricing methods to the valuation of firms. As a result, structural models not only facilitate asset valuation but also address the choice of financial structure for the firm. However, Wang (2009) points out that the main disadvantage of structural models lies in the difficulty of implementation. For instance, the continuous traceability assumption for corporate assets in structural models is unrealistic, and calibrating stochastic asset processes using publicly available information may be sometimes more difficult than anticipated. Furthermore, while improved structural models have addressed several shortcomings of earlier models, they tend to be analytically complex and computationally intensive.

Most of the contemporary structural models represent important improvements to Merton's original framework. New-look structural models are more realistic and able to better align with market data for example credit default swap (CDS) spreads for the betterment of the application of the model. In Merton's framework, a company could only default at its debt maturity date hence the model can be modified to allow for early defaults by specifying a threshold level such that a default event occurs when the asset value, falls below such a critical level. However, it has been observed that investors normally encounter two complexities when determining the optimal value of a firm using the structural asset valuation model (AVM). Investors normally depend on experts' judgments to determine the probability distributions of primary variables in an AVM. In practice, investors often subjectively describe the uncertainty they face in financial markets with implicit fuzziness also known as impreciseness. Implicit fuzziness can be expressed as, for instance, 'there is a good chance for a riskless interest rate of 10% on an investment in the following year, the riskless interest rate is very unlikely to go below 5% or it is most likely that it will be in the range of 2.5% to 7.5% (Zimmermann, 1980, Zadeh, 1965).

According to Zimmermann (1980), another example where market participants will describe market events using imprecise linguistic variables can be situations such as '--in a booming economy, there is about 80% probability that the riskless interest rate will grow by 10% in the following year'. The phrases 'booming economy' and 'about 80%' are implicitly taken to mean that the probability for the event of a '10% riskless interest rate' could vary for instance from 75% to 85%. An attempt to improve on shortcomings of structural and reduced form OPMs and AVMs has been made over the years to incorporate variables such as transaction costs in the valuation of firms. Merton's (1974) model has been extended for proportional transaction costs for pricing contingent claims of firms. The model includes commissions and spreads in the valuation of firm contracts such as options. Merton's model's continuous time theory is readily overcome by explicitly introducing transaction costs into the OPM but it is not extended to the estimation of risk metrics in banks. The traditional Merton PD model is restricted to the no transaction cost case as is the case in structural models making it unsuitable for use in markets characterised by friction and fuzziness.

The fuzzy theory introduced by Zadeh (1965) is a framework for modelling linguistic variables in mathematical, banking, and financial domains. Moller et al (2002) researched on modelling of fuzzy randomness and uncertainty. Therefore the concepts of vagueness and uncertainty are used interchangeably in mathematical modelling to represent *stochastic uncertainty. This type of uncertainty* is different from the other types of vagueness concerning the description of language variables such as meanings of events, phenomena, or statements themselves, which we shall *call fuzziness.* Fuzziness is found in many areas of daily life, such as meteorology (Cao and Chen, 1983), medicine (Vila and Delgado, 1983), manufacturing (Mamdani, 1981), and engineering (Brockley, 1980). Fuzziness is more prevalent in all areas in which human evaluation, judgment, and decisions are central for instance in all areas of decision-making, reasoning, and learning in real life. Zimmermann (1980) supports Zadeh (1965) by arguing that fuzzy set theory can be applied to solving real-life economic and financial problems.

Therefore fuzzy sets are designed for handling a particular kind of uncertainty, which is the degree of vagueness for a property that can be possessed by objects to varying proportions, for example, the volatility of stock returns in financial markets. Fuzzy models are efficient in the determination of approximate solutions to financial problems compared to systems of structural differential equations (SDE) (Bardossy, 1996). The solutions to financial problems by the Bardossy model were found to be almost the same for all practical purposes, given the inaccuracies and uncertainties in the input data drawn into the model used. Therefore a classical (crisp) set is a fuzzy set that restricts membership values of a set to [0;1] as endpoints of the unit interval as is the case in classical probability theory. Fuzzy set theory is different from classical probability theory in that it can model vague phenomena arising from human behaviours by assigning weights to any object based on the value of the membership function. The theory goes on to evaluate the extent to which the rule or object in a given set is judged to be true or false. However, this study proposes to extend the framework of CRM to cover financial domains that are characterised by linguistic variables and market friction which are realistic conditions in emerging financial markets, particularly those in Southern Africa.

The real beauty of Merton's (1974) model lies in its capacity to treat a company's equity as a call option on its assets, paving way for applications of Black-Scholes (1973) option pricing methods if corresponding modeling assumptions are made. As credit risk has become an increasing concern in recent years in economics and finance, various advanced valuation methods have been employed widely to measure credit risk exposures. Two sets of credit risk models have emerged namely structural and reduced-form models as primary classes of credit risk modelling approaches. Wang (2009) stresses that structural approaches aim to provide an explicit relationship between default credit risk and capital structure, while reduced-form approach models on the other hand treat credit defaults as exogenous events driven by a stochastic or random process (such as a Poisson jump process). Structural models, pioneered by Black, Scholes (1973) and Merton (1974), employ modern option pricing theory in the valuation of corporate debt. However, the Merton model was the first structural model for the valuation of debt and has served as the cornerstone for all other structural valuation models. However, both Black-Scholes and Merton models are criticised for being founded on unrealistic assumptions, for instance, frictionless markets yet in reality financial conditions in most economies such as those in Africa are seriously characterised by frictional markets as outlined above.

Because of the unrealistic nature of the theoretical assumptions on which structural models are based, attempts have been made to extend the Merton model along this direction pioneered by Black and Cox (2000). This group of structured models is often referred to as the First Passage

Time model. These models acknowledge that the constant interest rate assumption is not reliable, and hence a stochastic interest rate model can be incorporated into the Merton model or its extended versions. In this case, correlations between asset and interest rate processes can also be introduced as and when the need arises. It is further argued by Black and Cox (2000) that mapping all debts of a firm into a single zero-coupon bond is not always feasible. Research has shown that multiple debts with different characteristics can also be modelled using a structural approach. The Geske Compound Option model developed by Robert Geske is the first structural model of this nature.

Several more sophisticated structural models involving stochastic interest rates, volatility, jump-diffusion, and even regime-switching methods have also been proposed in the desire to move away from models based on frictionless to frictional market models. These new model applications can help explain market observations with a higher degree of accuracy, but they often involve a high level of analytical complexities.

2.6 CRMs and the Economic Values of Default (EVAs)

The economic value of default is presented in this approach as a put option written on the value of a firm's assets. The Merton (1974) model lays the foundation for credit risk modelling and assumes that financial markets are frictionless. Jubouri (2018) identifies a family of credit risk management indicators and their impact on stock market indices. The study postulated that factors such as capital adequacy ratios (KARs), non-performing loan ratios, loan-to-deposit ratios, total debt ratios for banks, return on risk-adjusted capital had significant effects on earnings per share (EPS), price-to-profit ratios, share turnovers and market values of private or international banks in Iraq. The systematic risk of China's stock markets was studied by Dai and Li (2019) under risk-neutral economic conditions and the study found that economy–wide factors measured market system risks more accurately than the traditional system for instance the value-at-risk (VaR) and expected shortfalls (ES) or ELs. The above findings reflect very well on the tabulated challenges above which were said to be responsible for the poor performance of banks in emerging economies such as Sub-Saharan Africa. Hence overheads, connections, poor corporate governance, and economic rents are three critical sources of market friction in banks in developing countries that must be incorporated into credit risk models in emerging countries before they are adopted and implemented.

Based on the magnitude of market friction facing banks in Southern Africa, policymakers face multiple challenges such as low incomes, financial unsoundness, demographic, and location of obligors that impact firm performance that is not included in current credit risk models (Vasanthi and Raja, 2011). These scholars identify the above gap, developed an Australian risk management model and tested its validity to serve as a tool for policymakers, government, and private lenders to assess the risk of default. They went further to develop appropriate financial management strategies for Australian banks for use in minimising credit risk. Their study found that the income, liquidity, and wealth constraints of borrowers should be factored into models to reduce the magnitude of credit risk. A logit model was developed based on adjustments for the above fundamentals for credit risk modelling by providers of credit in Australia. A default event occurs when the asset value of a firm falls below a certain threshold and the firm should be liquidated according to the first passage models (Duffie and Singleton, 2003). However new models for credit risk modelling postulate that default does not cause liquidation immediately but may represent the commencement of such a process, and may not translate into liquidation after it is completed.

The financial challenges chronicled by Fuchs and Uy (2010) above motivate this study to propose and investigate a CRM that banking corporations in Southern Africa can apply to turn their financial fortunes towards sustainable development in the 21st century. These challenges are a major impediment to the growth and development of banks but are not included in contemporary CRMs. Therefore the study proposes a CRM extended for market friction in the desire to accurately measure PDs, EADs, LGDs, and total ELs of banks in fuzzy emerging markets for the period 1997-2020. These risk metrics are estimated in the desire to improve the capitalisation of banks, broaden and deepen their products and services, accumulate assets, and grow shareholders' wealth. The application of the AVM in finance has always been considered the bedrock of contemporary structural and reduced-form models for the valuation of firms. However, the generic application of the models such as Merton AVM has always been constrained by its nature of not being suitable for fuzzy financial environments. Hence the need for reviewing the planning and decision-making processes of investors in financial markets to incorporate a feature of uncertainty that always affects their performance (Asset and Kazakhstan, 2015).

2.7 Empirical Evidence on CRM

A study by Lohmann and Ohlinger (2018) examines the non-linear relationships in a logistic model of default for a high-default installment portfolio. The model used was validated using data on consumer credit provided by a German retail and trading company. The Lohmann and Ohlinger (2018) model incorporated generalized additive models to analyze non-linear relationships and their effects on predicting the PD of a bank. The major findings of the study were that certain contracts and debtor characteristics had non-linear and non-monotonic effects on the challenges that led to a borrower's default on their consumer credit. The study recommends that financial analysts should include non-linear relationships in models they propose for predicting consumer default. However, the study ended by observing that the above consideration was likely to increase the complexity of models applied for the same purpose.

Jiri (2018) argues that the Basel II regulatory capital called on commercial banks to use an advanced internal-rating-based approach (IRBA) to estimate three key credit risk metrics, namely PD, LGD, and EAD for each credit exposure. The regulatory capital formula for retail products is given by C= [UDR (PD)-PD] LGD. EAD where C= regulatory capital. Bank capital is said to be sensitive to LGD and EAD such that a 10% error in LGD or EAD will lead to a 10% error in the final regulatory capital. However, the above works do not address the market friction issue in CRM, particularly in developing economies. Hence the need for this study to factor market friction and human psychology into existing credit risk models. These variables are incorporated in the proposed models in the quest to accurately measure the performance of firms in developing countries.

Virginia (1988) developed a transaction cost model and realised that transaction costs are mainly influenced by the amount of loan applied for, real interest rates, and land owned by the borrower. He further argues that dummy variables such as collateral security, delinquency of the loan, Central Bank policies, the borrower's distance from the bank, and the year in which the loan is borrowed are also part of transaction costs. According to Aymanns et al (2016), banks need a good understanding of the link between solvency and funding risks to be able to assess their fragility efficiently and effectively. According to Altman and Kuehne (2014), credit bubbles are becoming more common for several credit asset classes to which banks are exposed. They conclude that credit bubbles increase sharply with increases in corporate bonds and default on loans. It has also been observed that crises in credit and equity markets contribute to periods of unfavourable price movements and increases in volatility in asset classes (before the bursting of bubbles). Hence bank managers and boards of directors (BODs) need to manage the risks for the growth and development of banking corporations. Most banks in Southern Africa have gone through a lot of changes and challenges in the 21st century whose impact on the financial sector cannot be quantified and compared with other emerging economies (Zhang, et al, 2014).

Most contemporary CRMs have a lot of bias toward fuzzy estimation of option prices, their major findings, and shortcomings. Yu, Sun, and Chen (2018) for example, examine the application of the fuzzy estimates method to the pricing of European call currency options. The study uses fuzzy estimators for the standard deviation of exchange rates based on statistical data to obtain a fuzzy pattern of the G-K (German and Kohlhagen) model. A mathematical model is presented to obtain alpha-level intervals of the European call currency fuzzy price. The study concludes that financial markets have significant fluctuations, implying that there are frequent elements of vagueness and uncertainty in such markets. In foreign currency markets spot exchange rates fluctuated from time to time based on effects of markets that occurred imprecisely, contrary to the requirements of the assumptions of structural models.

Therefore it is natural to consider the existence of fuzzy foreign and domestic interest rates, exchange rates, and standard deviations in the valuation of fuzzy call currency options. The research is however limited in scope as it considers the valuation of currencies based on the European call option pricing model by Black and Scholes (1973), which are applicable under classical probability conditions. The Yun, Sun, and Chen (2018) model is restricted to the valuation of call currency options yet conditions in fuzzy markets called for adjustment for market friction to improve the reliability and validity of the models. Furthermore, new CRMs should have the flexibility and rigour to be applied to the valuation of firms and their risk metrics in various market structures and economies.

On the other hand, Ozari and Ulusoy (2017) use fuzzy logic and Merton's (1974) model to estimate bankruptcy probability using data drawn from USA firms. The Merton (1974) model is the first one that shows that the default choice of a company could be modelled by assumptions of the call option model by Black and Scholes (1973). The model is based on a new application of the traditional Merton model to the valuation of a company's bankruptcy probability independently from its sectors. The Merton model's underlying assumptions are based on the financial structure of a company. The level of default of a company is determined by the firm's market value of assets in conjunction with the structure of its liabilities. The study links the bankruptcy of a firm to variability in the value and views of its assets as a standard call option (Hull, 2010). The study is influenced by the views of Wang et al (2009) that the market value of a firm's assets is equal to the promised payment of its corporate debts. The study concluded that financial ratios were significant in understanding the bankruptcy situations of a firm. These ratios should be employed in the creation of new market indices. However, since financial ratios are many, firms can use factor analysis to eliminate some of the ratios from the proposed risk models.

Factor analysis is a branch of statistical sciences but its extensions in psychology make it mistakenly regarded as a psychological theory (Wang et al, 2009). The study used sectoral correlation analysis to examine the relationship between cluster variables because the results did not provide enough information to reduce the number of financial ratios. Factor and cluster analyses were used once and twice respectively to reduce the number of financial ratios drawn into the proposed model. The study uses the structure of the sample and constructed brand-new fuzzy bankruptcy indices using financial ratios. Factors that showed positive correlation were discarded because they explained information and results. Wang et al (2009) conclude that financial ratios signalled different meanings for different sectors due to sectoral level differences or reasons. A recent study on credit risk by Chen (2018) proposes a new loss-given default (LGD) model to address the missing and sample selectivity biases found in real-life experiences. Chen (2018) proposes a time-to-recovery survival model for the estimation of the LGD model with varying performance windows. Using an existing LGD data set, Chen (2018) performs five specification tests to evaluate the new approach to LGD modelling.

The study by Peng (2018) argues that a trade LGD model omits time to recovery and ignores censoring) was biased when applied to non-defaulted performing loans in which the time to recovery was unknown. This problem is addressed by proposing yet another new modelling approach that entails predicting both existing workout LGD data set comprising both censored and uncensored recoveries (Chen, 2018). Chen's (2018) model ensures that the new approximation model fits the given data well resulting in a higher LGD prediction and marginal sensitivity to triangles. It is important to note that several contemporary credit risk models such as Zhang, Lu, and Sang (2014) and Tang and Fang (2011) and references therein, compete to explain the factors that impact bank credit risk. The concept of bank credit losses is mainly influenced by three main traditional factors, namely PD, EAD, and LGD.

Although most banks of the world have implemented the Basel II Capital framework, more work needs to be done to improve credit risk management by building up rating systems and procedures for estimating loan loss parameters. This is because PD, LGD, and EAD systems are insufficient for preventing the financial system of a country from further crises such as liquidity and volatilities in loan exposures, interest, and exchange rates (Engelman and Raihmeier, 2011). Therefore, improvements are needed in regulatory frameworks and internal risk management (IRM) of all banking corporations of the world and in particular those in emerging countries such as those in Southern Africa characterised by high costs of transacting business and uncertainty. The Basel II Capital Accord framework broadens the capital bases of banks to achieve stability by creating capital buffers. The study by Engelman and Raihmeier (2011) discovers that the loan's collateral security is sufficient in the event of default to ensure that no losses are incurred by the lending institutions.

Moody's credit risk analytical model (CRAM) postulates the need for assessment and management of current and future credit risk exposures of firms across all asset classes. The model is built using a wide range of applications that include loan origination, risk ratings, credit loss reserving, stress testing, risk-based pricing, portfolio monitoring, and early risk warnings. The study concluded that Moody's model can be applied to the modelling of PD, LGD, expected default frequency (EDF), and EL at wholesale loan portfolio levels. It was also concluded that Moody's model is important in modelling regulatory compliance programmes and leveraging CRM of firm values and risk metrics (Iazzolino and Fortino, 2012). However, in a 2007-08 credit risk study, Moody's analytical model was found to be very weak in the area of corporate governance. Several firms applied Moody's model in trading derivative securities and failed to meet their set financial performance goals and objectives. Despite the model ending by recommending the use of sound corporate governance frameworks, it was not adjusted for variables such as corporate governance and uncertainty, which is an issue this research includes in the proposed models.

Huang and Huang (2002), argue that incomplete accounting information, managerial discretion, and jumps respectively must be incorporated into structural credit risk models to improve their estimation ability. Furthermore reduced and structural form models should be risk-adjusted for return on capital for purposes of distributing risk costs down to businesses themselves, products, customers, and individual transactions (Duffie and Lando, 2001). Furthermore, detailed evaluation of credit exposures allows lenders to accurately undertake marking-to-market of their investment portfolios. Corporate bonds and fixed-rate loans require models that measure both credit and interest rate risks accurately for efficiency and effectiveness in the financial performance of banks. Most credit risk models are based on risk management methods and systems excluding ratings of obligors, sectors of operation, terms and conditions, interest rates, and spreads, which were indispensable in accurately assessing and measuring banks' performance.

Virginia et al (1988) developed a transaction cost model and realised that transaction costs are mainly influenced by the amount of loan applied for, real interest rates, and land owned by the borrower. He further argues that dummy variables such as collateral security, delinquency of the loan, Central Bank policies, the borrower's distance from the bank, and the year in which the loan was borrowed are also part of the transaction costs. According to Aymanns et al (2016), banks need a good understanding of the link between solvency and funding risks to be able to assess their fragility efficiently and effectively. According to Altman and Kuehne (2014), credit bubbles are becoming more common for several credit asset classes to which banks are exposed. The two proceed to argue that credit bubbles increase sharply with increases in corporate bonds and default on loans. The study concludes that crises in credit and equity markets contributed to periods of unfavourable price movements and increases in volatility in the asset classes (before the bursting of bubbles). Altman and Kuehne (2014) recommend the need for firms to manage their credit risks for their growth and development.

Kurtz (2018) proposes a new model for the valuation of transaction costs such as capital charges for credit risk concentrations in banks. This model holds when economic capital measurements are conducted within a multifactor Merton (1974) asset valuation framework. According to Kurtz (2018), concentration charge is the impact of a particular sector on a bank investment portfolio's credit loss curve or profile. One of the study's main propositions was that the Monte Carlo simulation should be used in CRM in banks. This is because Monte Carlo simulation does not require the calibration of additional parameters and hence is easily applicable to banks that perform simulations. Secondly, the simulation method has a tractable analytical formulation that provides an efficient approximation because it is a simple and intuitive location of the resultant capital charges. Kurtz (2018) concludes that the simulation model is suitable for use in modelling capital charges for sector concentration risks, particularly under pillar II of the Basel II Bank Capital Accord.

In reality, financial investors are concerned with currency options price ranges rather than option pricing models. Malyaretz, Dorokhov, and Dorokhova (2018) concluded that economic efficiency indicators of bank activities should be stated as fuzzy quantities because they were not independent of human behaviours. They collected and analysed data from Ukrainian banks and their findings were tested for reliability, accuracy, and comparability with structural or regression analysis models. The application of fuzzy regression models was based on three proposed fuzzy models. The paper was first modelled based on the criterion of minimising vagueness also called the linear programming method (Tanaka, 1982). The paper then went on to apply fuzzy least squares (approximation by distance interval) and multiple-criterial methods. The study discovered that fuzzy least squares methods were the best because they were very flexible in solving bank financial problems using Search or Add-ons in micro-soft office models. Fuzzy least squares methods on the other made dual problems in banking corporations easier to solve than direct financial problems because they were able to reduce the number of calculations involved.

Research by Palma and Ochoa (2013) reveals that since the introduction of uncertainty theory in credit risk models, a new paradigm shift in economics and finance has been formed with specific reference to option pricing. This is based on the incorporation of new credit risk models that allow a greater degree of accuracy to the reality of the real environment facing firms based on fuzzy logic and theory. The fuzzy theory emphasizes the importance of uncertainty in financial markets provoking an increased need for the establishment of models to specifically determine the prices of Exchange Options through the use of triangular fuzzy numbers to exchange rate variables. This is intended to improve the determination of both domestic and foreign interest rates based on the classical Black-Scholes (1973) model.

A study by Abder-Kader and Dugdale (2001) presents the application of fuzzy set theory to the derivation of financial models. The paper proposed and implemented an AVM that was adjusted for uncertainty and human psychology as critical components that influence the estimation of the value of a firm. The findings of the study reveal that the model adjusted for uncertainty and human behavior was more realistic and practical in the valuation of firms in economics and finance. Therefore this research intends to come up with CRMs generated from Merton (1974)'s asset valuation model (AVM) that firms in Southern Africa can employ in their desire to grow towards self-reliance and sustainable development.

2.8 Conceptual Frameworks

The research at hand proposes CRMs extended for transaction costs for the valuation of banks and their risk metrics in fuzzy financial markets. Therefore there are two fundamental concepts on which the study is premised that is uncertainty with specific reference to fuzziness and market friction in the form of transaction costs (Dai and Li, 2019). The concept of uncertainty with specific reference to fuzziness or vagueness is used in mathematical, economic, and financial modelling to represent stochastic uncertainty. This type of uncertainty is different from the other type of vagueness concerning the description of language variables such as meanings of events, phenomena, or statements themselves, which we shall call fuzziness.

Financial market frictions have already been defined by Chen (2018) and need to be incorporated in credit risk modelling for three central reasons: In other words, financial market frictions can generate real costs to be incurred by investors. Recognising these investment costs helps in the understanding of the total costs of transactions faced by investors and deciding where to place a hedge, and even whether to make them at all in the first place for example capital gains tax (FRBA, 2007). Constantinides (1984) in FRBA (2007) shows that the option to assume or defer capital losses or gains has substantial value in the eyes of investors. The option's exact value in derivative markets and the corresponding optimal trading strategy normally depend on factors such as transaction costs, capital gains tax rates, and asset volatilities. Financial market frictions can also generate business opportunities such as investment in mutual funds, which relax wealth constraints and asset indivisibilities (DeGennaro and Kim, 1986). Financial market frictions are not exact and hence can change over time.

The Black-Scholes-Merton OPM alluded to earlier on is constrained as it is not extended to the valuation of banks and their risk metrics in the presence of transaction costs (Zimmermann, 1988). Fuzziness on the other hand has been incorporated in jump-diffusion models (JDMs) for option pricing. Two ways of studying fuzzy finance are the pricing of options based on stochastic stock models that use fuzzy set theory (Springer) and the use of fuzzy logic to challenge the use of probability theory in the pricing of options contracts. Liu's process has been employed in option pricing in fuzzy financial environments but has not been extended to the case of the valuation of banks and similar financial institutions. The Black-Scholes-Merton model has also been constrained as it is not extended to the inclusion of transaction costs and fuzziness in the valuation of firms.

Fuzziness is incorporated in the proposed models because it is found in many areas of daily life, such as in meteorology (Cao and Chen, 1983), medicine (Vila and Delgado, 1983), manufacturing (Mamdani, 1981) and engineering (Brockley, 1980). The economic value of default is presented in this approach as a put option written on the value of a firm's assets. The Merton (1974) model lays the foundation for credit risk modelling and assumes that financial markets are frictionless. Jubouri (2018) identifies a family of credit risk management indicators and their impact on stock market indices. The study postulated that factors such as capital adequacy ratios (KARs), non-performing loan (NPL) ratios, loan-to-deposit ratios, total debt ratios for banks, return on risk-adjusted capital had significant effects on earnings per share (EPS), price-to-profit ratios, share turnovers and market values of private or international banks in Iraq. The systematic risk of China's stock markets was studied by Dai and Li (2019) under risk-neutral economic conditions and the study found that economy–wide factors measured market system risks more accurately than the traditional system for instance the value-at-risk (VaR) and expected shortfalls (ES) or ELs.

Based on the magnitude of market friction faced by firms in Southern Africa, policymakers face multiple challenges such as income, financial soundness, demographic, and location of obligors into consideration in future credit risk modelling (Vasanthi and Raja, 2011). Their study argues that income, liquidity, and wealth constraints of borrowers should be factored into models to reduce the magnitude of credit risk. A logit model was developed based on adjustments for the above fundamentals for credit risk modelling by providers of credit in Australia. A default event occurs when the asset value of a firm falls below a certain threshold and the firm should be liquidated according to the first passage models. However new models for credit risk modelling postulate that default does not cause liquidation immediately but may represent the commencement of such a process, and may not translate into liquidation after it is completed.

2.9 Gaps in Structural CRMs

All structural approaches are asset valuation models (AVM) initiated by Kealhofer, McQuown, and Vasicek (abbreviated into KMV) based on the framework developed by Merton (1974). In the model, the default process of an obligor is endogenous and relates to the capital structure of a banking corporation. The Merton (1974) model is an extension of the theory of option pricing presented by Black and Scholes (1973). The model assumes that investors in financial markets assume no transaction costs in their trading and that stock prices are stable, that is there are no ups and downs in stock prices. However in practice, stock prices follow a Geometric Brownian motion and there are huge transaction costs, both monetary and non-monetary, faced by firms and investors in their day-to-day operations.

The Merton model offers an alternative to Credit risk migration or Metrics approach to CRM in banks and similar financial institutions. Merton model's valuation parameters, such as asset volatility are determined in the context of the classical probability theory and are assumed to be constant over time. However in reality financial markets are fuzzy in nature and far from being frictionless, precise, or certain, hence the need for research to relax some of the assumptions of structural models such as Merton (1974) and Black-Scholes (1973). Structural and reduced-form models are asset valuation models (AVMs) that are OPMs whose formulae are based on prices of underlying stocks, market values of assets, and their volatilities, making them precise or based on fixed numbers. However, in practice firms such as those in Southern Africa can be valued using human language which is dominantly used by investors to express returns and risks to investments as high or low.

According to Zadeh (1965 and Zimmermann (1988) human language refers to the use of qualitative variables used to describe the imprecise nature of terms used by investors to describe returns and risks faced in financial market investments, contrary to precise numbers on which all structural and reduced-form models are founded. Existing CRMs used in firm valuation are biased towards option pricing and thus are far from applying to the valuation of banks in frictional and fuzzy financial markets. These variables are practical conditions faced by banks in emerging economies such as those in Southern Africa. Hence the study has both contextual and methodological dimensions in that imprecise variables and transaction costs have not been applied to CRM and no theorists have gone into addressing the shortcomings of these models respectively. Investors in emerging economies such as those in Southern Africa often use language variables relative to precise numbers mainly due to the unavailability of quantitative data and this justifies the need for inclusion of market friction and fuzziness in the study.

This work proposes and investigates a generalised structural credit risk model. To put the research study on a firm foundation, we will let $(\Omega, F, \{F(t)\}\, t \geq 0, P)$ be a complete filtered probability space (Oksendal, 1998). The filtration $\{F(t)\}\, t \geq 0$ is a representation of the flow of information across financial markets in a given market. It is assumed that we have a homogeneous portfolio, of corporate loans subjected to default and we further assume that each credit exposure in the portfolio accrues an interest rate r(t), which is compounded continuously. Furthermore, it is assumed that the value process of a firm, X(t) at time, t, evolves according to the following stochastic differential equation;

$$dX(t) = b(X(t), t)dt + \sigma(X(t), t)\, dB(t) \tag{2.5}$$

$$X(0^{-}) = x \in IR \tag{2.6}$$

where; $b: IR^2 \to IR$; $\sigma: IR^2 \to IR$; ã$: IR^2 \to IR$, are functions satisfying the conditions for the uniqueness and existence and of a strong solution, $X(t)$ (Oksendal, 1998) for such conditions. The same source can also be consulted for a more extensive discussion of stochastic differential equations (SDEs) with applications to finance and investment analysis. Here, $B(t)$ is a 1-dimensional Brownian motion with respect to Ft. We assume that the time zero value of a bond or credit exposure is one unit of a currency.

In other words, transaction costs must be efficiently and directly allocated to individual loans issued about an individual lending institution's total debt and equity costs (Fuchs and Uy, 2010). Furthermore, detailed evaluation of credit exposures allows lenders to accurately undertake marking-to-market of their investment portfolios. Corporate bonds and fixed-rate loans require models that that measure both credit and interest rate risks accurately for efficiency and effectiveness in financial performance of banks. Most credit risk models are based on risk management methods and systems excluding ratings of obligors, sectors of operation, terms and conditions, interest rates, spreads, which were indispensable in accurately assessing and measuring banks' performance. Therefore it is in the interest of filling shortcoming of structural models that the study proposes a fuzzy Merton-Black–Scholes CRM extended for market friction and validates it using financial data from banking firms in emerging markets in Southern Africa. This is because most investors' decisions in all financial markets are based on human judgments more than the requirements of classical probability on which all structural and reduced-form models used worldwide are premised.

2.10 Summary

A review of literature on CRMs was presented but extended to include AVMs and estimation of risk metrics for banking corporations in the presence of market friction in frictional markets. Credit risk models reviewed and critiqued are mainly structural and reduced-form models used in estimation of risk metrics of firms namely probability of default (PD), exposure at default (EAD), loss given default (LGD) and expected loss (EL) and AVMs. It was noted that CRMs and AVMs are based on precise conditions emanating from classical probability theory, which were far from the truth and reality of what actual obtains in practice. Financial markets in reality are characterised by market friction and fuzziness that is imprecise conditions which were not factored into existing structural models. An attempt was made to assess the impact of both market share and friction on the growth and performance of banking corporations in emerging markets with specific reference to those in Southern Africa. The following chapter is devoted to detailing the research methodology used by the study on credit risk modelling in commercial banks in emerging markets, in the presence of market friction and fuzziness.

CHAPTER III

RESEARCH METHODOLOGY

3.1 Introduction

The methodology of the research study is broken down into six models for use in valuation of firms and their risk metrics in frictional and fuzzy financial environments. In this section the research proposes models based on objectives of stated and justified in chapter 1. The methodology starts with proposing a model for investigating the impact of extension of the Merton AVM for market friction on the value of banks in fuzzy financial environments. A fuzzy probability of default (PD) model for use in credit risk modelling in banks is then proposed as an extension to the Merton structural PD model. The study then proceeds to propose fuzzy models for valuation of other risk metrics namely exposure at default (EAD) and loss given default (LGD). The three risk metrics that is PD, LGD and EAD will then combined to give a market friction-fuzzy expected loss (EL) model for valuation of credit losses in banking corporations. The study's research methodology winds up by proposing a fuzzy model for assessment of performance of banks in emerging economies such as those in Southern Africa characterised by frictional and fuzzy financial markets.

3.2 Market Friction and the Merton Asset Valuation Model (AVM)

The structural Merton AVM is directly related to the Black-Scholes option pricing model (BSOPM). Lee, Tzeng and Wang (2005) derived the fuzzy-Black-Scholes option pricing model (FBSOPM). The proposed AVM by the study for bank valuations in frictional and fuzzy financial markets may be referred to as the fuzzy Merton-Black-Scholes model (FMBSM). This is because it blends option and asset valuation techniques and extends them for market friction and fuzziness in the estimation of the value of the firm and its risk metrics. Before the FMBSM is proposed and validated, the variables of the model are outlined and discussed as below.

3.2.1 Confidence level and fuzzy set theory

In this research stock volatility is modelled as a fuzzy variable and not a precise value as assumed under structural models. The application of models extended for fuzzy logic to firm valuations requires us to start by appreciating the role of confidence intervals in estimation of asset volatility. Fuzzy number theory is an extension of confidence intervals when all values in the 0 to 1 interval are considered instead of single or individual numbers. A fuzzy number is formed with a finite or infinite sequence of confidence intervals (CIs) represented by the [0;1]. This follows then that $A_{\propto}$ = $[a_1^{\propto}\ ;a_2^{\propto}]$ and the membership function of set A is given by μ_A (x) = $(A_1;\ A_2; A_3)$. For example set A = $(A_1;\ A_2; A_3)$ where $(A_1;\ A_2; A_3) \in R\ (Real\ numbers)\ and\ A_1 \leq A_2\ \leq A_3$ such that μ_A (x)= $\propto$.

$$\text{The term } \mu_A(\mathrm{X}) = \propto = \begin{cases} 0\ if\ x\ \leq A_1 \\ \frac{X-A_1}{A_2-A_1}\ if A_1 \leq X \leq A_2 \\ \frac{A_3-X}{A_3-A_2}\ if A_2 \leq X \leq A_3 \\ 0\ if\ A_3 \leq X. \end{cases} \quad (3.1)$$

where the variable X is expressed as a confidence interval bound by limits namely A_1, A_2 and A_3. For instance when the preceding and succeeding limits in the second last term of equation 3.1 is reorganised to make X the subject, we obtain the confidence interval given by the algebraic equation:

$$X = [A_1 + \propto (A_2 - A_1);\ A_3 - \propto (A_3 - A_2)]. \quad (3.2)$$

where $\propto$ assumes all rational values from 0 to1 and As are asset volatility values drawn from the banks. Traditional asset volatilities for the structural MAVM are fuzzified to reflect the importance of human language or perceptions in estimation of equity values of banks using the proposed model. Results obtained using the traditional and fuzzy models are then compared for precision, reliability, validity and consistency.

3.2.2 Estimation of firms' market values of assets and volatilities

The Merton (1974) model used in estimating the value of a firm gives rise to two simultaneous linear equations with two unknowns namely the value of the firm's assets (V_A) and their volatility (σ_A). The simultaneous linear equations for solving for the two unknowns are therefore given by two algebraic equations **namely** market value of the equity (VE), given by the formula,

$$V_E = V_A \times N(d_1) - Xe^{-\,\mu RE.T} N(d_2) \quad (3.3)$$

and the volatility or standard deviation of the equity of a firm,

$$\sigma_E = \frac{V_A}{V_E} \times N(d_1)\sigma_A \tag{3.4}$$

where, V_A = the value of the firm's assets, $\sigma_A =$ the standard deviation of the value of assets of the firm, T= the tenure of the firm's assets, μRE = the rate of return on the stock market, X= the exercise price of the underlying stock, which is represented by the liabilities of the firm, $N(d_1)$= and $N(d_2)$ = the cumulative normal probability distributions of the Z-Scores, d_1 and d_2 respectively to be estimated using the Black-Scholes model.

Two common approaches can be used to solve and these values are unobservable. The two Merton approaches are commonly used in the derivation of asset and volatility values of assets when they are not observable. The first is the Merton model as a single point calibration that requires equity values of firms, liabilities, and equity volatilities to be given to solve for the two unknowns using a 2-by-2 system of non-linear simultaneous equations. The second is the Merton time series approach which requires the use of time series data for the valuation of equity and all other model parameters. The study, therefore, employed the latter approach to solving for and because it is a direct method to the estimation of the values of assets and their volatilities based on asset, liabilities, and return values. The volatilities of the firms' assets are then converted into fuzzy variables because they were not observable and are based on human beliefs or perceptions.

3.2.3 Expected value of the fuzzy return on equity (ROE)

The study employed the firms' ROEs instead of risk-free returns because they were not readily available in the countries from which company financial data were drawn. In any case, ROEs were strong proxies for the risk-free rates of return because they were unique and directly related to the individual firms' equity bases and market financial performances. The firms' traditional ROEs were calculated using the formula,

$$\text{ROE} = \frac{Net\ Income\ (\ Earnings\ After\ Interest\ and\ Tax)}{Total\ Market\ of\ the\ Firm's Ordinary\ Equity} \tag{3.5}$$

Firms' ROEs were converted into fuzzy values because they were influenced by experts' perceptions. Most companies' experts employed ROEs as proxies for the risk-free rates of return that is Treasury-bill rates in their accounts and financial departments. This is because these equity returns could be forecasted for future periods by the states of the economy at hand.

3.2.4 The proposed fuzzy Merton equity valuation model

The research proposes a new look at Merton AVM for the valuation of default on a bank credit event. The dynamics of developing an understanding of the value process of a firm's assets is described by a Geometric Brownian motion of the form:

$$dV_t = \mu_v Vtdt + \sigma_v VtdB_t, \qquad V\ t = Vo, \tag{3.6}$$

where are constants and is a one-dimensional Geometric Brownian motion. The basic assumption of the model is that the no-arbitrage principle holds. From equation (3) above and using Ito's Lemma, we get:

$$V_t = V_o e^{\left(\mu_v - \frac{1}{2}\sigma_v^2\right)t + \sigma_v B_t} \tag{3.7}$$

Credit risk concerns the possibility that the process {Vt} on the maturity date, T will be less than the repayment value of the loan amount, F issued. Debt holders at the time, T either receive the value F (if VT>F) or the entire value of the firm (if VT and owners of the firm remain with nothing. The risk of default is explicitly linked to volatility in the firm's asset value. The above AVM is directly related to the Black-Scholes option pricing model (BSOPM). Lee, Tzeng, and Wang (2005) derived the fuzzy-Black-Scholes option pricing model (FBSOPM). According to Merton (1974), the value of equity of a firm is given by the formula for pricing a European call option on a non-dividend paying common stock, where firm value corresponds to stock price and F corresponds to the exercise price. The value of a European call option on maturity date is given by the formula:

$$E(V_1; O) = \max\{V - F, O\} \tag{3.8}$$

that is:

$$E(V_1 t) = V\emptyset(d_1) - Fe^{-rT-}\emptyset(d_2) \tag{3.9}$$

where; are standard normal distribution or Z values.

The research extends Merton's AVM above to the case for asset valuation in firms under uncertainty and fuzziness (Kim, 2005). The variables characterize all banks in emerging markets contrary to certainty and frictionless conditions under which structural credit risk models are applied. The transaction costs which are assumed to be zero in structural models are introduced in the model, represented by costs of capital. The cost that a bank faces in raising capital is estimated using Gordon's traditional zero-growth model. Based on the variables above, the research proposed a fuzzy AVM model to relate the value of a firm's equity, E and assets A, at any time before the maturity date, T. The proposed Merton AVM for the valuation of a bank's equity is given by the general equation:

$$V_E = V_A e^{-\mu RE \times T} \mathrm{N}(d_1) – V_L e^{-\mu RE \times T} \mathrm{N}(d_2) = e^{-\mu RE \times T}[V_A \times \mathrm{N}(d_1) - V_L \times \mathrm{N}(d_2)] \tag{3.10}$$

where
$$d_1 = \frac{[\ln\left(\frac{V_A}{V_L}\right) + \left(\mu RE - \mu CE + \frac{\sigma_A^2}{2}\right)T]}{\sigma_A \sqrt{T}}, \tag{3.11}$$

and
$$d_2 = d_1 - \sigma_A \sqrt{T}, \tag{3.12}$$

V_E is market value of the bank’s equity, V_A is market value of the bank’s assets, V_L is market value of the bank’s liabilities, T is maturity of the bank’s liabilities, σ_A is standard deviation of the bank's assets, μRE is return on equity and μCE is the total cost of capital.

The above equity valuation model (EVM) proposed by the research may be referred to as a fuzzy Merton-Black-Scholes model (FMBSM) because its return, cost, and volatility variables are fuzzy numbers.

The study employed the firms’ ROEs instead of risk free returns because they were not readily available in the countries from which company financial data were drawn. In any case ROEs were strong proxies for the risk free rates of return because they were unique and directly related to the individual firms’ equity bases and market financial performances. The firms’ traditional ROEs were calculated using the formula,

Firms’ ROEs were converted into fuzzy values because they were influenced by experts’ perceptions. Most companies’ experts employed ROEs as proxies for the risk free rates of return that is Treasury-bill rates in their accounts and financial departments. This is because these returns to equity could be forecasted for the future periods in accordance with the states of the economy at hand.

3.3 Derivation of Model for Estimation of Risk of Default in Banks in Southern Africa

According to the AVM by Li (2000) and Kalemanova, Schmidt and Werner (2007) default is triggered if the asset value of a firm falls below a certain threshold. The barrier to default is represented by the algebraic equation:

$$K = \emptyset^{-1}\ (1\text{-pk}) \tag{3.13}$$

where; The variable pk = The probability of default over the whole time interval in the market model. The term K = The threshold, normally taken to be a constant for a Collateralized Debt Obligation (CDO) type of contract. For the time dependent case, we can represent a time, t 'asset value' using the algebraic equation:

$$A(t) = \emptyset^{-1}(1\text{-pt}), \tag{3.14}$$

where default is triggered if the value, A(t) falls across the default barriers , K that is:

$$A(t)<K \text{ or } \emptyset^{-1}(1\text{-pt}) < \emptyset^{-1}\ (1\text{-pk}). \tag{3.15}$$

In other words, as ϕ^{-1} function decreases in value when p (t) becomes bigger, this means then that A (t) < K and p (t) > pk. Default occurs when a firm's value drops below some default barrier (DB) which in the Merton (1974) model is represented by the Future Value (*FV*) of Debt, F at its maturity value and hence *PD* = Probability (*VT* ≤ *F)* where PD=The Probability of Default.

According to Crouhy et al (2000) PD is a robust hypothesis confirmed by the actual delta. In this respect PD is stated in natural logarithmic form as;

$$Ln\ (\text{VT}) \approx \frac{\emptyset[LnV_0 + \left(\mu_v - \frac{\sigma_v^2}{2}\right)T]}{\sigma_V^2 T}. \tag{3.16}$$

$$\text{and PD=Probability } (\ln Vt \leq F). \tag{3.17}$$

Combining equations 13 and 14 above we obtain the final equation:

$$\text{PD} = \frac{\emptyset[Ln\,V_0 + (\boldsymbol{\mu_v} - \frac{\sigma_v^2}{2})T]}{\sigma_v\sqrt{T}} \text{ or PD} = \emptyset\ (-d_2^*). \tag{3.18}$$

The above PD model is criticised for only holding for a firm operating at the maturity stage of its growth or term, T, expected at t = 0 and t = T, when *Vo* is known with certainty. In general the term $\phi(d_2)$ is the probability that the European call option will be exercised by the equity holder and the company will not default on the obligation. The term $\emptyset\ (-\mathrm{d}_2^*)$ in the equation = The physical or real world PD while $\Phi(\boldsymbol{-}d_2)$ = The PD in the risk neutral world (from use of risk free rate of return on market traded instruments). The structural Merton model for estimation of a bank's PD is given by the general formula,

$$\text{PD} = \frac{N[ln\left(\frac{VA}{Xt}\right)+(r-\frac{\sigma_A^2}{2})T]}{\sigma_A\sqrt{T}}, \tag{3.19}$$

Because of the fact that the model is based on theoretical assumptions such as its application in frictionless markets, the research seeks to extend it to the case for fuzzy frictional markets. The study proposes an all-weather PD model that does not favour banks operating at maturity stage based on the AVM originated by Merton (1974) and extended by Li (2000), Kalemanova, Schmid and Werner (2007) and Crouhy et al (2000). The proposed DP model adjusted for market friction will be given by the general formula,

$$\text{PD} = \frac{N[ln\left(\frac{VA}{Xt}\right)+(\mu RE-\mu CE+\frac{\sigma_A^2}{2})T]}{\sigma_A\sqrt{T}}; \tag{3.20}$$

where; VA=Value of firm's Assets and VE=Value of the Firm's Equity, T=The tenure of the asset, μRE =The return on ordinary equity, μCE = The cost of ordinary equity (Market friction). On the other hand $N(d_1)$= The cumulative normal probability distribution of the Z-Score, d_1 and $N(d_2)$=The cumulative normal probability distribution of the Z-Score, d_2.

The extension of the structural PD model for friction and fuzziness is critical as it is suitable and precise to the financial circumstances facing banks in emerging economies.

3.4 Impact of Market Friction-Extended EL Model on Bank Financial Performance

After the estimation of PD in the preceding section, this section discusses the approaches that will be used in the estimation of two other risk metrics, namely LGD and EAD using credit risk metric models extended to include market friction. A bank's expected loss (EL) is calculated based on estimated market-friction risk metrics namely PD, LGD, and EAD. The PD values for banks are calculated as demonstrated under 3.3 above while those for EAD and LGD will be estimated as illustrated below. The reliability and validity of the risk metrics drawn from the new-look CRMs in the estimation of ELs of banks will be tested by comparing them with those drawn from traditional or structural models such as the Merton (1974) and Black-Scholes (1973) models. The PD model for use in the estimation of EL is as explained above while EAD and LGD values will be generated as demonstrated below.

3.4.1 Estimation of a bank's Exposure at Default (EAD)

This is the amount that a bank is expected to lose if the obligor will default on a loan obligation. According to the Bank for International Settlements (BIS) and Basel Committee on Banking Supervision (BCBS, 2009), EAD must not be lower than the book value of the Statement of financial position (SFP or balance sheet) receivables and should be calculated at the facility level. Under the internal ratings-based approach (IRB), EAD can be calculated using the Foundation approach (F-IRB) based on lines of credit and off-balance sheet (OBS) transactions (Zhang, Lu, and Sang, 2014). The traditional EAD is calculated using credit conversion factors (CCF) that are provided for in the Basel guidelines excluding collaterals and guarantees or securities.

The modelling of firms' EADs will also be performed based on calculated credit conversion factors (CCFs) and loan equivalences (LEs) for given loan portfolios as well as for banks' credit portfolios. The EADs specify the exposure the banks have to their corporate borrowers. A bank's exposures consist of out-standings and commitments where out-standings are the portion of the exposures of a lending institution already drawn by the obligors. Therefore, the EAD is the quantity that specifies the exposure that a bank has in the hands of its borrowers, and in the case of the borrower defaulting, the bank is exposed to the total amount of the out-standings. The commitments are the exposures the bank promises to lend to the obligor (borrower) and comprises drawn and undrawn components, in the time before default. Under the FIRB, we define calculate EAD as a function of out-standings (OUTST) and commitments of the loan using the formula:

$$\text{EAD} = \text{OUTST} + y\ \text{COMM}, \tag{3.21}$$

where y = CCF.

On the other hand, the EAD of a firm can also be estimated using the advanced approach (A-IRB) which allows banks to use their models. In other words, A-IRBs accord banks the flexibility to generate models for use in calculating their EADs. Under the CCFs, the amounts owed by borrowers to the bank at time T =EADs (Elizalde, 2005) (fixed or variable exposures). Fixed exposures are exposures in that banks have not made commitments to provide credit in the future and on-balance sheet (OBS) values such that EAD=Drawn Credit Lines that is EAD =The Current Amount Outstanding on a firm's balance sheet and hence no modelling is required for Basel II Requirements (Zhang, Lu, and Sang, 2014; Tang and Fang, 2011). On the other hand, variable exposures are exposures under which banks will provide future commitments in addition to the current credits that such exposures have both on and off BS values. Therefore we define the proposed EAD as a function of out-standings (OUTST) and commitments of the loan (COMM) and market friction (MNC) using the formula:

$$\text{EAD} = \text{OUTST} + \text{y COMM} + \mu\text{CWB} + \sum \sigma\text{MNC} + e_{i,}, \tag{3.22}$$

given that y is the expected portion of commitments likely to be drawn prior to default, μ *and* σ. are other regression coefficients to be calculated In practice, banks can evaluate their credit exposures based on the creditworthiness of borrowers (CWB) and various types of market friction (monetary and non-monetary transaction costs, MNC). The research, however, assumes that EAD is a random variable comprising both quantitative and qualitative variables, and EAD like the PD can be extended to capture qualitative market friction variables to improve efficiency in prediction and robustness. In practice EAD is assumed to be a deterministic quantity, hence the reason for the study to deal with income or monetary variables ignoring the underlying random variable where,

$$\text{CCC} = \frac{Increase\ in\ Exposure\ Until\ Default\ Day}{Maximum\ Possible\ Increas\ in\ Exposure\ Until\ Default\ Day.}. \tag{3.23}$$

Calculated CCFs must be checked for appropriateness for current macroeconomic scenarios in banks in emerging economies before being used in the calculation of EADs of firms (Zhang, Lu, and Sang, 2014; Tang and Fang, 2011). The study at hand intends to adjust the above EAD model for market friction in the form of corporate governance costs to enhance its robustness in the estimation of EADs for banks in Southern Africa. Poor corporate governance for instance sitting allowances for boards of directors (BOD) and ethics are central sources of leakages and poor financial performance of banks in most emerging markets.

3.4.2 The Formula for calculation of the LGD of a bank

A bank is said to have incurred a loss when a company to which it has lent out money or entered into a contract defaults on its payments usually both the principal and interest components. According to the BIS, default on an obligation is said to have occurred when one or more of the following events have taken place:

. The obligor is past due more than 90 days on a credit obligation.

. The obligor has filed for bankruptcy or similar protection from creditors and

. The LGD is the percentage loss rate on the EAD given the obligor's defaults.

The actual loss incurred by the bank=LGD EAD (Zhang, Lu, and Sang, 2014; Tang and Fang, 2011). The components of loss to be incurred by the bank are the loss of the principal, carrying costs, and workout expenses. It should however be noted that firms' LGD values are known for varying with economic cycles namely cyclical LGDs (Point in time LGDs), long-run LGDs (Throughout the cycle LGDs), and downturn LGDs. Cyclical LGDs are based on recent data and depend on economic cycles while long-term LGDs are average long-term LGDs corresponding to noncyclical variables that do not depend on the time at which the LGDs are calculated. Downturn LGDs represent the LGDs of firms at the worst time of the economic cycle, say at the lowest peak of a recession.

The Basel II Framework (See Basel Committee on Banking Supervision, BCBS, 2009) requires that LGDs of firms must reflect downturn conditions wherever it is necessary to capture relevant risks facing the organization. It is also recommended that banks should use downturn LGDs when credit losses for given asset classes are expected to be higher than the averages. Therefore under the F-IRB approach, senior claims on sovereigns, corporates, and banks that are not secured by acceptable collaterals are given higher LGD values of 45%, and subordinated claims are given LGD values of 75%. Under the A-IRB approach, LGDs should be estimated using any of the following internal rating methods:

.The market LGD, is based on market values of defaulted bonds or loans.

.Workout LGD, based on cash flows from a firm's workout processes.

.Implied LGD, based on the market prices of non-defaulted bonds or loans and

.Statistical LGD, based on regression techniques on LGDs and facility characteristics for example qualitative forms of market friction such as spreads and macroeconomic environment.

It is argued further that market and implied LGD methods are less computation intensive and normally work well for liquid market instruments. Banks are therefore advised to use market or implied LGD approaches to estimate their LGDs under the above conditions and employ workout LGD methods when they hold illiquid and non-marketable instruments, which is usually the case in most emerging economies (Zhang, Lu, and Sang, 2014; Tang and Fang, 2011). However, under conditions of large exposures, banks should apply techniques that make it possible to estimate more precise LGDs. For forecasting of LGDs statistical LGD methods should be used as long as it is possible to establish dependent and independent linear relationships. It should also be noted that the LGD under the workout approach,

$$\text{LGD} \quad = \frac{EAD_T - PV[\sum R_t) + PV(\sum C_t)}{EAD_T}, \qquad (3.24)$$

where PV (R_t) and PV (C_t) are recoveries and costs incurred during workout prices and processes respectively.

It is also noted that implied LGDs that are based on observed market information such as stock prices can be calculated using both structural and reduced-form models, for instance, the Merton model, as specified in this study. On the other hand, the statistical LGD approach stipulates that a firm's LGD lies between values of 0 and 1 (Bluhm, Overbeck, and Wagner, 2003). Hence the LGD can be transformed into the variable X extended for market friction, to give a logit model of the form,

$$X_t = \text{Log}\left(\frac{LGD}{1 - LGD}\right). \qquad (3.25)$$

The results drawn from the logit model (3.25) are compared with those estimated using a logistic model given by,

$$X_t = e^{\alpha_0 + \alpha_1 y_1 + \alpha_2 y_2 + \cdots \ldots\ldots\ldots\ldots + \alpha_n y_n}. \qquad (3.26)$$

The above financial model is applicable when:

. Only significant variables are incorporated into the model.

.The variables used have economic meaning in explaining the variability in firms' LGDs.

.Independent variables are able to explain the LGDs significantly and

.The financial data collected should be properly processed leaving out all outliers.

Therefore it is the interest of this study to apply statistical techniques to estimate the LGDs of banks in Southern Africa under frictional and fuzzy financial markets. Hence such a model can go a long way to improve the rigour and accuracy needed in estimation of LGDs of banks characterised by use of uncertain or imprecise nature of human behaviours in financial market planning and decisions (Oksendal and Sulem, 2009).

3.4.3 Proposed model for estimation of expected losses of banks

After estimation of PDs, EADs and LGDs, extended for market friction and fuzziness, the ELs of banks are then estimated using the general formula,

$$\mathrm{EL} = \mathrm{PD} \times \mathrm{EAD} \times \mathrm{LGD}, \tag{3.27}$$

where all three independent risk metrics are fuzzy variables estimated through methods articulated above.

3.5 The Impact of Market Friction on the Performance of Banking Corporations in Emerging Economies

The overall purpose of the research is to examine the relationship between expected loss and market friction and the financial performance of banks in Southern Africa using inferential statistical tests such as regression and correlation analyses. Correlation analysis will be used to determine the existence of a relationship between expected loss and market friction (independent variables) and bank financial performance (dependent variable) and each of the firm-specific and macroeconomic factors (independent variables) drawn into the model and bank performance (dependent variable). Regression analysis will then be conducted to obtain statistical evidence describing the nature of the relationship between the dependent and a family of independent variables mentioned above and the market friction assumed to be suitable for explaining credit risk modelling in banks in Southern Africa. The financial data set for this study contains cross-sectional dimensions (several banks) and longitudinal dimensions (several periods from 1997-2020). The combination of cross-sectional and longitudinal quantitative surveys will enable the investigation of constructs of the research study, that is, factors influencing credit risk modelling and financial performance of banks in Southern Africa.

3.5.1 Variables and measures

The impact of firm-specific and macroeconomic factors on bank financial performance is most frequently analysed through panel data regression models. The study will employ a panel data model because it allows multiple phenomena obtained over multiple periods to be observed simultaneously, increases the degrees of freedom from error, and reduces co-linearity among variables leading to improved efficiency and consistency. Both firm-specific and macroeconomic variables faced in financial markets are the bedrock of bank financial performance and hence should be modelled using dynamic panel data models to assist in dealing with endogeneity problems found in the real world of financial investment.

Table 3.1: Measurements of Firm Specific Factors Affecting Bank Performance

Variable	Measurement	Formulae / Proxy
Bank Profitability	PROF	$\frac{Operating\ Profit}{Sales}$
Size of Bank	SIZE	Natural logarithm of total assets
Asset Tangibility	ATAN	$\frac{Tangible\ Lloans}{Total\ Assts}$
Growth of Bank	GROB	% Change in total assets
Business Risk	RISK	Standard deviation of operating profit / total assets
Bank Loans	BANL	$\frac{Total\ Bank\ Loans}{Total\ Assets}$
Liquidity Status	BLIQ	$\frac{Current\ Assets}{Current\ Liabilities}$
Bankruptcy Probability	PBCY	$\frac{Iinterest\ Expense}{Operating\ Profit}$
Government Taxes	GOVT	$\frac{Ccorporate\ Taxation}{Earnings\ Before\ Tax}$
Board of Directors	CBOD	BOD size , Composition and Remuneration

Table 3.1: Measurements of Macroeconomic Factors Affecting Bank Performance

Variable	Measurement	Formulae / Proxy
Exchange Rates	EXR	Prime Exchange Rate
Credit ratings	CRR	Country's Credit Ratings (as per 3 ratings agencies)
Economic growth	GDP	Country's GDP (%)
Inflation Rate	INFL	Country's Consumer Price Index
Unemployment	UER	Annual Unemployment Rate
Corruption index	CPI	Country's Corruption Perception Index
Interest rates	INT	Prime Interest Rate

Table 3.2: Measurements of Bank Expected Loss (Book Values)

Variable	Measurement	Formula / Proxy
Total Bank Expected Loss	BEL	$\frac{Total\ Expected\ Loss}{Total\ Assets}$
Long-term Expected Loss	LEL	$\frac{LT\ Expected\ Loss}{Total\ Assets}$
Short-term Expected Loss	SEL	$\frac{ST\ Expected\ Loss}{Total\ Assets}$

Table 3.3: Measurements of Bank Financial Performance

Fuzzy Variable	Measurement	Formulae / Proxy
Return on equity	ROE	$\frac{Profit\ After\ Tax}{Equity}$
Return on assets	ROA	$\frac{Operating\ Profit}{Total\ Assets}$
Return on investment	ROI	$\frac{Profit\ After\ Ttax}{Total\ Assets}$

The impact of firm-specific and macroeconomic factors on bank financial performance is most frequently analysed through panel data regression models. The study will employ a panel data model because it allows multiple phenomena obtained over multiple time periods to be observed simultaneously, increases the degrees of freedom from error and reduces co-linearity among variables leading to improved efficiency and consistency. Both firm specific and macroeconomic variables faced in financial markets are the bedrock of bank financial performance and hence should be modelled using dynamic panel data models to assist in dealing with endogeneity problems found in the real world of financial investment.

3.5.2 Proposed market friction and bank performance model

The values of the dependent variables (various returns to the firm) can be used as regressors to account for their impact on bank profitability. Under such a scenario the general form of the multiple regression model can be specified as:

$$R_{it} = \propto \ + \rho\, R_{it-1} + \beta X_{it} + \mathcal{E}_{it} \tag{3.28}$$

The proposed model for assessing bank financial performance is a transformation of the model adapted from previous empirical studies by El-Sayed Ebaid (2009); Fosu (2013; and Chadha and Sharma (2015). The model of the study captures firm-specific factors as well as market variables as other factors that can influence bank financial performance though they are uncontrollable. Thus the proposed research model is given by:

$$R_{it} = \propto_0 + \rho R_{it-1} + \ \beta X_{it} + \sum_{k=1}^{N} \theta_k\, Z_{kit} + \mathcal{E}_{it} \tag{3.29}$$

where R_{it} is a measure of bank financial performance (ROA, ROI and ROE) in year $_t$, X_{it} is a measure of uncontrollable economy-wide factors, for instance interest and exchange rates, Z are the controlled variables which include company size, BOD and asset tangibility. The lagged profitability, (R_{it-1}) is included the regression model because the profitability in the previous year influences the current year's profitability. Where R_{it} is bank profitability measured by ROE, ROI and ROA for firm, i in year t, Xit is the set of exogenous observable firm-specific and macroeconomic variables as listed in Tables 1 and 2 of the firm above, ρ and β are regression parameters to be estimated and $\mathcal{E}$ is the error term. The lagged profitability measure, (R_{it-1}) is included in the multiple linear regression model because the level of profitability of a bank in the preceding year influences its level of profitability and growth potential in the current financial year.

3.5.3 VAR Estimation techniques

The research postulates the use of dynamic profitability measurement models based on short-term asset growth rates of banks. These growth rates are described by a system of Stochastic Differential equations (SDEs) which considers the asset values of banks as evolving processes from time to time. Previous studies concerning measurement of bank performance based on both firm-specific and macroeconomic variables are known for suffering serious estimation errors and hence the adoption of SDE to improve accuracy and precision in estimation based on practical financial circumstances facing banks in emerging economies. In light of this, the VAR technique developed by VAR (1980) was employed to estimate the model. The VAR model system employs an extra advantage of the lagged first difference of the dependent variable to enhance the efficiency of the estimator and curtail the problem of weak instruments in the difference VAR technique. Taking first differences in VAR eliminates the firm-specific effects on the dependent variable.

The proposed VAR model is also appraised for its capacity to address problems of endogeneity from the relationship between dependent and independent variables. In the presence of the above considerations, the study employed an E-Views 8 program to conduct the main regression procedure connecting market friction and the financial performance of banks in Southern Africa. Where multiple regression techniques are used, Pearson's product-moment correlations and coefficients of determination were conducted using ANOVA and Chi-Square tests. We used panel financial data of 16 banks conveniently drawn from Southern Africa over a period of 24 years (1997-2020) to validate the proposed log-VAR model for assessing bank performance in the presence of market friction. The discussion below details the major findings of the research on all countries drawn into the study after their financial data under different currencies were harmonised through the log-VAR model above.

3.6 Summary

The research methodology of the study was broken down into several fuzzy models that are meant to be adopted by firms to improve accuracy in the valuation of their assets, equities, and debts. The models adopted were traditional structural models that were extended to the case for incorporate fuzzy environments in the presence of market friction. In this section, study models were proposed and justified starting with an investigation of the Merton AVM as applied to modern markets characterized by market friction in fuzzy markets. The proposed Merton AVM was also extended to the case for a fuzzy jump-diffusion (FJD) for comparison purposes. A new probability of default (PD) model adjusted for fuzzy variables for the valuation of firms was proposed as an extension to the structural CRM. The study then went further to propose other fuzzy risk metrics valuation models such as exposure at default (EAD) and loss-given default (LGD). The risk metrics adjusted for fuzzy variables are meant to be combined into a new look market friction-adjusted EL model compatible with market conditions of banks in emerging financial markets. The study's methodology ends by proposing a CRM for evaluation of the impact of market share and friction on the growth and performance of banking corporations in Sub-Saharan Africa in fuzzy financial markets.

CHAPTER IV

IMPACT OF TRANSACTION COSTS ON BANK EQUITY IN FUZZY FINANCIAL MARKETS

4.1 Introduction

Fuzzy theory was introduced by Zadeh (1965) as a framework for modelling uncertainty with specific reference to use of linguistic variables in mathematical domains. The fuzzy theory models vague, imprecise and ambiguous phenomena by assigning weights to any object based on such investor or expert judgments. The option pricing models (OPMs) by Merton (1974), Black-Scholes (1973) and Bachelier (1964) are some of the major groundbreaking models that have beendeveloped to exploit the power of probability theory in modelling uncertainty in financial markets. In recent researches, several studies have been developed to effectively handle intrinsic uncertainty which is prevalent in the social sciences, such as economics, banking and finance. Cordova et al. (2017) and Hryniewicz (2010) argued that the theory of fuzzy numbers is the best description of uncertainty and risks faced by investors in financial markets. As such application of fuzzy mathematics in finance has the ability to provide rigorous results and conclusions on asset valuations in banks. While a lot of literature has been generated on the use of fuzzy theory and numbers in valuation of options and assets, none has captured market friction. It is on the basis of this observation that the study proposes and validates an equity valuation model that captures both uncertainty and transaction costs to determine the rigour of results and conclusions arrived at.

4.2 Background to the Study

Banks worldwide operate in financial markets characterised by friction and uncertainty (fuzziness). Zimmermann (1980) noted that existing AVMs are only suitable for valuations where market friction or transaction costs are very minimal or negligible. In practice, investors often subjectively describe the uncertainty they face in financial markets with implicit fuzziness, also called impreciseness. According to Zimmermann (1980) and Zadeh (1965), implicit fuzziness describes events using imprecise linguistic variables with descriptions such as '--in a bullish economy, there is about 70% probability that the riskless interest rate will grow by 10% in the following year'. The phrases 'bullish economy' and 'about 70%' implicitly mean that the probability for the event of'10% riskless interest rate' could range for instance from 65% to about 75%. According to Zebda (1989) and Zimmermann (1980), the example illustrates that operational language in financial markets is intrinsically imprecise contrary to the underlying assumptions behind structural models commonly used in firm or bank valuations.

From the above expositions it can be deduced that investors use both probabilistic and fuzzy tools to assess and characterize the uncertainty inherent in financial markets. In other words, firm investment behaviours are directly related to bank performance, profitability and stock market liquidity (Vengesai and Kwenda, 2020). However, the precondition of probabilistic and stochastic structural models is that probability used for decision analysis is a 'precise' number determined from repeated samples and relative frequency distributions (Kolmogorov, 1933). Kolmogorov's classical probability theory is different from fuzzy theory, which was derived in accordance with the 'degree of belief' set by experts. Therefore, it is difficult to employ structural models under uncertainty for fuzzy decision-making processes (Bellman and Zadeh, 1970). This research thus proposed and validated an equity valuation model extended for both market friction using financial data of banks that were conveniently sampled from southern Africa.

Problem statement

The research is motivated by the need to develop a new equity model for valuation of equity of banks in frictional and fuzzy financial markets. Although traditional structural financial models such as the Merton AVM and Black-Scholes option pricing model (OPM) are the bedrock of all asset valuation models they are criticized for being restricted by their nature of not being suitable for application in frictional and fuzzy financial environments. Most contemporary option and firm valuation models have been extended for uncertainty (that is fuzziness), but none have included market friction. Hence, by including market friction in contemporary bank valuation models, the effects of variables, such as sovereign ratings, capital adequacy and working capital, can be accurately or precisely measured. Hence it is on the basis of the above shortcomings of structural AVMs that the research proposes an equity valuation model that captures both transaction costs and uncertainty. It is believed that the inclusion of transaction costs and uncertainty in the proposed model will go a long way in making it realistic and rigorous for valuation of banks in emerging economies and markets.

Objectives of the study

The research study sought to:

- Investigate the effects of the structural KMV model on equity values of banks in modern markets characterized by friction in fuzzy financial markets.
- Propose a new KMV model extended for market friction for use in valuation of equity of banks in fuzzy financial markets.
- Compare equity values drawn from the structural and transaction cost extended KMV models in terms of consistency, rigour and accuracy.

Hypothesis of the study

The study is carried out under the following hypothesis:

Null hypothesis (H0): Transaction costs and uncertainty have no effect on the equity values of banks in emerging economies.

Alternative hypothesis (H1): Transaction costs and uncertainty have an effect on the equity values of banks in emerging economies.

Significance of the study

Although structural models such as Merton (1974) and Black-Scholes (1973) are the bedrock on which all firm valuations are based, they are criticized for being premised on unrealistic assumptions such as constant volatility and frictionless markets. Banks in reality operate in frictional and fuzzy financial markets, contrary to the assumptions under which structural and stochastic models are applicable. By factoring transaction costs and market friction into the KMV equity valuation model, estimation accuracy, efficiency and effectiveness are made possible (Palma and Ochoa, 2013). The purpose of research work at this level is to extend frontiers of knowledge in the valuation of banks' equity. Consequently, in banking and finance, newly proposed models must always be premised on rigorous, detailed and technical derivation. The theoretical derivation plays the role of justification for validation of the proposed model. The derived financial model can then be applied in a practical context using either empirical or simulated data.

Scope of the study

The study was carried out in Southern Africa as a case study where both transaction costs and uncertainty characterize financial markets (Fuchs and Uy, 2010). Panel used were drawn from audited financial statements of eight regional banks for the period 2008-2020. The study extends the Merton (1974) model for bank valuation to the case for market friction and uncertainty to test its rigour and accurate compared to traditional models.

Organization of the study

The paper is divided into five sections. After the introduction, Section 2 presents a literature review which examines classical, transaction and fuzzy theories. Section 3 examines the variables used in the derivation of the fuzzy financial model. Section 4 uses data drawn from banks in Southern Africa to validate the proposed equity model. Conclusions and recommendations of the study are presented in Section 5 of the paper.

4.3 Literature Review

This section presents literature review based on AVM, KMV modelling, classical probability, transaction cost and fuzzy theories.

4.3.1 Geometric Brownian Motion and the asset valuation model (AVM)

The classical structural AVM is based on the dynamics of the value process of a firm's assets as described by a geometric Brownian motion of the general form (Oksendal, 1998):

$$\mathrm{dV_t} = \mu_v V_t \mathrm{dt} + \sigma_v V_t \mathrm{dB_t},\ V_0 = \mathrm{X} > 0, \tag{4.1}$$

whereV_t is the asset value of a firm at time t, μ_v and σ_v are constants representing the return and standard deviation of assets, respectively and B_t is a one-dimensional geometric Brownian motion.

According to Bluhm, Overbeck and Wagner (2003) geometric Brownian motion has gained extensive applications in financial circles because of its ability to capture uncertainty in financial markets. Elizalde (2005) noted that the Merton (1974) model is popular and critical as it allows for direct application of the theory of European option pricing by Black-Scholes (1973) to asset valuation in banks. The models also rule out the possibility of early default on an obligation, regardless of what happens with the value of the firm before maturity of the debt. The assertions that default event can only happen at the maturity and is predicted with increased precision as the maturity of the credit exposure draws nearer are unrealistic. Therefore, new AVMs need to be extended to include market frictions and human variables to make them more practical and realistic (Sundaresan, 2013; Ely, 2012 and Graff and Williamson. 2002).

4.3.2 The KMV and valuation of firms

The essence of the KMV model is a kind of OPM which is heavily rooted in the option pricing theory by Black-Scholes. The KMV assumes that the value of a company follows a geometric Brownian motion (GBM), an assumption shared with the Black-Scholes option pricing formula (Kollar and Gondzarova, 2015). New firm valuation methods have been introduced in somewhat unique structure in fuzzy financial markets. Hence emphasis needs to be laid on Kealhofer-Merton-Vasicek (KMV) approach for valuation of banks. KMV method makes the valuation offered by Moody's KMV rigorous as detailed in the research by Crosbie and Bohn (2003). The paper used KMV based on daily market capitalization and quarterly updated debt levels for three banks to obtain asset value estimates. It concluded that the KMV depends on implied asset values of firms and hence cannot be used to obtain unknown parameters in capital structure, a viewpoint shared with Bharath and Shumway (2008).

4.3.3 Transaction Cost Theory

The concept of transaction cost was formally proposed by Ronald Coase in 1937 to explain the existence of firms. According to Palma and Ochoa (2013), Thavanswaran et al (2007). Williamson (1981) and Coase (1937), transaction costs are costs incurred in sourcing for capital and constructing market investments, agents or brokers' commissions, fees and spreads. While recent structural models have attempted to adjust for uncertainty in firm valuation, no attempt has been made to put transaction costs into consideration. Duffie and Singleton (2003) argue that banks face huge direct and indirect costs in transacting business. The transaction theories by Coase and Williamson are used for analyzing market friction, uncertainty and other organizational factors such as board size and insider lending. Challenges faced by banks are worsened by their lending to shareholder connections, non-creditworthy borrowers and poor application of corporate governance and ethics, which result in huge transaction costs (Fuchs and Uy, 2010). They called for innovations in banks in emerging economies to attain financial stability and soundness, liquidity, growth and development.

4.3.4 The theory of fuzzy sets and valuation of financial companies

The origin of the mathematical theory of fuzzy numbers is due to Zadeh (1965) but many results in this area have been achieved by Dubois and Prade (2000). Fuzzy theory by Zadeh (1965) goes further to evaluate the extent to which the rule or object in a given set is judged to be true, false or vague. Most financial models in derivative pricing define market uncertainty or fuzziness through stochastic evolution of the price of the underlying assets where constant parameters are used. It is believed that extra value in bank equity valuation may be achieved through simultaneous adjustment of structural models for uncertainty and transaction costs (Roger, Alfonso and Pedregosa, 2019). By assuming that the proposed financial model's asset value and volatility parameters are fuzzy it can show that their membership functions fairly reflect both characteristics and personal judgments of investors about the behaviours of the parameters themselves (Zebda, 1989).

The concept of vagueness, also known as indeterminacy, is inherently a major factor that affects the structure and direction of decisions in economics, banking and finance. Zimmermann (1980) postulated that models adjusted for linguistic variables reflect real market situations, implying that markets are often not crisp and deterministic due to a lack of information and cannot be described precisely. Although fuzziness is not simple to handle, fuzzy models are efficient in the determination of solutions to financial problems compared to systems of structural differential equations (Zetriuslita, 2020 and Bardossy, 1996). Precise or crisp sets are special fuzzy sets that restrict own membership values to rational numbers with [0;1] as end points of the unitary interval as is the case in probability theory. Therefore, fuzzy set theory has an edge over structural models because they are able to model ambiguous or vague phenomena arising from human behaviours by assigning weights to objects based on the values of the membership function (Palma and Ochoa, 2013 and Ross, 2010). Nowadays many phenomena in human sciences are fuzzy, but are treated as if they were crisp or precise in decisions of firms yet both business and consumer bankruptcy are imprecise and ambiguous (Korol, 2008).

4.4 Derivation of the Proposed Market Friction-Fuzzy Equity Model

The motivation for the study is based on our conviction that given the volatile nature of market parameters, use of fuzzy theory in frictional market valuations deserves deeper and broader investigation. Zimmerrmann (2010) and Zadeh (2008) address the concept of fuzziness in the pricing of European options but restricted it to discrete stochastic settings and fuzzy numbers. In Thavaneswaran, Appadoo and Paseka (2009), Generalised Autoregressive Conditional Heteroskedasticity (GARCH) discrete models were analysed from a fuzzy context. In the study, the authors modified the threshold values for positive and negative information with fuzzy rule and many empirical investigations. They studied centred moments and kurtosis for class of fuzzy coefficient autoregressive (FCA) and fuzzy coefficient volatility (FCV) models. Dubois, Prade and Somari (1993) used non-linear fuzzy partial differential equations (PDE) to price European options using fuzzy extension principle and option prices which have proved very useful in real markets characterized by frictions in a range of support (or a different α-cut). Another contribution of the authors' work is the idea of applying a rolling estimation method for crisp model parameters in order to determine fuzzy parameters which are consistent with empirical market observations.

4.4.1 Architecture of fuzzy systems and estimation of fuzzy model variables

Most structural models used in the valuation of banks involve some degree of uncertainty, which arises from lack of knowledge or inherent vagueness. Of late, there has been growing interest by researchers to use fuzzy numbers to deal with vagueness and imprecision (Appadoo, 2006). More authors including Cherubini (1997) have gone on to deal with randomness in OPMs. They have managed to extend their framework to the case for estimation of prices of corporate debt contracts and providing a fuzzified version of the Black-Scholes model using a family of fuzzy variables. Ghaziri et al (2000) introduced a critical artificial intelligence approach to pricing of options using neural networks and fuzzy logic and compared their results to those obtained using the Black-Scholes model. These authors note that the Black-Scholes OPM is a mere approximation model which leads to a considerable number of errors. Trenev (2001) came up with a refined model for options pricing and discovered that because of the fluctuation nature of financial markets over time some of the parameters of the Black-Scholes model may not be expected in the precise or exact sense.

Authors such as Yun, Sun and Chen (2011) and Thavaneswaean, Appadoo and Paseka (2009) used fuzzy application for the Black-Scholes OPM and deduce that the model is far from being realistic. This is because it is based on precise variables such as efficient and frictionless markets and constant asset volatilities. Most of these structural stochastic models are solved using classical and fuzzy set theories but are not extended for transaction costs which are a huge cost to investors in emerging financial markets. In the process of managing functions of real variables the fuzzy extension should result in the correct application of the extension principle (Talamanca, Guerra and Stefanini, 2012). Assuming we are given an exact relationship function of the general form,

$$y = F(x_1; x_n; \text{------------------}; x_n) \tag{4.2}$$

of n real variables given by x_1; x_n;------------------;x_n. The above multiple linear relationship function's fuzzy extension can be obtained to evaluate the effects of both transaction costs and uncertainty on the variable, x_j, modelled by the corresponding number, u_j for each level, α in the interval [$u^-_{j,\alpha}$; $u^+_{j,\alpha}$,], given the poossible values of xj. Suppose we are also given another variable, v = f (u_1; u_n;------------------;u_n) which denotes the fuzzy extension of a continuous function, f. The continuous function, f is characterized by n variables for each level of α, resulting in the interval [v^-_α; u^+_α], which represents the propagation of uncertainty from all variables x_j to the variable, y (Ross, 2010).

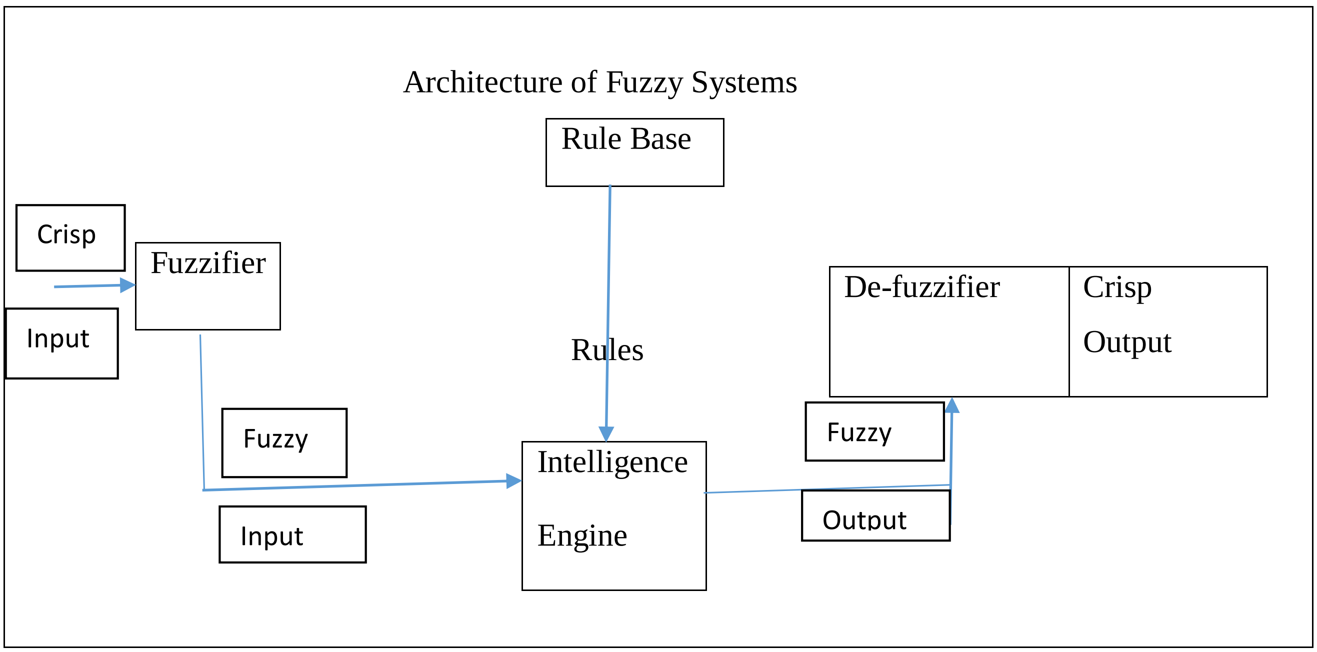
It should be noted that if uncertainty on the original variables of a model is denoted by, y which is also modelled by linear numbers, the y-variable will still be a fuzzy number, starting from a single value (at α =1.00) to the most uncertain interval level (at level α=0.00) but it loses its linearity property in the process of such transformation. This also follows that the parametric representation of the variable is also necessary when input variables are triangular fuzzy numbers in order to apply the extension principle and represent the non-linear output fuzzy numbers (Talamanca, Guerra and Stefanini, 2012). To obtain fuzzy extension of fuzziness to normal semi-continuous fuzzy intervals, we have to compute the α–cuts [v^-_α; u^+_α] of v, defined as the images of α-cuts of (u_1; u_n;------------------;u_n) that are then obtained by solving the following constrained optimization problems for α ∈[0;1]

$$(\text{EP})\alpha = \begin{cases} (v^-_\alpha = min\{f(x_1; x_2; --; x_n)\ xk \in [uk^-_\alpha; uk^+_\alpha], k\ = 1; 2; ---; n] \\ (v^+_\alpha = max\{f(x_1; x_2; --; x_n)\ xk \in [uk^-_\alpha; uk^+_\alpha], k\ = 1; 2; ---; n] \end{cases} \tag{4.3}$$

Source: (Talamanca, Guerra and Stefanini, 2012)

Only in simple cases can the optimizing problems above be solved analytically. In general, the solution of the above equations is complex and computationally expensive to determine for each $\alpha \in [0;1]$, hence, we require global solutions of the two non-linear problems for all model variables. Ross (2010) presents the architecture of fuzzy systems used in translating input variables such as asset values and standard deviations into fuzzified parameters:

Figure 4.1 Showing Architecture of Fuzzy Systems by Rose et al (2010) Used by the Study



Source: Rose et al (2010)

Fuzzy logic is taken to be a computing technique that is based on the degree of truth or it can be taken as a method of reasoning that resembles human reasoning or logic. The approach of fuzzy logic imitates the way of decision making in people that involves all intermediate possibilities between digital values, Yes and No. Therefore, fuzzy logic works on the levels of possibilities that are associated with the input to achieve a definite output. Fuzzy logic is thus a basic control system that relies on the degrees of state of the input/s. The state of the input used determines the nature of the output to be achieved. In other words, a fuzzy logic system operates on the principle of assigning a particular output depending on the probability of the state of the input. The principal components of a fuzzy logic control system are fuzzification, rule base and evaluation, aggregation of the rule outputs and defuzzification which are detailed below.

The Fuzzifier

This is a component of the system that transforms the raw inputs into fuzzy sets. Fuzzification of input variables can be achieved by outlining the various known precise and deterministic quantities as totally uncertain and nondeterministic. This uncertainty variable may have emerged mainly because of imprecision and vagueness, which may then lead to the representation of the input variables by a membership function as they could be fuzzy in nature. For instance, if we postulate that the cost of capital is 20% per annum, an investor could then convert the crisp variable into a linguistic variable such as moderate, high or low cost.

Fuzzy rules, base and evaluation

The fuzzy rules are composed of input and output variables, which draw values from their term sets with meanings that are associated with each linguistic concept. Exact or crisp model variables are fed into a fuzzifier for conversion into fuzzy variables under a clearly defined rule base. The rule base is made up of all the rules and membership functions that regulate decision-making processes in the fuzzy logic system. The base system also contains the "If-Then" decision conditions which are used in conditional programming and controlling the whole fuzzy logic system. The rules evaluation is the process used to assess the criteria and return model values based on a defined dynamic configuration process. The evaluation framework gives users the space to configure model inputs for application scoring, approving flows, credit bureaus or insurance.

Aggregation of the rule outputs

The aggregation of the rule outputs is a technique by which the fuzzy sets representing the outputs of each rule base are combined into a composite fuzzy set. It is a technique that only occurs once for each output variable, and takes place before the final defuzzification process is undertaken. The outputs of the aggregation process are finally converted into one fuzzy set for each given output variable.

Defuzzification

It is the opposite of the process of converting the crisp results into fuzzy variables. That is, the mapping done here is converting the fuzzy results into crisp results. This process is thus capable of generating a non-fuzzy control action which illustrates the possibility distribution of an inferred fuzzy control action. The de-fuzzification process can also be taken to be the rounding off process, where a fuzzy set having a group of membership values on the unit interval is transformed into a single scalar quantity.

Differences between fuzzification and defuzzification processes

The table below summarizes the main differences between these two concepts of the architecture of fuzzy systems.

Table 4.1 Showing Main Differences Between Concepts of Fuzzification and Defuzzification

Comparison Variable	Fuzzification	De-fuzzification
Definition	It is the process of converting crisp quantities into fuzzy quantities or variables.	It is the inverse process of fuzzification where the transformation is done to convert the fuzzy results into crisp results or output.
Basic data	Precise data are converted into imprecise data.	Imprecise data are converted into precise data.
Example	Turning cost of equity into a fuzzy variable	Turning fuzzy cost of equity into a precise variable
Methods used	Uses intuition, inference, rank order, angular fuzzy sets and neural network	Uses maximum membership principle, centroid approach, weighted average method and centre of sums
Complexity	It is quite simple to apply	It is quite complicated to apply
Purpose	It can apply If-Then rules for fuzzifying the crisp values	It can use concepts such as the central limit methods to find the centre of the sets.

Source: Authors

4.4.2 Calculation of the traditional cost of equity and ROEs of banks

The research estimates the cost of equity of a bank using Gordon's traditional zero growth model given by the formula,

$$k_e = \frac{D_0}{P_o}, \tag{4.4}$$

where, k_e = The observed cost of ordinary equity, D_0 = The constant dividend per share at time, $t = 0$ and P_0 = The market price per share at time t=0. It is the cost of equity, a fairly huge transaction cost in emerging markets that is added to the proposed KMV model in order to improve both accuracy and rigour in estimation of equity values of banks. The study also used ROEs of banks instead of Treasury-bill rates because they represent their net return after all obligations have been settled and which measures growth rate in earnings and future investments. The banks' traditional ROEs are calculated using the algebraic formula,

$$\text{ROE} = \frac{Net\ Income\ (Earnings\ After\ Interest\ and\ Tax)}{Total\ Market\ \ Value\ of\ the\ Firm's\ Ordinary\ Equity} \tag{4.5}$$

4.4.3 The proposed bank equity valuation model

Extensions to existing structural models such as KMV, AVMs for transaction costs, and uncertainty enable investors to improve precision and robustness in the estimation of equity values of banks. According to the KMV model, the equity of a firm is represented by a call option on its reference assets mainly because at the maturity of debt, bondholders receive their debt dues and equity holders realize the rest. The model is applicable only if we are respectively given observable and unobservable equity and asset values and their corresponding volatilities. The assumptions on which the KMV model is founded are:

.The debt is homogeneous with time to maturity, T;

.The capital structure of a firm is given by the equation,

$$V_A(t) = D(t) + V_E(t), \tag{4.6}$$

where V_A(t) =The value of assets, D(t) = The value of debt and V_E(t) =The value of ordinary equity;

.The markets are perfect and ignore coupons and dividends and there are no penalties incurred by investors for short selling; and that:

.The dynamism of a bank's assets follows a geometric Brownian motion as in equation (1) above.

Under the Black-Scholes OPM the market price of a call option is given by the formula:

$$C(t) = S(t)\,N(d_1) - e^{-r(T-t)}.KN(d_2) \qquad (4.7)$$

where S(t) =The stock price, r =The risk free rate, K =The strike price of a call option, T =Term to maturity of the bank's assets and liabilities, t= Time today, N(d_1) and N(d_2) = The cumulative probabilities of the Z-values,d_1 and d_2 respectively.

It follows then that the traditional values of equity of banks can be determined from the structural model:

$$V_E(t) == V_A(t).\,N\,(d_1) - De^{-r(T-t)}.N(d_2), \qquad (4.8)$$

where $d_1 = \frac{\log(\frac{VA\,(t)}{D} + (r + \frac{\sigma_A^2}{2})\,(T-t)}{\sigma_A\sqrt{(T-t)}}$ and $d_2 = d_1 - \sigma_A\sqrt{(T-t)}$, $r =$ The risk free rate of return, t=The time now and T =The term to maturity of the bank investment.

Using Ito's Lemma we can demonstrate that values of a bank's equity and assets and their volatilities can be connected through the formula:

$$V_E = \frac{VA}{VE}.\frac{dVE}{dVA}\sigma_A \qquad (4.9)$$

To be able to solve for the unobservable variables of the equation, VA and σ_A we should solve a system on nonlinear equations given by:

$$\begin{cases} f_1(V_E;\ \sigma_E) = V_A(t).\,N\,(d_1) - e^{-(r-k_e)(T-t)}.\,DN(d_2); V_E(t) = 0 \\ f_2(V_E;\ \sigma_E) = \frac{VA}{VE}.\frac{dVE}{dVA}N\,(d_1)\sigma_A;\ \sigma_E = 0. \end{cases} \qquad (4.10)$$

The solutions to the above system of equations are unique as:

$\frac{df_1}{dV_E}$ = N(d_1) which is analogically to changes in the Black-Scholes model and $f_1$is an increasing function of the value of assets, V_A $that\ is\ f_1(V_A)$ has a unique solution. On the other hand $f_2(\sigma_E)$ also has unique solution as well. Since banks operate under conditions of limited liability, their equity values(V_{Es}) at maturity are determined using the simple algebraic equation:

$$S\,(T) = \text{Max}\,(V_T\text{-F};0) \qquad (4.11)$$

Hence V_E at time, t$\leq T$ can be valued through the Black-Scholes OPM to become:

$$S(V_t,\sigma) = V_t N(d_1) - F.\,e^{-r(T-t)}.N(d_1 - \sigma_A\sqrt{(T-t)}, \qquad (4.12)$$

where model parameters are defined as in equation 10 above.

The proposed KMV model for estimation of the equity of a bank is the extension of the above model for transaction costs, which is given by:

$$V_E = V_A e^{-(ROE-k_e)T} \mathrm{N}(d_1) - V_L e^{-(ROE-k_e)T} \mathrm{N}(d_2) = e^{-(ROE-k_e)T} [V_A \times \mathrm{N}(d_1) - V_L \times \mathrm{N}(d_2)] \quad (4.13)$$

where $d_1 = \frac{[\ln\left(\frac{V_A}{V_L}\right) + \left(ROE - k_e + \frac{\sigma_A^2}{2}\right)T]}{\sigma_A \sqrt{T}}$, and

$$d_2 = d_1 - \sigma_A \sqrt{T},$$

V_E= The market value of the bank's equity, V_A= The market value of the bank's assets, V_L = The market value of the bank's liabilities, T= The maturity of the bank's liabilities, $\sigma_A =$ The standard deviation of the bank's assets, $ROE =$ The return on equity and k_e= The transaction costs that is costs of ordinary equity of banks.

4.4.4 Challenges faced in derivation of the proposed model

Most commonly used structural models in bank valuations are based on precise or crisp assumptions, which are far from being realistic. Rigorous fuzzy extension principle estimations are employed to transform unobservable model asset values and volatilities parameters into fuzzy variables. Market friction and uncertainty are emerging concepts and hence membership functions and fuzzy theory are fairly presented based on the architecture of fuzzy systems by Ross (2010). We test variables for no or correlation using Hausman fixed and random tests. In the absence of correlations, both Hausman Fixed and Random Effect tests are consistent and the Fixed Effects alone would be inefficient (Fergusson, 2011 and Herring, 2005). However, in the presence of correlation between regressors and effects, the Fixed Effects test is consistent while the latter is inconsistent. Financial data for two of the eight banks had gaps for the period 2008-10 and we used time-series forecasting to generate scores for such gaps.

4.5 Validation of the Proposed Model, Results and Discussion

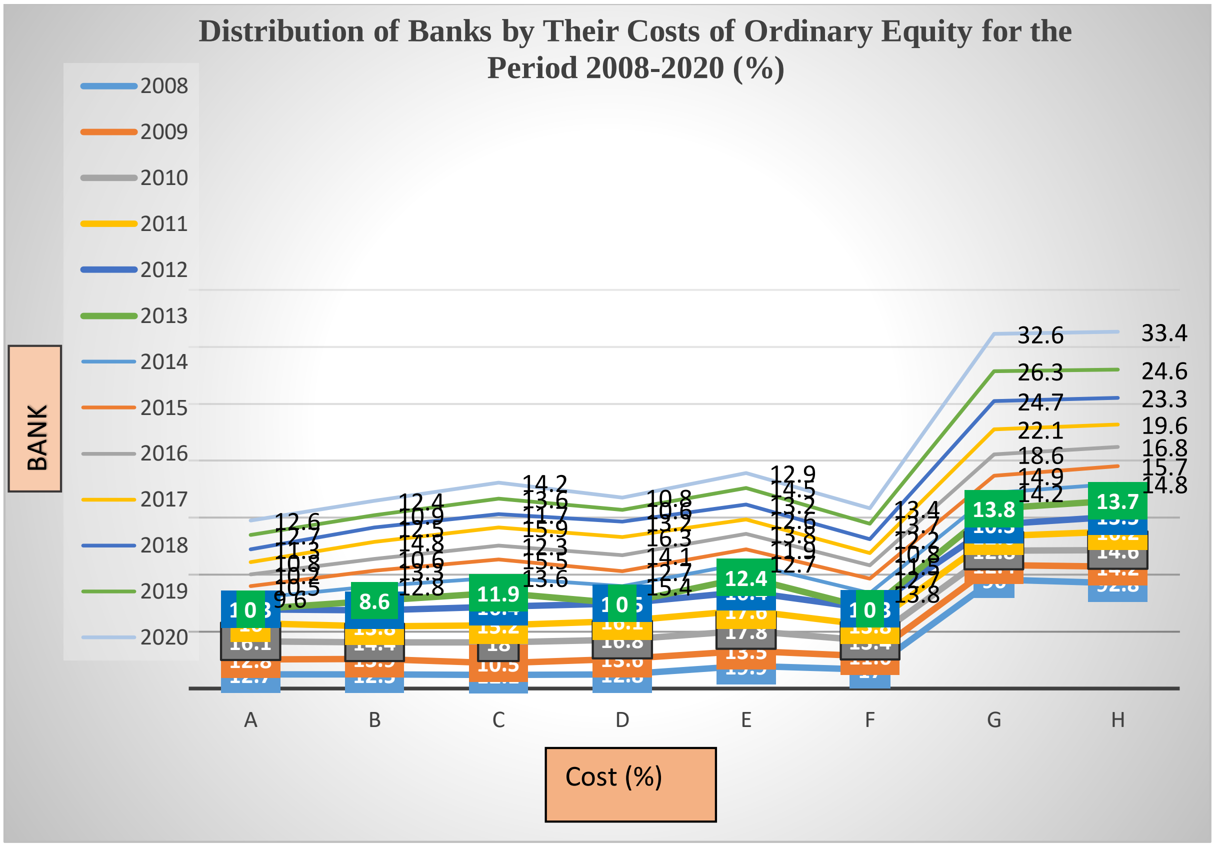
The study used panel financial data of eight banks conveniently drawn from seven economies of Southern Africa for validation of the proposed KMV equity valuation model in equation (13) above. The countries of the region from which banks used were drawn are South Africa, Zambia, Botswana, Namibia, Mauritius and Zimbabwe. This sample fairly represented well-performing, improved and constrained countries of the region. The results obtained using the proposed KMV model were then compared with those realized from the use of the structural model in equation (8) above. The table below summarizes the costs of equity of banks calculated from the raw panel data based on a time horizon of a year.

Table 4.2 Showing Distribution of Banks by Their Transaction Costs (Costs of Ordinary Equity) for 2008-2020 (%)

Year	08	09	10	11	12	13	14	15	16	17	18	19	20
A	12.7	12.8	16.1	16.0	12.3	8.7	9.6	10.5	10.2	10.8	11.3	12.7	12.6
B	12.5	13.9	14.4	13.8	14.2	11.6	12.8	13.3	10.6	14.8	12.5	10.9	12.4
C	12.1	10.5	18.0	15.2	16.4	11.9	13.6	15.5	12.3	15.9	11.7	13.6	14.2
D	12.8	13.6	16.8	16.1	15.5	14.3	15.4	12.7	14.1	16.3	13.2	10.6	10.8
E	19.9	13.5	17.8	17.6	16.4	12.4	12.7	11.9	13.8	12.6	13.2	14.5	12.9
F	17.0	11.6	13.4	13.8	14.3	14.1	13.8	12.7	11.5	10.8	12.2	13.7	13.4
G	96.0	12.4	12.8	12.6	10.3	13.8	14.2	14.9	18.6	22.1	24.7	26.3	32.6
H	92.8	14.2	14.6	16.2	13.5	13.7	14.8	15.7	16.8	19.6	23.3	24.6	33.4

Source: Authors

Figure 4.2 Showing Distribution of Banks by Their Costs of Equity for 2008-2020



Source: Author

Costs of equity for about 25% of the banks investigated (G and H) were fairly high and not easily sustainable. These high costs are attributed to the nature and forms of repressed or administered, shallow and frictional financial systems and markets in which the banks are established and operated. The study discovered that traditional market values of equity of banks were very low compared to their liabilities, and hence their exposure to high costs of capital. Hence, unlike the case for frictionless markets on which structural models are founded, banks in Southern Africa are over-exposed to lenders and very high costs of capital. All banks drawn into the study accumulated assets out of debt rendering them vulnerable to take-overs by such lenders. Banks' high costs of capital erode their ROEs leading to poor capital formation, high costs of capital and interest on debt and riskiness of their businesses.

4.5.1 Distribution of banks by their asset standard deviations for 2008-2020

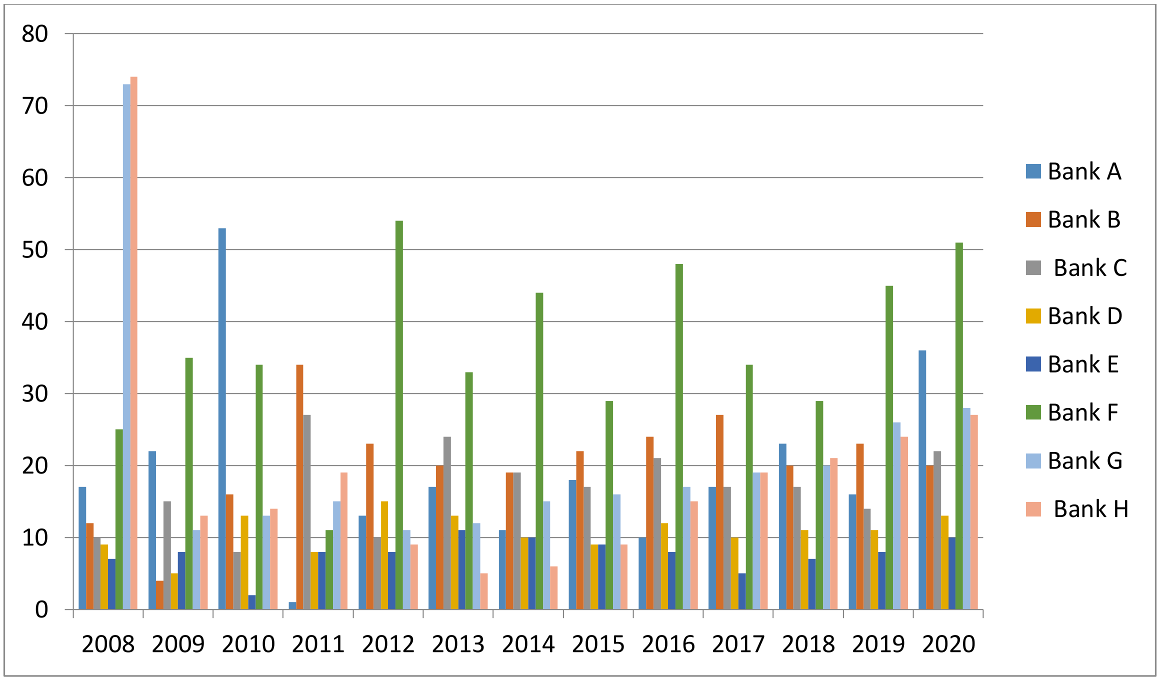
The research incorporated transaction costs and uncertainty (that is fuzziness) into the proposed equity valuation model before calculation of fuzzy standard deviations of asset values. We then proceeded to estimate both traditional and fuzzy standard deviations of the assets of the banks as tabulated below.

Table 4.3 Showing Banks' Traditional and Fuzzy Asset Standard Deviations for 2008-2020 (%)

Bank	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A-T	16.8	21.6	53.1	0.6	12.6	16.5	11.2	18.3	9.5	16.8	22.7	15.6	36.4
F	18.4	20.8	34.4	10.3	16.2	17.6	13.8	21.6	11.7	18.4	25.2	17.3	28.8
B - T	11.5	4.3	15.8	34.0	23.2	20.0	18.9	21.6	24.2	26.8	19.5	22.7	20.4
F	15.8	12.2	17.9	27.0	21.6	20.0	19.3	17.5	19.6	17.8	20.2	18.4	18.6
C-T	9.8	15.4	8.2	27.4	10.0	24.3	18.6	16.5	20.8	17.2	16.8	13.6	22.4
F	14.9	17.7	14.1	23.7	15.0	22.2	19.8	17.3	17.5	15.4	18.5	16.8	17.4
D-T	8.7	5.2	12.8	7.7	14.8	12.6	10.2	9.4	11.6	9.8	10.7	11.3	12.8
F	14.4	12.6	16.4	13.9	17.4	15.6	13.7	12.9	12.6	11.7	12.9	12.3	15.2
E-T	6.8	8.4	1.7	7.6	8.0	11.2	9.5	8.8	7.6	4.9	6.7	7.8	9.6
F	13.4	14.2	12.5	13.8	14.0	5.60	15.9	15.6	13.8	12.5	14.9	13.7	15.8
F-T	24.6	35.02	33.8	11.0	54.0	32.6	44.4	28.9	48.2	33.7	28.8	44.5	50.8
F	23.0	27.5	26.9	15.5	37.0	27.8	32.6	26.4	36.3	27.1	25.8	32.4	37.6
G-T	72.6	10.5	12.5	14.6	11.2	12.3	14.9	15.7	16.5	18.6	20.4	25.7	27.5
F	68.5	11.6	15.8	16.5	13.3	14.8	15.3	16.7	17.4	19.6	21.9	26.2	28.7
H-T	74.4	12.6	14.3	18.6	8.7	5.4	6.3	9.2	14.8	18.7	20.7	23.5	26.8
F	71.6	14.8	16.8	20.4	8.8	5.6	7.8	10.8	15.1	18.7	22.6	25.5	27.8

Source: Author

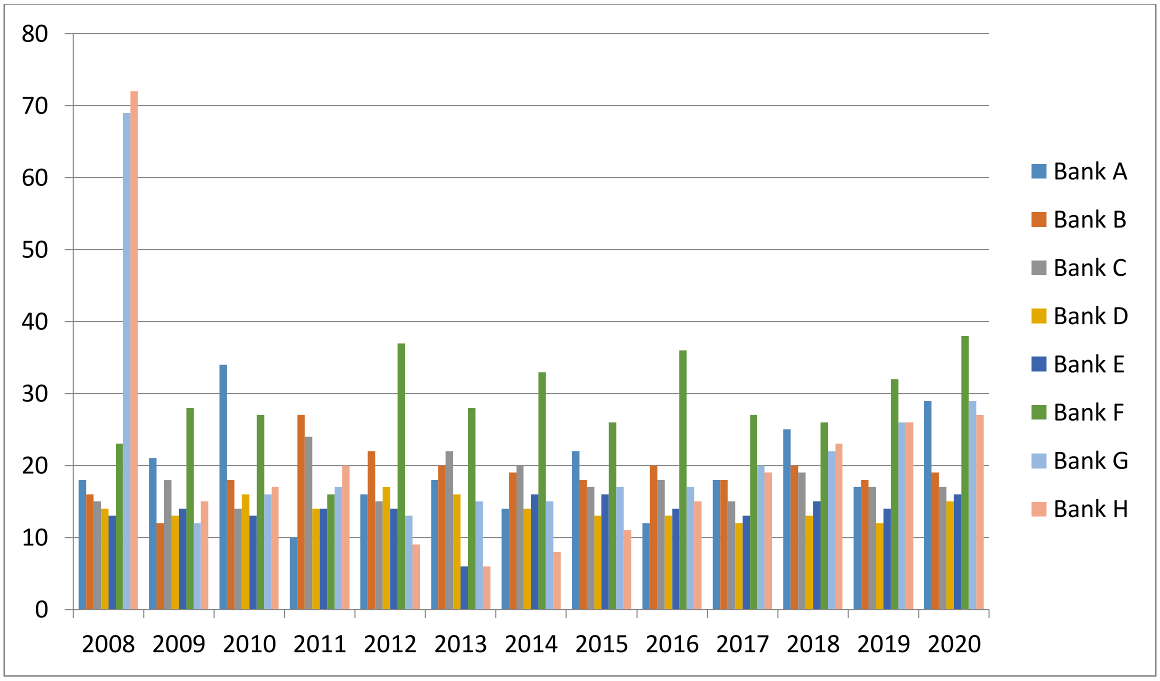
Figure 4.3 Showing Distribution of Banks by Their Traditional Asset Standard Deviations for 2008-2020 (%)



Source: Author

The study revealed that 37.5% of the banks (H, G and F) dominated all others in terms of high traditional asset volatilities, which were above 40% during the period investigated. Otherwise, 62.5% of the banks had fairly low traditional asset volatilities which ranged between 15 and 30%. The traditional asset volatilities of banks are compared with those realized from the proposed financial model extended for market friction and uncertainty (fuzziness). The asset volatilities calculated using the new look model are as illustrated by the component bar graph below.

Figure 4.4 Showing Distribution of Banks by Their Fuzzy Asset Standard Deviations for 2008-2020 (%)



Source: Author

Banks with high traditional asset volatilities also had highest asset volatilities in the range 20%-40% under the new look equity-financial model. The study realized that the estimated fuzzy asset standard deviations are less fluctuating than the traditional values. This implies that banks' asset and standard deviation values are fairly low and consistent or stable when calculated under uncertain market conditions. This means that uncertainty is pertinent factor that must be incorporated in modern day financial models when it comes to accurate estimation of asset and let alone equity values of banks.

4.5.2 Distribution of banks by their traditional ROEs for 2008-2020

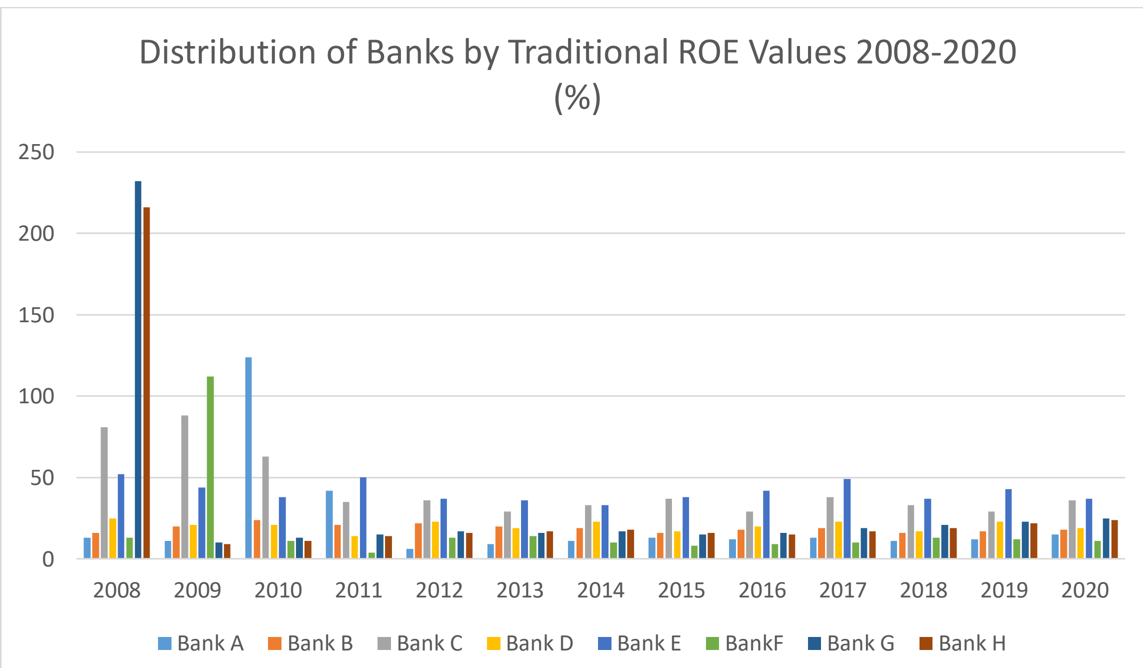
The study compares the effects of both traditional and fuzzy ROEs on the equity values of the banks. Both sets of estimated ROE values are tabulated by bank for the period 2008-2020.

Table 4.4 Showing Distribution of Banks by Their Traditional ROEs for the Period 2008-2020 (%)

Year	08	09	10	11	12	13	14	15	16	17	18	19	20
A-T	13.4	11.0	124.2	42.3	5.6	8.7	10.5	12.8	11.7	13.4	10.8	12.3	14.8
B - T	16.1	20.1	23.5	20.7	21.9	20.0	18.6	16.4	17.7	18.9	15.8	16.5	17.6
C-T	81.4	87.3	63.1	35.0	36.4	29.0	32.7	36.8	28.6	37.5	32.6	28.8	36.4
D-T	25.1	20.8	21.3	13.7	22.6	18.6	22.7	16.9	19.5	23.4	17.2	22.6	18.8
E-T	51.7	44.2	37.6	50.0	37.1	36.3	32.8	37.6	42.4	48.7	36.9	43.2	36.5
F-T	12.7	112.2	10.7	3.6	12.5	13.8	9.8	7.6	8.7	10.4	12.5	11.8	10.6
G-T	232.0	9.6	12.8	15.0	16.5	15.8	16.8	14.6	16.2	18.7	20.5	22.5	24.8
H-T	216	8.8	10.7	13.6	15.6	16.7	17.8	16.4	14.6	17.3	19.3	21.8	23.5

Source: Author

Figure 4.5 Showing Distribution of Banks by Their Traditional ROEs for 2008-2020 (%)



Source: Author

The study found that traditional ROEs of all banks were randomly distributed as they followed no defined pattern. However banks A, C, F, G and H, constituting 62.5% of banks investigated dominated others in the period 2008-10, in terms of traditional ROEs. These ROE results reveal that bank investors have high hopes or are optimistic in stable and boom economic conditions and the converse also holds for recession conditions.

4.5.3 Distribution of banks by their traditional and fuzzy equity-asset value ratios for 2008-2020 (%)

The study started by expressing equity values of banks as percentages of total assets to be able to compare their performances across the different emerging economies of Southern Africa. The study then used a STATA package to estimate both traditional and fuzzy equity values of the banks based on four independent variables namely annual average values of ROEs, costs of equity, asset values and their standard deviations. The table 1.5 below displays the Equity-Asset ratio results of the banks.

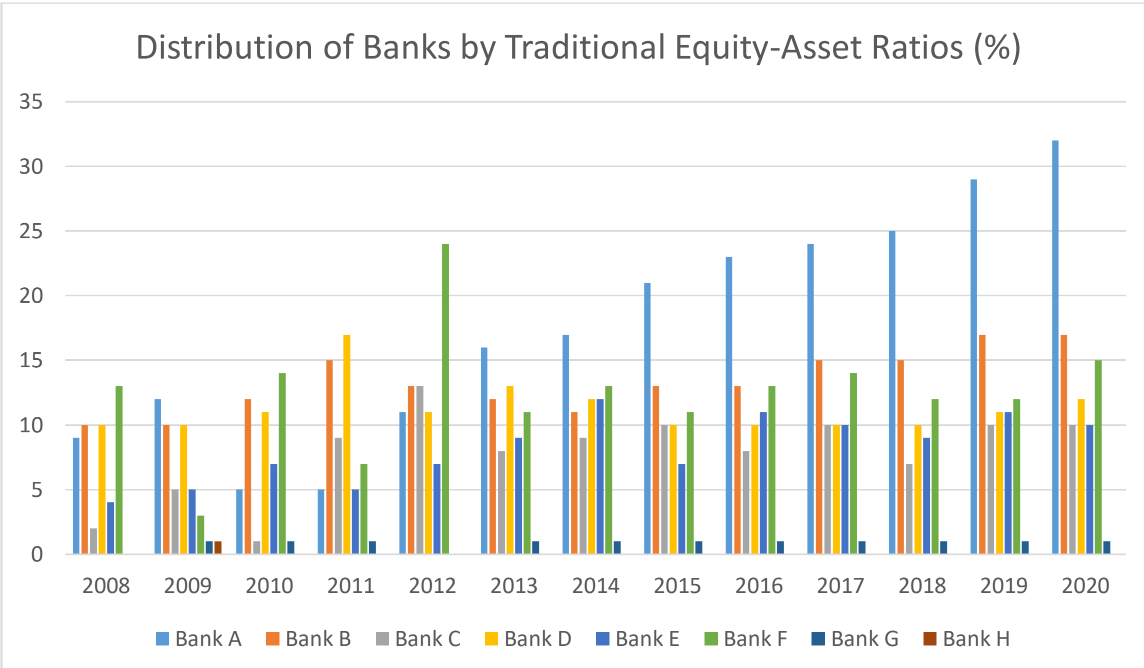
Table 4.5 Showing Distribution of Banks by Their Traditional and Fuzzy Equity –Asset Ratios for the Period, 2008-2020 (%)

Bank	Ratio	08	09	10	11	12	13	14	15	16	17	18	19	20
A-T	$(\frac{TE}{TA})\%$	9.0	11.6	5.3	5.3	11.2	15.5	16.7	20.7	23.4	23.6	25.0	28.9	31.7
F	$(\frac{FE}{TA})\%$	6.8	4.8	0.5	3.7	7.2	6.8	6.4	7.8	7.5	6.6	7.6	8.5	7.8
B -T	$(\frac{TE}{TA})\%$	10.0	9.7	11.8	14.8	12.5	11.9	11.4	12.7	12.8	15.1	15.4	17.3	17.3
F	$(\frac{FE}{TA})\%$	4.2	5.2	4.3	5.4	5.0	7.3	4.6	5.4	4.9	6.7	5.8	6.4	6.3
C-T	$(\frac{TE}{TA})\%$	2.3	5.4	0.9	9.1	12.8	7.7	9.2	10.3	7.8	9.8	7.1	9.5	10.3
F	$(\frac{FE}{TA})\%$	0.3	0.1	0.9	1.6	1.5	2.0	2.8	2.2	3.5	4.6	3.2	4.3	4.8
D-T	$(\frac{TE}{TA})\%$	9.5	10.1	10.9	17.2	10.6	13.3	12.4	9.6	9.5	10.2	10.1	11.4	12.3
F	$(\frac{FE}{TA})\%$	8.6	5.0	4.7	11.2	3.9	4.2	4.8	7.6	6.4	4.1	4.3	7.2	5.7
E-T	$(\frac{TE}{TA})\%$	3.8	4.8	6.5	5.0	7.4	8.5	11.8	7.4	11.0	9.9	9.4	10.8	10.3
F	$(\frac{FE}{TA})\%$	1.6	0.9	1.7	1.3	1.2	1.8	2.5	1.4	2.3	1.9	2.4	3.2	2.8
F-T	$(\frac{TE}{TA})\%$	12.8	2.5	14.4	7.1	23.6	11.3	12.5	10.6	13.4	13.9	11.6	12.2	15.3
F	$(\frac{FE}{TA})\%$	8.0	0.10	0.10	8.8	26.9	9.3	8.6	12.4	0.80	0.72	8.8	7.5	10.7
G-T	$(\frac{TE}{TA})\%$	0.38	0.64	0.74	0.63	0.40	0.66	0.61	0.65	0.63	0.61	0.61	0.62	0.58
F	$(\frac{FE}{TA})\%$	0.32	0.60	0.68	0.58	0.36	0.62	0.55	0.58	0.58	0.55	0.55	0.56	0.52
H-T	$(\frac{TE}{TA})\%$	0.46	0.60	0.37	0.23	0.21	0.20	0.17	0.17	0.17	0.15	0.15	0.13	0.12
F	$(\frac{FE}{TA})\%$	0.42	0.56	0.33	0.18	0.17	0.16	0.16	0.16	0.16	0.14	0.14	0.12	0.11

Source: Author

The above traditional and fuzzy equity values of the banking firms were graphed using Excel to give the illustration below.

Figure 4.6 Showing Distribution of Banks by Their Traditional and Fuzzy Equity-Asset Ratio Values (%) for 2008-2020

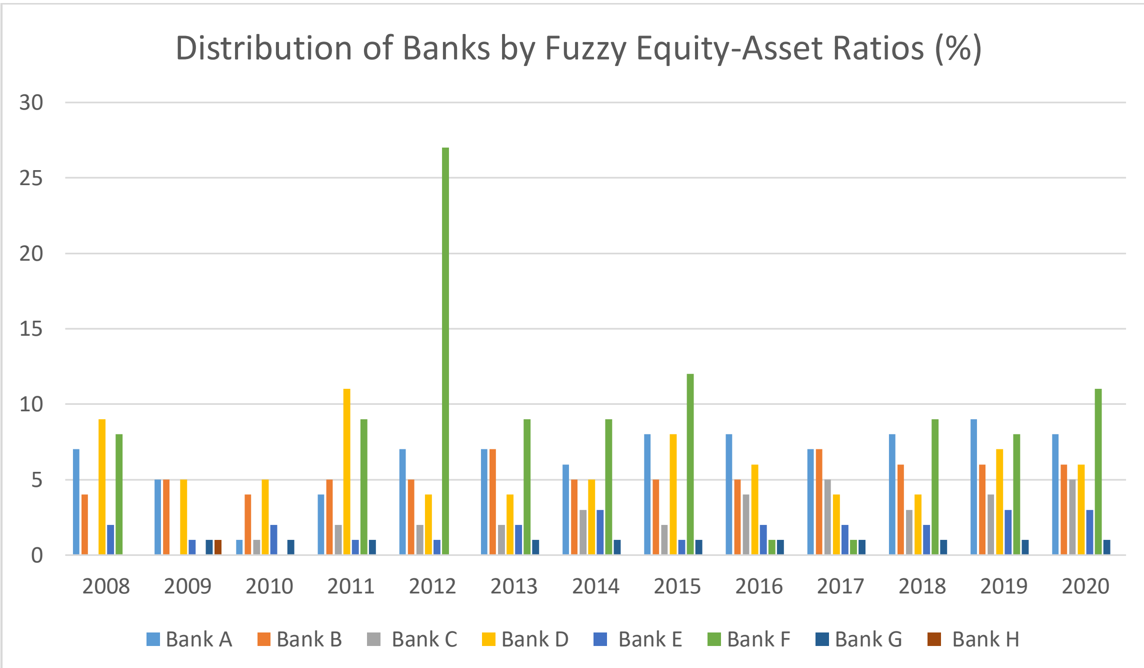


Source: Author

The research discovered that all banks had very low equity-asset ratio values especially in the period 2008-11, which later improved from 2012-20 for banks such as A, B, E and F. This development could be attributed to the impact of the global financial crisis on financial systems of the world (2007-2009). Conversely the above finding implies that overall all banks used in the study are poorly capitalized measured in terms of debt-equity market value ratios of their traditional values. The financial of about 75-80% of all banks' capital bases through debt is a sad development which shifts the ownership of the banks from ordinary shareholders to the lenders. Despite all the banks being over-borrowed or owned by foreigners all the capital generated was used effectively as it was translated into assets for use in generating income for the businesses. The banks' traditional equity values were directly and indirectly related to their asset and liability values respectively. The study revealed that transaction costs or market friction had a strong negative impact on traditional equity-asset ratios of banks which overall were very low across all banks investigated. These traditional ratios of banks also postulated positive and negative relationships with ROEs and costs of ordinary equity respectively.

The graph below represents the distribution of all banks by their fuzzy equity-asset ratios to make them comparable.

Figure 4.7 Showing Distribution of Banks by Their Fuzzy Equity-Asset Ratios for the Period 2008-2020



Source: Author

Fuzzy and traditional equity-asset values or ratios were different across all banks investigated that is fuzzy equity-asset ratios are lower than traditional ratios for all banks. On the other hand, transaction costs and uncertainty or fuzziness drawn into the proposed equity-valuation model had a strong negative relationship with equity values of banks. Banks' fuzzy equity values showed direct and inverse relationships with ROEs and costs of ordinary equity respectively.

4.6 Conclusions and Recommendations

The study proposed and analysed the valuation of equity of banks based on a model combining transaction costs and uncertainty arising from the random evolution of asset prices, imprecision and vagueness. The model was proposed after noting that financial markets in which banks operate are far from being efficient and perfect. Based on the research findings above the study concludes that banks in emerging economies are heavily geared and characterized by high debt-equity ratios. The study also concludes that these banks are poorly capitalized, regulated and supervised, over-borrowed or foreign owned. The future of banks lies in their ability to negotiate for turning debt into equity.

The study also concludes that transaction costs and expert judgments are pertinent and hence need to be added to contemporary models to attain fair valuations. Both sets of equity values are directly related to ROE values, and inversely related to market friction and total costs of equity. Overall the study concludes that traditional AVMs are not suitable for application in frictional and fuzzy financial markets. Based on the above conclusions, the study recommends that transaction costs and uncertainty must be adjusted for in existing financial models to make them more rigorous, reliable and precise in estimation of bank values. The study overall recommends that the proposed model can be adopted banks in frictional and fuzzy emerging markets in order to come up with fair valuation of their equities and financial performances.

CHAPTER V

FUZZY STRUCTURAL RISK OF DEFAULT MODEL FOR BANKS IN SOUTHERN AFRICA

5.1 Introduction

The Merton (1974) structural probability of default (PD) model revolutionised research and practice in credit risk modelling in banking firms, and over the years academic researchers and practitioners have extended Merton's theory in various directions. For instance, structural credit risk models are based on the assumption that the non-stationary structure of the loan obligation leads to the termination of operations on a fixed date and default can only happen on a specified date. Geske and Delianedis (2003) extend the Merton asset valuation model (AVM) to the case of the valuation of tradable bonds with different terms to maturity. Their study concludes that it is incorrect to assume that the firm's value of assets can be traded in financial markets. In reality, the value of the assets of a firm and its related parameters may not even be directly observed as articulated in most structural valuation models. This is one of the major drawbacks of structural credit risk models (CRMs) such as the Merton (1974) and Black-Scholes (1973) models that new models must seek to address.

Geske and Delianedis (2003) conclude that interest rates which are assumed to be fixed in the traditional PD model are random or stochastic. They go further to conclude that the yield spread curve in calibrated versions of the Merton model remains essentially zero for months, contrary to the observations made in real financial markets. On the one hand, structural models for the valuation of banks are based on the assumption that financial markets are perfect and frictionless. However, in practice, these markets are characterised by some degrees of friction and uncertainty such as fuzziness, vagueness, and ambiguity (Zimmermann, 1980, Zadeh, 1965). Therefore the study proposes a KMV risk of default model extended for transaction costs for estimation of the risk of default in banks that operate in fuzzy financial markets.

Background to the study

Banks worldwide operate in financial markets characterised by some degrees of uncertainty, particularly in the forms of friction and fuzziness. Zimmermann (1980) and Zadeh (1965) demonstrate that most contemporary CRMs are suitable for the estimation of PDs of banks where transaction costs are negligible, minimal, or equal to zero. In reality, financial investors often subjectively express the uncertainty they face with some degree of implicit fuzziness also known as impreciseness (Zebda, 1989). For instance, investors may express market costs or returns as average, high or low, which terms are not explicit but implicit numbers. According to Zebda (1989), Zimmermann (1980) and Zadeh (1965) the investors' use of operational language or semantics in market valuations is intrinsically imprecise contrary to some of the assumptions underlying most structural CRMs alluded to above.

However, the prerequisites of structural models such as Merton (1974) and Black-Scholes (1973) are that probability used in decision analysis is a 'precise' number determined from repeated samples and relative frequency distributions (Kolmogorov, 1933). Kolmogorov's classical probability theory differs significantly from fuzzy theory which is derived from the 'degree of belief' set by financial experts and investors. Therefore, from the above observation, it becomes difficult to apply structural models to the valuations of risk metrics of banks under conditions of uncertainty in fuzzy decision-making processes (Bellman and Zadeh, 1970). In other words, we can conclude that investors use both probabilistic and fuzzy valuation tools to characterise the uncertainty that is inherent in financial market operations.

Problem statement

Contemporary literature has shown that traditional structural models such as the Merton (1974) are the cornerstones of all structural CRMs but are criticised for being constrained by their nature of not applying to frictional and fuzzy financial environments and underscoring the risk of default in good or boom economic times. Although some contemporary structural CRMs have been adjusted for fuzziness, especially in option pricing, none of them have been extended for market friction (Tsunga et al, 2021, Obeng et al, 2021). Hence this research is motivated by the need to incorporate market friction and fuzziness in the valuation of the risk of default of banks to fairly reflect on practical conditions faced by investors. Banks in economies in Southern Africa are used because this is a region characterised by unique economic, business, and financial environments compared to other continents such as Europe and America.

Aim and Objectives of the study

This study aims to improve the ability of structural models in the estimation of risk metrics concerning the modelling of the risk of default of banks. The objectives of the research study are to:

.Examine the impact of a KMV risk metrics model extended for both friction and fuzziness on the risks of default of banks in emerging financial markets.

.Validate the proposed KMV risk of default model using financial data drawn from banks in emerging markets of countries in Southern Africa.

.Compare the results estimated using the KMV risk of default model of banks with those generated from both hazard function and structural risk models.

Significance of the study

Although structural CRMs such as Merton (1974) and Black-Scholes (1973) are the benchmarks on which all bank and other firm valuations are based, they are usually criticised for being founded on unrealistic assumptions such as constant risk-free rates of return and frictionless markets. In practice, banks operate in frictional and fuzzy financial markets, contrary to the assumptions on which most structural CRMs are applied. Therefore by extending contemporary structural models for market friction and fuzziness banks can attain precision in the estimation of their risk metrics (Palma and Ochoa, 2013).

Research hypothesis

The study is carried out under the following hypothesis:

Null hypothesis (H0): Market friction and fuzziness have no effects on the DPs of banks in emerging economies.

Scope of the study

The study is carried out in Southern African countries mainly characterised by frictional and fuzzy financial markets. The study extends the existing structural models for market friction and fuzziness which are practical factors investors face in the valuation of banks and their risk metrics such as the risk of default (Tsunga et al, 2021, Obeng et al, 2021).

Organisation of the study

The paper is divided into five sections that is the introduction above is Section 1 and Section 2 reviews literature on structural PD models, transaction costs, fuzzy set theory, and its applications. Section 3 proposes a new look KMV framework, Section 4 validates the model using financial data of eight banks from countries in Southern Africa and conclusions and recommendations of the study are then presented in Section 5 of the paper.

5.2 Literature Review

This section examines the various theoretical frameworks underlying credit risk modelling with specific reference to the use of the hazard function and structural models in the estimation of the distances-to-default (DTDs) needed in the valuation of risk metrics of banks such as the risk of default. The evolution of CRMs starts by presenting the hazard function model by Cox (1972) before examining structural models such as Black-Scholes (1973) and Merton (1974) models and other relevant contemporary models on which the study leans.

5.2.1 Cox (1972)'s semi-parametric hazard function model and risk of default

The Cox proportional hazards model (Cox, 1972) is mainly a regression model that is commonly used in medical science research for modelling the association between patients' survival data or times and one or more predictor or explanatory variables. This study starts by noting that there are two main classes of models that can be applied to credit risk modelling which are structural and reduced-form models. Structural models are used to calculate the probability of default for a form or bank based on its values of assets and liabilities. A firm defaults if its market value of assets falls below the debt obligations it has to pay or settle. A firm defaults if the market value of its assets is less than the debt it has to pay. The hazard rate function in the context of credit risk modelling is defined as the rate of default calculated at any time, assuming that the borrower has survived up to that point in time (Boland, Neweihi, and Proschan, 2016). The other name for the hazard rate is the marginal default probability (MDP) which is different from classical probability and survival analysis in credit risk modelling. Survival analysis has been introduced into credit scoring or rating in recent years. It is an area of statistics that deals with the analysis of lifetime data, where the variable of interest is the time of the event.

The difference between hazard and survival functions is that a hazard function focuses on failing or an event occurring while a survival function focuses on an event not failing (Sawadogo, 2014). Therefore in some sense, a hazard function can be perceived as the converse side of the information provided by a survival function. The survival analysis model attempts to estimate the survival probability over the entire data set given. By comparison, risk represents probability while hazard represents settings, situations, or physical objects or phenomena. Risk can be expressed in degrees whereas hazards cannot be expressed in degrees. Hazard rates unlike classical probability theory can value values greater than 1 and technically cannot be a probability value. The hazard function can be interpreted as the conditional probability of the failure of a device at age, X, given that it did not fail before the attainment of age X (Mendes, 2014). In other words, the interpretation and boundedness of a discrete hazard rate are different from those under the continuous probability case, such as the normal distributions. In reality, there are four main groups of work hazards faced namely chemical, ergonomic, physical, and psychosocial, which can cause harm or adverse effects to humans in the workplace.

Hazards must be identified first before a company undertakes a risk assessment, which implies that these two variables are different (Boland, Neweihi, and Proschan, 2016). In principle, a hazard is anything that could cause harm to a person or environment while the risk is a combination of two things which are the chance that the hazard will cause harm and the seriousness that the harm would cause. The hazard rate is sometimes referred to as the failure rate which is a rate that only applies to items or objects that cannot be repaired (Sawadogo, 2018). It is therefore fundamental to the design of safe systems in organisational applications and is often relied on in disciplines such as engineering, insurance, economics, finance, and banking and regulatory industries. The Cox (1972) model is presented in this section briefly to pave way for comparing results from its application in the study to those obtained from the traditional structural model and the proposed KMV risk of the default model.

5.2.2 Structural models for valuation of risk metrics of banking firms

Although the history of structural models backdates to before the Black-Scholes (1973) option pricing model (OPM), the models of management of credit portfolios are pioneered by Merton (1974). These models are then, developed further by Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996), and Jarrow (2011). Although the Merton (1974) model is the benchmark on which all credit risk models are based it is said to be limited in that it is only applicable when asset values and volatilities of firms are not directly observable. Secondary parameters of the Merton model such as μ and σ (drift and asset volatility respectively) are unknown and should be estimated from observed model parameters such as equity values and volatilities. Several estimation methods for the two parameters are applied in economics, business, banking, and finance but not enough attention has been given to their theoretical and empirical shortcomings.

New models for the valuation of risk metrics of banks have been introduced in a somewhat unique liability structure in financial practice. Hence emphasis needs to be laid on KMV methods and maximum-likelihood approaches to the valuation of risk metrics of banks. The KMV approach makes the Credit Analytics Services (CAS) offered by Moody's KMV rigorous as detailed in the research by Crosbie and Bohn (2003). The two researchers start by describing KMV as a σ restriction method without referring to Ronn and Verma's (1986) model. However, they later take it as an iterative approach made up of the following steps:

.Application of σ to obtain a time series of implied asset values and hence compound asset returns continuously.

.Use of time series of continuously compounded asset returns to obtain updated estimates for the unknowns, μ, and σ.

.Not going back to the initial stage with the updated σ values unless convergence has been attained.

The KMV approach uses fixed maturity at one year and sets a default point to the sum of the short-term and half of the long-term debt obligations. According to Crosbie and Bohn (2003), the KMV suggests that firm defaults when its asset value falls somewhere between short-term default and the value of the total liabilities. The paper by these two uses KMV based on daily company market capitalisation together with quarterly updated debt levels for three banks to obtain the estimates needed. The study concludes that the KMV approach is an improvement in the distance-to-default (DTD) estimation models although it has its shortcomings. For instance, the KMV depends on the implied asset values and hence cannot be used to obtain unknown parameters in the capital structure of a firm. It also does not provide clear information on statistical estimations based on the Merton (1974) model. Bharath and Shumway (2008) argue that since Merton's DTD model is not an econometric model it is not clear how its parameters can be estimated using alternative techniques. Furthermore, it has been observed as unclear how standard errors for data forecasts can be calculated for the Merton DTD model whose application is premised on inaccurate or unrealistic assumptions.

Huang and Huang (2003) believe that there is no consensus from existing credit risk literature on how much observed corporate yield spreads can be explained by credit risk approaches. Their research concludes that using calibrated historical data, it is possible to obtain consistent estimates across credit spreads across all economic considerations within the credit risk structural frameworks. However, it has also been realised that credit risk explains just a small fraction of observed investment bond returns of all maturities together with a smaller fraction of short-term and higher fractions for junk bonds (Duan and Huang, 2012). Different structural DP models which generate high credit spreads are good in predicting fairly similar spreads under empirically reasonable choices of parameters, leading to the robustness of conclusions of the results of the paper by Huang and Huang (2003).

Bharath and Shumway (2008) examine the accounting and contributions of Merton's (1974)'s DTD bond pricing model in comparison with the naive Merton model which cannot be applied in solving the implied probability of default (PD) model. Their research reveals that forecasting variables and naïve predictor models are better than hazard function models for in and out-of-sample forecasts than both Merton DTD and reduced-form models that are based on sample inputs or variables. Fitted values drawn from expanded hazard models outperform Merton DTD-PD out of sample models. On the other hand implied DTDs from credit default swaps (CDS) and corporate bond spreads are said to be weakly correlated with Merton DTD probabilities after adjusting for agency ratings and bond characteristics (Masatoshi and Hiroshi, 2009). The Merton DTD model is however criticised for not producing a sufficient statistical model for estimation of DPs of banks, making its functional form remain only useful for forecasting default events.

5.2.3 Estimation of DTDs for banking, financial and non-financial firms

Duan and Huang (2012) investigate several empirical applications to the estimation of DTD of banks. The DTD measures the extent to which a limited–liability corporation is away from a default event. Conceptually a firm's asset values evolve according to a stochastic dynamic process where a debt (D) contract is honoured when the value of assets, A is greater than the promised payment to be settled in the future. Otherwise, if the above condition does not hold, a firm's debt level, D and its debt holders can only recover a partial amount of what is left of the firm. When the current value of a company is much higher than its promised future obligations, the likelihood of the default event is small (Duan and Huang, 2012, Bharath and Shumway, 2008). This is because the firm has enough buffers to absorb losses in its asset values based on its corporate financial leverage (the debt-to-asset ratio) and the converse also holds. Since asset values move randomly due to external shocks, leverage ratios may not be good enough to fairly and adequately capture the notion of the DTD of a company. Some of the methods proposed in the literature used to estimate the unknown model parameters are:

.The volatility restriction method by Jones, Mason, and Rosenfeld (1984) and Rom and Verma (1996).
.The transformation-data maximum likelihood technique by Duan (1994; 2000).
.The KMV iterative method described in Crosbie and Bohn (2003) and
.The market value proxy method is used in Brockman and Turtle (2003) and Eom, Hellweg, and Huang (2004).

The model by Crosbie and Bohn (2003) for instance uses financial firms to illustrate the shortcomings of the KMV-DP estimation model. The study concludes that financial firms have large proportions of liabilities for instance policy obligations of insurance firms which cannot be accounted for by the KMV approach. The maximum-likelihood models by Duan (1994) are modified by Duan et al (2012) and Duan (2000) to deal with financial firms and are very appropriate and flexible techniques for use in the estimation of DTD. According to Duan et al (2012), the CRMs' application of DTD makes it very unrealistic because its strict applications are at odds with empirical discount rates. Most financial academics argue that the DTD approach is highly informative about defaults but must be applied together with other variables to achieve good bank financial performance. According to Duan (2000) calibration through reduced-form models such as logit and logistic regression analyses should be a must take in all bank financial practices. However, it is ironic to argue that the DTD as a structural credit risk model (CRM) must be calibrated by a reduced-form model to give rise to good, precise, or accurate results. Based on the Merton (1974) model it is assumed that firms are financed by equity, E with its value at the time, t, denoted by S_t, and one single pure discount bond (denoted by, D_t) with a maturity date, T and principal, F.

The asset value of the firm, V_t is assumed to follow a geometric Brownian motion (GBM) given by the equation:

$$dV_t = \mu_A V_t(dt) + \sigma_A V_t W_t \tag{5.1}$$

where V_t =A standard Brownian motion, σ_A= The volatility of assets, μ_A = The drift, and dW_t = The Wiener process.

Due to banks' operations under limited liability, the value of equity, V_E at maturity is given by:

$$S\,(T) = \text{Max}\,(VA,_T\text{-F; 0}). \tag{5.2}$$

Hence V_E at t$\leq T$ can be valued through the Black-Scholes OPM to become:

$$S\,(VA_t, \sigma_A) = VA_t N(d_1) - e^{-r(T-t)}.F \times N(d_1 - \sigma_A\sqrt{(T-t)}). \tag{5.3}$$

where,

$$d_1 = \frac{ln(\frac{VA(t)}{F} + (r + \frac{\sigma^2}{2})\ (T-t)}{\sigma_A\sqrt{(T-t)}}. \tag{5.4}$$

S =The stock price, V_A = The market value of assets, σ_A = The asset volatility, r =The instantaneous risk-free rate of return, F =The strike price of a call option, T =The term to maturity, t= Time today, N(d_1) and N(d_2) = The cumulative probabilities of the Z-values,d_1, and d_2 respectively.

Due to the nature of diffusion models, μ cannot be estimated with high precision using frequency data over several years. This is true in financial econometrics because μ is accompanied by a time factor, d_t whereas σ is a time factor of $\sqrt{d_t}$, as implied in dWt. The frequency of data sampled is known for being less informative about μ than σ because $d_t < \sqrt{d_{t.}}$ (Duan, 2020). When the value of d_t is small, we can avoid the use of μ in DTD estimation particularly when this measure is used as an input in a reduced-form model that needs to be calibrated. Hence the need to reduce sampling errors through the use of the alternative DTD formula given by:

$$DTD_i^* = \frac{ln(\frac{Vt}{F})}{\sigma\sqrt{T-t}}. \quad (5.5)$$

In this respect, DTD_i^* amounts to setting, $\mu = \frac{\sigma^2}{2}$ in Equation 4, making DTD_i^* to be more stable than the traditional DTD.

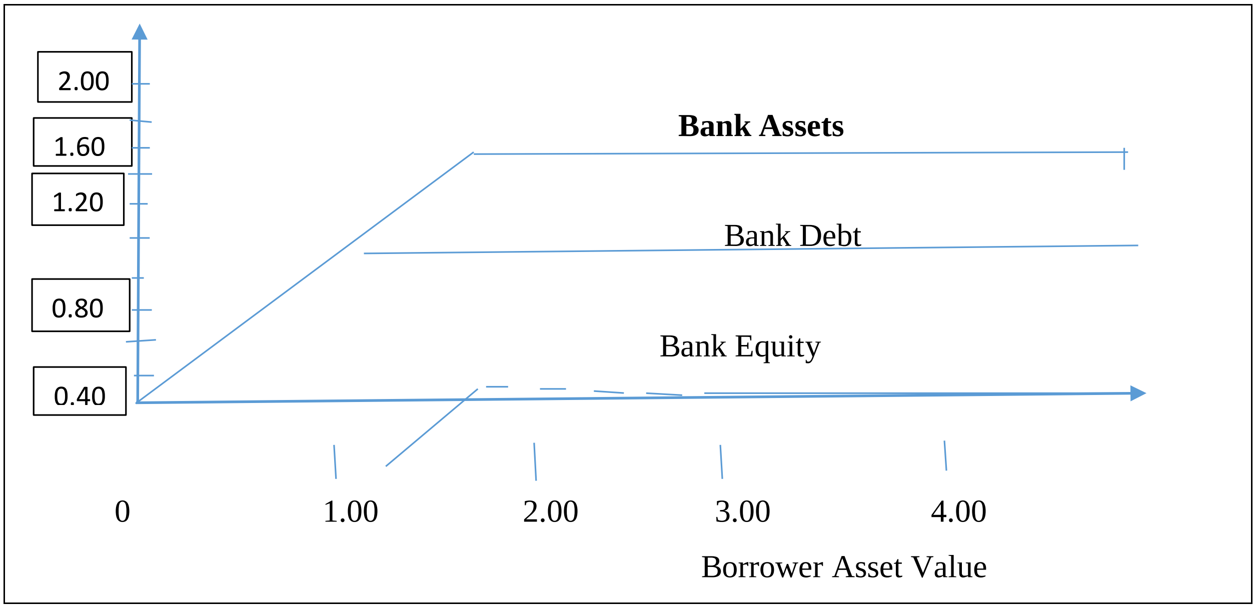
5.2.4 Classical models for valuation of banks' probabilities of default (PDs)

Nagel and Purnanandam (2020) point out that the distress faced by banks during the 2007-08 global financial crisis brought an urgent need for understanding and effective modelling of their PDs. Assessment of default risks for banks is fundamental to investors, risk managers, and regulators when it comes to the measurement of bank performances and failures. In all these applications investors and risk analysts rely on structural models of default risk where equity (E) and debt (D) values are seen as contingent claims on the assets owned by the firms. For example, Merton's (1974) standard approach or model assumes that the value of assets of a firm follows a log-normal process that is options embedded in the firm's E and D values which can be valued using the Black-Scholes (1973) model.

Extensive financial literature employs the Merton (1974) model to price deposit insurance in particular (Rennacchi, 1988, Roun and Verma, 1986, Marcus and Shaked, 1984). It is argued that Merton's log-normally distributed asset values may provide some useful approximation for the asset value process of non-financial firms. However, this assumption is very problematic for banks that include debt claims such as mortgages whose upside payoffs are limited contrary to the requirements of the log-normal distribution which postulates that the upside is unlimited. The model by Nagel and Purnanandam (2020) uses capped upside of a bank's assets. The study uses the log-normal distribution assumption on the assets of borrowers of a bank as collateral security. The bank's assets are modelled as a pool of zero-coupon bonds whose repayments depend on the value of the borrowers' collateral assets at loan maturity as postulated in Vasicek (1991).

The model by Nagel and Purnanandam (2020) also stipulates that loans issued have variable maturities which means that every time a fraction of a loan portfolio matures, the bank issues repayment proceeds as a set of new loans at a fixed initial loan-to-value ratio. If a loan becomes delinquent, the pledged collateral is replenished to the extent that the initial loan-to-value ratio is then satisfied. Lastly according to Nagel and Purnanandam (2020) and Vasicek (1991) the bank's assets, A, E and D are assumed to be contingent claims on the borrowers' collateral assets. According to the study, all loans issued have values of 1.60 and debt has a face value of 1.20. This implies that the maximum that the bank can recover in the event of default is the loan value (1.60). This is capped when the borrower's assets value falls below 1.60 because the assets of the bank are sensitive to the borrower's asset values as demonstrated below.

Figure 5.1 Showing Assets of a Bank Sensitised by Those of the Borrower



Source: Author

It can be observed that the value of the bank's assets cannot have a log-normal distribution. This is because its borrowers maintain the upside of their assets' values above the face value of the loans. The bank's equity payoffs do not resemble a call option written on an asset with an unlimited upside, but rather a mezzanine claim comprising two kinks. These kinks have implications for the risk dynamics of the equity of a bank and the estimation of its risk of default (Nagel and Purnanandam, 2020). Under a capped upside, the volatility of a bank's assets will be very low in good economic times such as booms when asset values are high. At maturity, asset values will end up running parallel to the right where banks' equity payoffs are insensitive to variabilities in the borrower's asset values. The standard Merton (1974) model in which the equity of a firm is assumed to be a call option on an asset with significant variability, will miss all these non-linear risk dynamics. In times of high asset values, banks may show many asset volatilities away from default, a conclusion that could be very misleading. This is because it ignores the fact that bank asset volatilities may dramatically rise if the values of its assets fall. Merton's standard model could end up making misleading predictions about the riskiness of the values of the bank's assets, debt, and equity.

5.2.5 Option pricing models (OPMs), uncertainty and estimation of PDs of banks

Nowadays many phenomena in economics, business, banking, and finance are uncertain or fuzzy but are treated as if they were crisp or precise in all decisions of banking firms and similar financial institutions. However in practice prediction of both business and consumer bankruptcy is imprecise and ambiguous (Korol, 2008). The financial performance and PD modelling of banks are affected by both internal and external factors that cannot be precisely and unambiguously defined. The mere allegation that a company is at risk of bankruptcy must not be considered imprecise. In economic reality, there are more firms or persons than can be considered 100% bankrupt. It is difficult to accurately determine the degree of bankruptcy of a bank using structural statistical methods such as multivariate discriminant analysis (MDA). Fuzziness is defined by Zimmermann (2001) and Zadeh (1980) as a market condition in which returns to financial market investments are not precisely defined as expressed in probability theory but in linguistic terms such as high, average, or low. With the use of fuzzy logic and vague and ambiguous concepts, banks' risk metrics can be defined as "high" or "low" risk of bankruptcy (Korol, 2008, Zimmermann, 1980).

The precisions on which structural CRMs are based are influenced by deduction and thinking processes that use a non-binary logic with fuzziness. The Merton (1974) and Black-Scholes (1973) models for instance do not put fuzziness into consideration when dealing with all the aforementioned financial problems. Banks' CRMs such as the one by Appadoo and Thavanaswaran (2013) are structural valuation models that involve uncertainty that arises from lack of knowledge, inherent vagueness, or imprecision. Of late there has been growing interest by researchers to use fuzzy numbers to deal with vagueness and imprecision facing investors in financial markets (Appadoo et al, 2006). More researchers such as Yu, Sun, and Chen (2011) and Cherubini (1997) deal with randomness in European OPMs. These models are extended to the case for pricing corporate debt contracts and provide a fuzzified version of the Black-Scholes (1973) model using a variety of input variables. Results realised from the application of their models have improved estimation precision, robustness, and consistency in the pricing of options in fuzzy financial markets.

Ghaziri et al (2000) use an artificial intelligence approach to the pricing of options using neural networks and fuzzy logic and compare their results to those obtained using the Black-Scholes stock indices. The study concludes that the Black-Scholes OPM is an approximation model which leads to a considerable number of errors. Trenev (2001) comes up with a refined option pricing formula and discovers that because of the fluctuating nature of financial markets over time, some of the parameters of the Black-Scholes model may not be expected in the precise or exact sense. Thavaneswaran et al (2008) apply fuzziness to the Black-Scholes OPM and conclude that it is far from being practical. Although the Black-Scholes OPM is flexible and can be applied to the valuation of banks and their risk metrics, it is criticised for being premised on precise variables such as frictionless markets.

Chang and Huang (2020) apply a fuzzy credit risk assessment model in banks and conclude that financial institutions need to regularly update their assessment models to maintain correct assessment results. They note that every update involves a lot of numerical experiments using multiple systems or software packages for the evaluation of the effects of different sampling techniques and classifiers to construct suitable models for the updated data sets. Chang and Huang (2020) used the latest web-based technology to develop a fuzzy decision support system (DSS) that used logistic regression as the classifier combined with various sampling methods and model threshold settings to come up with a model fitting process that is more structured and efficient. Therefore by fuzzifying the independent variables of the proposed DP model, the study intends to make the new model efficient and accurate. A study by Zhao and Dian (2017) argues that fuzzy credit risk models must be premised on stable conditions and effectiveness to reduce the conservativeness of the imperfect circumstances found in the real markets. On the other hand, Sakai and Kubo (2011) argue that institutional investors attain estimation rigour by assessing credit risk by using a combination of quantitative information for example OPMs and qualitative assessments.

Although structural OPMs can be easily constructed, they are mainly suitable for the assessment of long-term credit risk where the probability of bankruptcy varies directly with the timing of the assessment. Sakai and Kubo (2011) propose a new set of assessment models for long-run credit risk that does not use stock prices and incorporates business cycles. Values estimated from these models are effectively usable in the calculation of risk spreads such as DTDs although rating biases may exist in the credit risk assessment of the markets. Schultz, Tan, and Walsh (2017) employ Merton's PD as a continuous ex-ante measure of the likelihood of a firm's failure to perform on a loan obligation. They apply a dynamic generalised methods of moments (GMM) to characterise the link between corporate governance and the chance of default by a financial institution.

By using the GMM technique the research overcomes the limitations of discrete proxies used in previous similar studies. Initial test results demonstrate that there is a significant relationship between a bank's PD and corporate governance measured in terms of executive remuneration, board, and ownership structures. However, when endogenous factors are accounted for in the model above relationship ceases to exist. Malyaretz, Dovkhov, and Dorokhova (2018) substantiate the need to incorporate efficiency indicators of bank activity as fuzzy quantities to fairly demonstrate the actual conditions faced by institutional investors in emerging markets. They propose a fuzzy multivariate regulatory analysis model for the assessment of the competitiveness of Ukrainian commercial banks. The results of their research show that there is the expediency of the application of the model to determination f the competitiveness of banks drawn into the study.

According to Lee, Tzeng, and Wang (2005), the Black-Scholes option model developed in 1973 has always been considered the cornerstone of all OPMs. However, its practical applications have always been constrained by its nature of not being suitable for fuzzy financial market environments. Hence since the planning and decision-making processes of investors in the area of option-pricing are always with a degree of uncertainty, it is pertinent that new structural models for estimation of risk metrics be adjusted to improve their precision ad accuracy (Zimmermann, 1980, Zadeh, 1965). In reality, option pricing methods depend on a person's deduction and thinking process which employs a non-binary logic with fuzziness which is not considered by classical probabilistic models such as the Black-Scholes model. Lee, Tzeng, and Wang (2005) adopts a fuzzy decision tree and Bayes' rule as a base for measuring fuzziness in options analysis and pricing. Their study also employs a fuzzy decision space comprising four dimensions namely fuzzy state, sample information, action, and evaluation function to describe investors' decisions to derive a fuzzy Black-Scholes OPM under a fuzzy financial environment. The results of the research by Lee, Tzeng, and Wang (2005) show that the market risk-free rate of return, stock price, and call prices in the money (ITM) and at the money (ATM) are over-estimated while that of the out of the money (OTM) option is under-estimated. It is the flexibility demonstrated by the Merton (1974) model that exploits the flexibility of the Black-Scholes OPM to come up with an asset valuation model (AVM) on which all contemporary structural credit risk models are premised.

5.2.6 Contemporary credit risk models for estimation of PDs of banks

According to Engelmann and Rauhmeier (2011), the use of internal rating-based approaches (IRBA) of the new Basel Capital Accords is critical as it allows banks to use their rating models for the estimation of PDs provided their systems meet the set minimum requirements. Statistical theory provides a variety of methods for building and assessment of credit rating models. These methods include linear regression modelling, time-discrete techniques, binary response analysis, hazard models, and non-parametric models such as neural networks and decision trees such as the Bayes' theorem. The benefits and drawbacks of the above models are interpreted in light of the minimum requirements of the IRBA. Engelmann and Rauhmeier (2011) discover that although there are difficulties such as data collection processes and procedures, model building, and time and effort took in model validation, IRBA is efficient and maintains its predictive power in the estimation of the risk of default of a bank. In other words, contemporary IRBA must maintain their linearity assumption to be always reliable and valid in their application to the valuation of credit exposures of banks.

Kholat and Gondzarova (2015) compare characteristics and mutual relationships among contemporary CRMs based on the importance of credit risk issues in the global economy and business sector operations. Their study examines Credit-Metrics and Credit Risk + models whose parameters are based on differences in computational procedures and model techniques used in the quantification of input parameters. They use variables such as risk definitions and sources, characteristics and data requirements, credit events, returns, and numerical model designs across various CRMs to come up with their models. They realise that effective models to do with credit risk need to be based on default-mode and marking-to-market (MTM) models, a result consistent with that by Gisho and Khestik (2013). Default mode is a technique used in the prediction of bank losses caused by default events arising from failure and non-failure model variables. This is the case for Credit-Risk+ and Moody's KMV) models which are improvements to the structural risk metric models by Merton (1974) which is drawn from the Black-Scholes (1973) OPM.

MTM models on the other hand contain the Credit-Metrics approach and focus on the market values of loans. These models use rating systems (rating migration) to determine changes in the loan quality of a potential borrower. It is noted that default mode models are only measured by changes in the debtors' assessments which emanate from their failure to honour their loan obligations. The models compare the possible paths of loan default rates under both Credit-Metrics and Credit-Risk+ models (Musankova and Koisisova, 2014b). Credit-Metrics and KMV models are based on the traditional Merton (1974) model and hence banks' assets and their volatilities are central data variables that are needed in the estimation of the risk of default of a bank. On the other hand, the major variables under Credit-Risk+ models are default risk levels and asset volatilities. The paper by Musankova and Koisisova (2014b) discovers that KMV data inputs are a time series of asset values comprising risk liabilities, stock prices, and asset correlations. The characteristics of credit events can also be used to compare CRMs, for example, the creditworthiness of bondholders. In this respect, the KMV determines a credit event of a bank as a change in the distance-to-default (DTD) which leads to changes in the expected default frequency (EDF) (Gavlakova and Khestick, 2014).

The Credit-Metrics model on the other hand characterises credit events as states in which there are migrations of default events from one grade to another. Empirical evidence suggests that EDF values respond to changes in the credit quality of borrowers faster than changes in their rating classifications. Therefore we can conclude that credit events are more prevalent in the KMV than in Credit-Metrics models and hence the need to adopt the former for its ability to estimate prevalent credit events. Under Credit-Risk+, credit events are based on default-state because they are unique. Default mode models and changes in default rates may imply decreases in the credit quality of a borrower. The estimation of the DP of a bank and the distribution function of the default probability of σ of a credit event in the Credit-Metrics approach is represented by the default probability modelled based on its annual historical data (Kliestik et al, 2015). Under the KMV approach, the expected frequency of failure changes in response to variabilities in market values of assets and their standard deviations.

On the other hand, their paper determines that under Credit-Risk+, default probability is represented by a measure of the risk of default (Gavlakova and Kliestik, 2014). The recovery rates under default events are exogenous parameters for each sub-portfolio of loans used in the Credit-Risk+ approach. However, under the Credit-Metrics approach, these recovery rates are captured as random variables with beta distributions and modelled using the Monte Carlo Simulation approach. The simple KMV considers recovery rates to be constant model parameters while the KMV model assumes that these rates follow a beta data distribution. Hence it is based on gaps in contemporary structural credit risk models presented above that this study proposes a KMV-DP estimation model extended for market friction to make it reflective of the practical conditions faced by investors and banks in fuzzy financial markets.

5.2.7 Market friction and variables of the proposed KMV default probability model

This sub-section intends to provide the link between the literature detailed above and the proposed KMV risk of default model's variables which are market values and volatilities of banks and return on equity (ROE) and market friction.

The concepts of market efficiency and friction

Calin (2016) defines market efficiency as the extent to which the price of an asset or stock fully reflects all information available and hence this makes financial markets frictionless. However, economists disagree on how efficient markets are attained and frictionless. Followers of the efficient market hypothesis by Fama (1952) hold that markets efficiently deal with all information on a given security and reflect it in the stock price immediately. Therefore technical and fundamental analysis methods of stock pricing and/or any speculative investing based on these methods are useless. A frictionless market can be defined as a theoretical trading environment where all costs and constraints associated with transactions are non-existent. (Downey, 2019). However, the primary observation of behavioral economics, for instance, holds that investors make decisions on imprecise impressions and beliefs, rather than rational analysis. This, therefore, renders financial markets somewhat inefficient to the extent that they are affected by the decisions of investors and people in general (Zadeh, 1973 and 1972).

Akbas et al (2016) argue that markets are frictionless when efficiency in capital markets requires that capital flows are sufficient to eliminate arbitrage anomalies. The authors examine the relationship between capital flows to a quantitative (quant) strategy. This is a strategy based on capital market anomalies and the subsequent performance of the financial institutions after the implementation of this strategy. When capital flows are high, holders of quantitative funds can implement arbitrage strategies more effectively, leading to lower profitability of market anomalies in the future, and the converse is also true. This means that the degree of cross-sectional equity market efficiency varies across time factors with the availability of arbitrage capital. Frictionless markets are used in theory to support investment research or financial trading concepts. In speculative investments, many financial performance returns will assume frictionless financial market costs.

Investors must view both friction and frictionless analyses for a realistic understanding of a security's return on the market of operation. Pricing models such as Black-Scholes (1973) and other methodologies also make frictionless market assumptions which are important to consider since actual transaction costs will be associated with real-world financial applications (Fuchs and Uy, 2010). In economic theory, a frictionless financial market is defined as a market without transaction costs to be faced by investors (Downey, 2019). Friction on the other hand is a type of market incompleteness. Every complete market is frictionless, but the converse does not hold. In a frictionless market, the solvency cone is the half-space normal to the unique price vector. The Black-Scholes OPM is based on the assumption of a frictionless market that is a market where investors incur no transaction costs in all their banking and investment endeavours (Durbin, 2002).

De-Young (2007) defines market frictions as all transaction costs, taxes and regulations, asset indivisibility, nontraded assets, and agency and information problems that negatively impact the performance of banks and other players. These costs are not provided for in contemporary CRMs making all asset, equity, and risk metric estimates from banks' financial data to be inaccurate and not true reflections of their actual financial performances. These costs facing banks are made very high when additional hazards such as regulatory and supervision and corporate governance and ethical challenges are factored into the estimation of risk metrics. Fuchs and Uy (2010) note that a lack of technology and innovations bring huge barriers to further outreach and development, including high transaction costs and lack of access to long-term sources of finance.

Although structural models assume that markets are frictionless, in reality, financial markets are characterised by friction as evidenced by changing asset standard deviations and returns to investments. It is observed that costs of making financial transactions are very huge and not constant as purported by popular CRMs by Merton (1974) and Black-Scholes (1973). De-Gennaro and Robotti (2007) define market friction as anything that interferes with investors' trading and can exist even in efficient markets. They go further to argue that financial market frictions generate business opportunities and costs to investors and change over time.

Relationship among variables of the proposed KMV DP model

Traditional and asset value volatility movements must play an indispensable role in the determination of the likelihood of default events (Mason and Rosenfeld, 1984). This is because the same level of the buffer may not be sufficient to withstand potential losses when the value of a firm's assets is highly volatile. Under good economic times, a good DTD value must be a leverage ratio adjusted for the trend and a standard deviation of the value of a firm's assets. Duan and Huang (2012) introduced the Merton model to define the DTD of banks above. The DTD is an appropriate concept used in the estimation of default risk. However, it is challenged because it is computed only when we know the market values of assets and parameters governing both trend and asset volatilities. The asset values and volatilities are not observable when used under the standard Merton (1974) model. In the absence of a time series of observed asset values, it is complex to estimate model parameters that define both trends and movements in standard deviations of assets.

Default probability (DP) or risk of default is the likelihood that a bank will fail to perform on its interest and principal obligations when they fall due. Default risk occurs when the market values of banks' assets lose value in the financial markets (Sakai and Kubo, 2011). In other words when assets default, a bank must reduce the values of such assets on its books, following its laid down accounting standards and rules, and decrease its capital values and earnings. DP is a very important risk metric that must be efficiently or effectively estimated to improve precision and robustness in the assessment of bank financial performance (Engelmann and Rauhmeier, 2011). Effective DP estimation can also enable banks to come up with critical policies and strategies for reducing the quantity of non-performing loans (NPLs) and financial performance. In the study, the risk of default is the dependent variable, and market friction (cost of equity), bank liabilities, asset values, and their volatilities, time to maturity, and return to equity are the independent variables. The discussion below details all the independent variables of the proposed model, and how they are estimated from the raw data before they are fuzzified using contemporary models presented in sections 3 and 4 below.

Market values of banks and their volatilities

Using Ito's Lemma we can demonstrate that the values of equity and assets of a bank and their volatilities can be connected through the general formula:

$$\sigma_E = \frac{V_A}{V_E} \times \frac{dV_E}{dV_A} \sigma_A \tag{5.6}$$

To be able to solve for the unobservable variables of the equation, namely V_A and σ_A, we should be able to solve a system of non-linear equations. The famous Merton (1974) PD model as applied by Garcia, Herrero, and Morillas (2021) gives rise to two simultaneous linear equations (7 and 8) with two unknowns which are V_A and σ_A:

$$V_E = V_A \times N(d_1) - e^{-rT} X \times N(d_2)\,; \tag{5.7}$$

The volatilities of equity and assets of a banking firm are connected through the equation:

$$\sigma_E = \frac{V_A}{V_E} \times N(d_1)\sigma_A; \tag{5.8}$$

Asset volatilities represent how large a company's assets swing around the mean market price that is it is a statistical measure of the returns to a bank. The values of a bank's assets and their volatilities are unknowns to be estimated from the financial fundamentals of the bank under structural models 5.7 and 5.8 above.

Estimation of the liabilities of a bank

The basic accounting equation used in the estimation of a bank's liabilities is Assets (A) = Liabilities (L) + Owner's equity (S), such that Liabilities, L = A-S. In the model a bank's Total assets = Non-current assets+ Current assets, Liabilities = Long term Liabilities+ Current liabilities, and Owner's equity = Number of shares issued$\times$ Marke price per share. The current liabilities of a bank include all current payments on long-term loans such as mortgages and deposits by its clients.

Return on equity and its estimation

Return on equity (ROE) is defined as the company's average residual or net income per share in the issue. According to Burns, Sale, and Stephan (2008), the ROE is a better way or measure of bank profitability than the risk-free rate of rate which is just a nominal and not effective measure of bank performance. Most structural models of default risk are pinned on the risk-free rate of return, which is far from reflecting fairly the actual performance of a bank. Hence the study employed the banks' ROEs as these fairly represented a true reflection of their actual position from all their trading and investment activities over a given time frame. ROEs are strong proxies for the risk-free rates of return because they are unique and directly related to the individual bank's issued shares and market financial performances. The banks' ROEs are calculated using the traditional formula,

$$\text{ROE}=\frac{Net\ Income\ (Earnings\ After\ Interest\ and\ Tax)}{Total\ Market\ of\ the\ Firm's\ Ordinary\ Equity}. \tag{5.9}$$

Banks' ROEs like all other risks of default model variables are converted into fuzzy values because they are also influenced by experts' perceptions.

Estimation of the cost of equity

One of the major contributors to the risk of default in banks is the cost of equity which is not factored into most contemporary structural models (Besley, Roland, and Reenen, 2020). Cost of equity can be defined as the return that a company needs for a project or investment or equity investment or pays out to ordinary shareholders as dividends. According to Zabai (2019) in conjunction with Madan and Unal (1996) banks must manage their default risks through efficient, effective ad prudential management of their costs of equity. The study, therefore, incorporates market friction in the form of the cost of equity, in the proposed KMV risk of default model to represent costs incurred by banks in constituting their capital bases. We used Myron Gordon's constant growth model to estimate the bank's cost of capital or the required rate of return (RRR).

The traditional RRR or cost of equity is calculated using Gordon's constant growth model or formula,

$$\text{RRR}=\frac{D_0\,(1+g)}{P_0}+g; \tag{5.10}$$

where D_0 = The dividend per share paid today, P_0 = The market price per share today, and g = The constant growth rate in earnings and dividends.

5.3 Research Methodology

Section proceeds by first illustrating the fuzzy logic, concepts, and approach to the estimation of risk metrics of banks before presenting the proposed KMV risk of default model, assumptions, sources of data and input variables, and their justification.

5.3.1 Fuzzy approach to estimation of the risk of default of a bank

This section commences by presenting the basic fuzzy concepts of fuzzy sets and systems before proposing the model to be used in calculating the risk of default of banks.

Fuzzy concepts and their definitions

The following are the basic definitions and properties of fuzzy set theory, systems, and numbers with their relevant operations.

Definition 5.1 A fuzzy set, A in X∈R (real numbers), is a set of ordered pairs A ={(x; μ(x): x ∈ X) where μ(x) is the membership function, grade of membership, degree of compatibility, or truth of x∈ X which maps x ∈ X onto the real interval $[0;1]$ (Chen and Pham, 2001, Zadeh, 1972).

Definition 5.2 A fuzzy set A in R^n is said to be a convex fuzzy set if its $\gamma-[level\ set\ A\ (\gamma)$ are (crisp) convex sets for all $\gamma\in[0;1], \mu_A\,(\lambda x_1+\lambda\, x_2)\geq Min\,\left(\mu_A(x_1);\mu_A(x_2)\right)$(Appadoo and Thavanswaran, 2013).

Definition 5.3 A fuzzy number, u= $\{a^{-}, a, a^{+})$is specified by its core a $\in R$ *and a membership function*, $\mu: R \rightarrow [0;1]$with support in $[a^{-}; a^{+}]$ defined as:

$$\mu(x) = \begin{cases} L(x) \text{ if } a^{-} \leq x \leq a \\ R\,(x) \text{ if } a \leq x \leq a^{+} \text{ for } x \in R, \\ 0 \quad Otherwise, \end{cases} \tag{5.11}$$

where L(x) is an increasing function with L(a^{-}) =0, L(a) =1 and R (x) is a decreasing function with R (a) =1, R $(a^{+}) = 0$. Functions L(.) and R (.) are the left and right shape functions of u, and they are assumed to be differentiated. (Chen and Pham, 2013).

Definition 5.4 For values of $\propto \in [0,1,$ *the* $\propto -cut$s are defined to be the compact intervals $[u]_{\propto}$= $\{x|\mu(x) \geq \propto\}$, which are "nested" closed intervals (Stefanini, Sorini, and Guerra, 2006).

Fuzzy approach systems and their applications

Fuzzy approach systems have not only been used in a variety of practical problems, but also in regulatory risk and credit analysis, as well as for evaluation of bankruptcy and default prediction. The fuzzy approach combines easy designs based on both experts' opinions and data history in the estimation of the risk of default of a bank, leaving out market friction. Fuzzy logic arises as a good tool for emulating expert rules since they do not require too much effort to modelling risk metrics as other traditional methods do (Chen and Pham, 2001). A fuzzy system can emulate rules of the common type: IF (Conditions) THEN {Consequences}, where conditions and consequences are fuzzy propositions built by linguistic expressions or semantics:

1. x is Low
2. y is Not High
3. x is Low AND y is High
4. x is Low OR y is High.

Propositions 1 and 2 (expressions) define immediate propositions while 3 and 4 define combined propositions (Soares, Neto, and Barbosa, 2013). Since these terms operate over fuzzy variables, we need to define them in linguistic terms or fuzzy sets, as covered or defined in section 3.0 above.

Fuzzy expressions to do with semantics such as NOT, OR, and AND, which are combined to form relations, R are detailed in section 4 below.

5.3.2 Proposed KMV-risk of default model for commercial banks

The study proposes a risk of default model that incorporates the variables of the distance-to-capital (DTC) model but is based on the DTD valuation model. The DTC takes precedence over the DTD because the latter model does not involve complexities that are related to the firm's numerical fundamentals. According to Larsen and Mange (2008) a firm's DTC, at the time, t can be computed from the general formula:

$$DTC(t) = \frac{log(\frac{V_A(t)}{\rho X(t)} + (r - \frac{\sigma_A^2}{2})\ (T-t)}{\sigma_A \sqrt{(T-t)}}, \tag{5.12}$$

Where V_A(t)= The market value of the bank's assets at the time, t, X(t) = The market value of debt, r= The risk-free rate of return, T = The term to maturity of assets or debt, $\sigma_A =$ The asset volatility, $\rho = \frac{1}{1-PCAR}$ where PCAR = The capital requirements of a bank according to the Capital I Accord set at 8% of the risk-weighted assets of a bank. It should however be understood that for the calculation of the DTD we take $\rho = 1.00$ and PCAR = 0. The conversion of the Merton-KMV-DTD model to a risk of default model calls for deducting transaction costs from the risk-free rate of return obtained in the economy. Therefore the proposed KMV model for the valuation of the risk of default of banks drawn from the above equations is given by:

$$\text{DP} = \frac{N[ln\left(\frac{V_A\ (t)}{X(t)}\right) + (\mu - k_e + \frac{\sigma_A^2}{2})T]}{\sigma_A \sqrt{T}}; \tag{5.13}$$

where $\ln =$ The natural logarithm, $\mu =$ The return on equity, and $k_e =$ The cost of capital.

The study uses the return on equity (ROE) instead of the risk-free rate of return because it is more characteristic of the fair returns to banks than the latter. Following the Merton (1974) model we can show that the risk of the default probability of a bank at the time, T evaluated at the time, t is given by:

$$\text{DP = N (-DTD)}, \tag{5.14}$$

where DTD at the time, t is given by:

$$DTD_t = \frac{ln(\frac{V_A(t)}{X_t} + (\mu - \frac{\sigma_A^2}{2})\ (T-t)}{\sigma_A \sqrt{(T-t)}}. \tag{5.15}$$

where μ= The expected market rate of return.

Since the Standard Normal Distribution (0;1) function is universal, the side factor that determines the DP of a bank is the DTD. The DTD is simply the logarithm of the leverage ratio shifted by the expected return (u-$\frac{\sigma^2}{2}$) (T-t) and scaled by the volatility-time term, $\sigma\sqrt{(T-t)}$. If we have two firms with identical leverage ratios and asset σ deviations but the asset value of one firm is expected to increase at a faster rate than that of the other, it means that results will depend on the sign attached to the numerator (Duan et al, 2012). When the numerator is positive the assets of the firm will cover default and lower, σ_A, making the firm less likely to default and vice versa. The implication is that when economic agents are risk-neutral, the expected investment return will be R_f that is μ should be replaced by R_f. In theory, DP is not physically experienced but DTD is, hence the need for suitable estimates for both expected return and σ.

Two models adopted for comparison with the proposed KMV model are the traditional structural PD and hazard function models whose formulae are outlined below:

The structural probability of default (PD) model for comparison with the proposed model (equation 5.13) is given by the formula:

$$\text{PD} = \frac{N[ln\left(\frac{V_A\,(t)}{X(t)}\right)+(\mu+\frac{\sigma_A^2}{2})T]}{\sigma_A\sqrt{T}} \tag{5.16}$$

The Cox (1972) proportional hazard function model discussed in section 2.1 of the literature is based on two major assumptions which are (1) survival data curves for different strata should have hazard functions that are proportional over the time frame, t and (2) the relationship between the logarithmic hazard and each covariate are linearly related, which can be verified with residual data plots. It has been shown that when the hazard ratio is > 1.00, the treatment group would have a shorter survival span than the control-referenced group (Sawadogo, 2018). On the other hand, if this ratio is < 1.00, it implies that the group of interest is less likely to have a shorter time to the event than the reference group. The ratio is however criticised for not being able to quantify the magnitude of the difference between the two groups. While the logistic regression model tests whether a risk factor affects the odds of a disease or not, the Cox (1972) proportional hazards model tests whether a risk factor affects the age of onset of the disease in humans. Any hazard ratio above 1.00 would generally mean that the treatment group healed faster or had a slower time to an event. A hazard ratio of 1.00 means that both groups (control and treatment) would be experiencing an equal number of events at any given point in time (Mendes, 2014).

According to Sawadogo (2018), the Cox (1972) semi-parametric model, is a regression approach for human survival data that provides an estimate of the hazard ratio and its corresponding confidence interval, which is an area of interest to credit risk modelling under frictional and fuzzy financial markets. The study uses the Cox (1972) proportional regression model which fits data with a constant covariate, xx that is data that do not vary over time to a hazard function of the general form given by:

$$h(t|x)= h_0(t)\exp[\beta_1 x]\ h(t/x)=h_0(t)\exp[\beta_1 x]; \quad (5.17)$$

where we can estimate the unknown value of β1 and h0(t) is the baseline hazard, which is a non-parametric and unspecified value that depends on the variable tt and not on xx. Therefore for particular xx values, we will be able to estimate the survival function if we have an estimate of the baseline survival function, S_0 (t) S_0^(t) (Bolnd, Neweihi, and Proschan, 2016). The estimated survival function for an individual human with covariate value xk turns out to be given by:

$$S(t/xk)=[S_0(t)]\exp(\beta_1 xk) S_0(t/xk) =[S_0(t)]\exp(\beta_1 xk), \quad (5.18)$$

The study adopts the form of the hazard model above to determine the hazard rates of banks when it comes to their failure to meet their debt obligations. Cox's (1972) model is used mainly for comparing results with those drawn from the structural and proposed KMV risk of the default model. It should however be noted that there are usually magnified differences between hazard and credit risk model ratios because risk ratios do not care about the timing of the event but are only concerned about the occurrence of the event by the end of the day.

Sources of data

The study used audited and published financial data of eight banks conveniently drawn from six Southern African countries. The countries from which the banks are drawn are South Africa, Namibia, Botswana, Malawi, Tanzania, and Zambia. The audited financial statements of the banks cover the period 2008-2020 and are from the World Development Index (WDI, 2020). This is a credible source of data and researchers can easily download data directly from the website for research purposes. Although the available financial data are drawn from various economies of Southern Africa, the research concentrates on financial data of banks listed on Stock Exchanges of the same countries. Panel data are popular for their ability to reduce the co-linearity among explanatory variables, and hence improve the efficiency of econometric estimates. The study employs unbalanced panel data that have been checked and screened for apparent coding errors and missing variables. It is the interest of the research to conveniently draw various countries from the region in the quest to compare and contrast the performances of banks in large and small countries such as South Africa and Mauritius respectively.

Input variables for the proposed model and their justification

The data variables for the estimation of the risk of default for a bank are market values of its assets (A) and debt (D), asset volatility, the return on equity (μ), and cost of capital ($\mathrm{k_e}$). Extension to existing CRMs such as KMV DTD-PD for market friction enables investors to improve the accurate estimation of their banks' PDs. According to KMV-DTD model, the equity of a firm is perceived as a call option on its underlying assets because at the maturity of debt, bondholders receive their debts and equity holders realize the rest. The model operates only if we are given observable equity and unobserved asset values and their corresponding volatilities. Based on the Black-Scholes OPM where the value of assets is represented by the price of a call option (C) and the value of equity by the value of the stock (S), debt (D) is taken as the strike price (K).

Assumptions of the KMV risk of default model

The assumptions on which the KMV model is founded are two that is:

.Debt is homogeneous with time to maturity, T;

.The Capital Structure of a firm is given by the general equation,

$$V_A(\mathrm{t}) = \mathrm{D(t)} + V_E(\mathrm{t}) \tag{5.19}$$

where V_A(t) =The value of assets, D(t) = The value of debt and V_E(t) =The value of equity;

.The markets are perfect and ignore coupons and dividends and there are no penalties incurred by investors for short selling; and

.Dynamism of a bank's assets is that they follow a geometric Brownian motion as alluded to earlier on under equation 1.

Under the Black-Scholes OPM, the market price of a European call option is given by:

$$C(t) = S(t)\,N(d_1) - e^{-r(T-t)}.KN(d_2) \tag{5.20}$$

where S(t) =The stock price, r =The risk-free rate, K =The strike price of a call option, T =Term to maturity of the bank's assets and liabilities, t= Time today, N(d_1), and N(d_2) = The cumulative probabilities of the Z-values,d_1, and d_2 respectively.

Using Ito's Lemma we have demonstrated that the values of equity and assets of a bank and their volatilities can be connected through the general formula above (See equation 6). To solve equation 6 for the unobservable variables of the equation, namely V_A and σ_A , we should be able to solve a system of non-linear equations given by:

$$\begin{cases} f_1(\,V_E;\,\sigma_E) = V_A(t).\,N\,(d_1) - e^{-(r-k_e)(T-t)}.\,DN(d_2);\; V_E(t) = 0 \\ f_2(\,V_E;\,\sigma_E) = \frac{V_A}{V_E}.\frac{dV_E}{dV_A}N\,(d_1)\sigma_A;\; \sigma_E = 0 \end{cases} \tag{5.21}$$

The solutions to the above system of equations (13) are unique as:

$$\frac{df_1}{dV_E} = N\,(d_1); \tag{5.22}$$

which is analogically due to changes in the Black-Scholes model and f_1 is an increasing function of the value of assets of a bank, V_A *that is* $f_1(V_A)$ has a unique solution. On the other hand, $f_2(\sigma_E)$ has a unique solution as well.

5.3.3 The structure of a fuzzy system

According to Soares, Neto, and Barbosa (2013), a fuzzy system is usually characterized by the following components:

.Input variables (made up of their respective fuzzy data sets)

.Output variables (these are the diagnostic values)

.Rule base (this determines the output for each combination of input values)

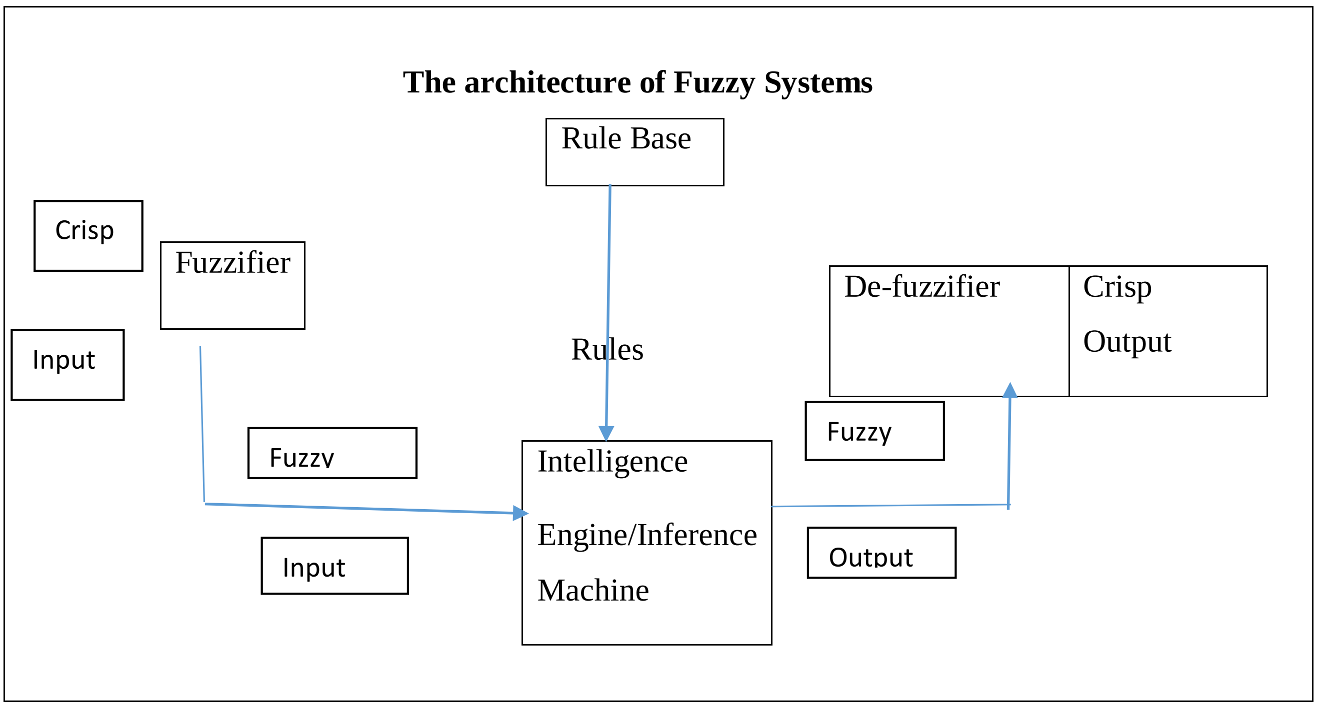
.Inference machine (which applies fuzzy operations to transform crisp variables into fuzzy variables)

.Fuzzy sets (these are linguistic terms for each input variable drawn into the model)

.Crisp values (these are numerical values taken from the real financial world)

The four model variables namely expected return on equity (μ), cost of equity (k_e), asset values, and volatilities are fuzzified through the architecture of fuzzy systems given below, developed by Ross et al (2010).

Table 5.1 Showing the Architecture of Fuzzy Systems by Rose et al (2010)



Structure of Fuzzy Systems (Source: Ross et al, 2010)

5.3.4 Concepts used in the architecture of fuzzy systems

The principal components of fuzzy logic control systems are the fuzzifier, rule base, and evaluation, aggregation of the rule outputs, and de-fuzzification as detailed below.

Fuzzification

This is the process of transforming real-world crisp quantities into fuzzy quantities or variables (Ross et al, 2010). This can be realised by identifying the various precise and deterministic quantities as totally non-deterministic and very uncertain in practice. These variables' uncertainty could have emerged as a result of imprecision and vagueness which may then lead to the representation of the input variables by a membership function as they can be fuzzy. For example, if we argue that the return to equity of a bank is 25% per annum, an investor could convert this crisp input into a linguistic variable such as moderate, high, or low return (Rose et al, 2010). The study transforms the four input variables, ROE, cost of equity, asset values, and volatilities into fuzzy parameters for use in the estimation of the risks of default of banks. The fuzzifier translates these crisp variables into fuzzy inputs which are then directed into the intelligence engine for further transformation. The intelligence engine is responsible for converting the input variables into fuzzy outputs. All the fuzzy outputs obtained are then directed into the de-fuzzifier where they are finally turned into crisp outputs.

Fuzzy operations, rules, base, and evaluation

This is the application of fuzzy operations, Minimum and Maximum in input variables according to the available rules to determine if they should be inclusive (AND) or exclusive (OR). The fuzzy rules comprise input and output variables that assume values from their term sets, with meanings associated with each linguistic concept (Soares, Neto, and Barbosa, 2013). Crisp or exact model inputs are fed into the fuzzifier to transform them into fuzzy variables under a clearly defined rule base. The rule base is characterised by all the rules and membership functions that regulate decision-making processes in fuzzy logic systems. The base also contains the “If-Then” conditions which are used for conditional programming and controlling the whole fuzzy logic system. Rules evaluation is a technique that is used to assess the criteria and return model values based on a dynamic configuration process (Rose et al, 2010). The evaluation system gives users the possibility to configure model inputs for application scoring, approving flows, credit insurance, or bureaus.

Aggregation of the rule outputs

The aggregation of the rule outputs is the process by which the fuzzy sets representing the outputs of each rule are combined into composite fuzzy sets. It is a process that only occurs once for each output variable, and happens before the final de-fuzzification process is undertaken. The output of the aggregation process is converted into one fuzzy set for each given output variable.

De-fuzzification

This is the opposite of the process of converting the crisp results into fuzzy variables. The mapping done here is the conversion of the fuzzy results into crisp variables. This process is very capable of generating a non-fuzzy control action which illustrates the possible distribution of an inferred fuzzy control action or process (Ross et al, 2010). The de-fuzzification process can also be considered to be the rounding-off process, where a fuzzy set having a group of membership values on the unit interval is transformed into a single scalar quantity.

5.4 Validation of the KMV-Risk of Dealt Model, Findings, and Discussions

The validation of the proposed risk of the default model follows several steps as detailed below.

5.4.1 The fuzzy extension principle

It has been observed that most structural stochastic models are solved using classical and fuzzy set theories but are not extended for market friction which is a huge transaction cost to investors, especially in emerging financial markets. In the process of managing functions of real variables, the use of the fuzzy extension framework should result in the correct application of the extension principle by Talamanca, Guerra, and Stefanini (2012). We start by assuming that we are given an exact relationship function of the general form,

$$y = F(x_1; x_n; \text{------------------}; x_n) \tag{5.23}$$

of n real variables given by x_1; x_n;------------------;x_n. The above multiple linear relationship function's fuzzy extension can be obtained to evaluate the effects of both transaction costs and uncertainty on the variable, x_j, modelled by the corresponding number, u_j for each level, α in the interval [$u^-_{j,\alpha}$; $u^+_{j,\alpha}$,], given the possible values $of\ x_j$. Suppose we are also given another variable, v = f (u_1; u_n;------------------; u_n) which denotes the fuzzy extension of a continuous function, f. The continuous function, f is characterised by n variables for each level of α, resulting in the interval [v^-_{α}; u^+_{α}], which represents the propagation of uncertainty from all variables, x_j to the variable, γ (Rose et al, 2010).

It should be noted that if the uncertainty on the original variables of a model is denoted by, γ, which is also modelled by linear numbers, the γ-variable will still be a fuzzy number, starting from a single value (at α =1.00) to the most uncertain interval level (at level, α =0.00) but it loses its linearity property in the process of such transformation (Soares, Neto and Barbosa, 2013). This also follows that the parametric representation of the variable is also necessary when input variables are triangular fuzzy numbers to apply the extension principle and represent the non-linear output fuzzy numbers (Talamanca, Guerra, and Stefanini, 2012). To obtain the fuzzy extension of fuzziness to normal semi-continuous fuzzy intervals, we have to compute the α–cuts $[v_\alpha^-; u_\alpha^+]$ of v, defined as the images of α-cuts of $(u_1; u_n$;-----------------; $u_n)$ that are then obtained by solving the following constrained optimisation problems for α ∈[0;1]:

$$(\text{EP})\alpha = \begin{cases}(v_\alpha^- = min\{f(x_1; x_2; --; x_n)\ xk \in [uk_\alpha^-; uk_\alpha^+], k\ = 1; 2; ---; n] \\ (v_\alpha^+ = max\{f(x_1; x_2; --; x_n)\ xk \in [uk_\alpha^-; uk_\alpha^+], k\ = 1; 2; ---; n]\end{cases} \quad (24)$$

Source: (Talamanca, Guerra and Stefanini, 2012)

Only in simple cases can the optimising problems above be solved analytically. In general, the solution of such a system of equations is complex and computationally expensive to determine for each $\alpha \in$ [0;1]. Hence we require global solutions to the two non-linear problems for all model variables. Therefore Ross et al (2010) present the above architecture of fuzzy systems which we used in translating input variables of the KMV risk of the default model into fuzzified parameters.

5.4.2 Settings and implementation of the fuzzy system

The fuzzy system elaborated above is implemented using Mamdami's (1975) system as the central inference machine. This is because Mamdami's system is very simple for use in processing rules and values of model variables. The system is set up as tabulated below for fuzzification and de-fuzzification of model variables:

Table 5.2 Showing the Process of Fuzzification and De-fuzzification of Model Variables

Variable/Approach	**The System Applied**
Input fuzzy sets	Gaussian
Output fuzzy sets	Triangle

Implication method	AND/OR
Aggregation method	Product (AND)
De-fuzzification method	Centre of gravity

5.4.3 Fuzzy logic and DP modelling in banks

Fuzzy logic has been defined as a computing technique based on the degree of truth. It may also be taken to be a method of reasoning that resembles human logic or reasoning in financial investment decisions. The approach of fuzzy logic imitates the way of decision-making in humans that involves all intermediate possibilities between two digital values, which are “Yes and No” (Chen and Pham, 2001). Therefore fuzzy logic operates on the levels of possibilities that are associated with the input variables to attain a definite model output. Fuzzy logic is thus a basic control mechanism that depends on the degrees of the state of the input variable/s. It is the state of the input used in modelling that determines the nature of the output to be realised. A study by Sabounchi, Triantis, and Liu (2011) uses fuzzy logic that incorporates linguistic variables in the valuation of a default event. The study concludes that the use of the max-min operator system diversifies inconsistencies among fuzzy rules and defuzzified model variables behaved reasonably for defuzzified default risk estimation methods. In other words, a fuzzy logic system works on the principle of assigning a particular output depending on the probability of the state of the input variable to be used. There are many reasons given for the application of fuzzy logic in banking and finance such as:

.It is relatively simple to create and deploy since it’s fully based on human experts’ opinions and evaluations.

.It is used in fields of study involving decision-making processes that require some sort of human judgment.

.Human mind abstracts real-world variables in an imprecise manner to constitute semantic networks.

.These semantic variables or networks define relations that can be expressed in linguistic terms just as experts do in various disciplines (Soares, Neto, and Barbosa, 2013).

Hence any business activity that requires expert opinions or judgments can be modelled through fuzzy logic rules without the need for an existing theoretical model to base on. By taking into account all the above sets of fuzzy sets, numbers, and logic or literature into consideration, we designed a model for predicting the risk of default of banks that could be accurate, robust, and practical. The methodology used in this design is the same as that which was developed and employed by (Pereira, Oliveira, and Soares, 2012). The research methodology we use comprises the following procedure or variables running from stages A to C:

Table 5.3 Showing the Three Stages of the Study Procedure or Variables

A. Definitions of Variables	B. Definitions of Rules	C. Definitions of Inferential Machine Settings
.Sets of Definitions	.Operational Research	.Aggregations
.Membership Functions	.Database Formations	.De-fuzzification

Source: Author

According to literature, the default event is influenced by many borrower internal and external conditions most of which could be unknown to the loan provider unless they are declared. However simple models of DP can yield good results using statistical measures or techniques (Ross et al, 2010). To make the system more applicable we took all the crisp variables of the model and fuzzified them before the validation of the proposed KMV DP model.

5.4.4 Results from validation of the KMV-risk of default model

The study proposes a new look KMV model for estimation of the DP of a bank in emerging financial markets. It investigates the effects of asset values, liabilities, returns on equity and cost of equity or market friction on the DPs of banks in fuzzy financial markets. The model is validated using financial data drawn from banks in emerging markets of countries in Southern Africa. It also aims to compare the results estimated using the KMV model with those generated from other approaches such as hazard function and structural risk models. The research employs a STATA Package to come up with three sets of results for all eight banks based on structural, fuzzy KMV, and hazard function models for comparison purposes. The table below summarises the risks of default results from the structural and KMV models (in Scientific form, $A \times 10^{-3}$) and Hazard Ratios (as decimals).

Table 5.4 Showing Distribution of Banks by Their Annual Structural (S) and Fuzzy KMV (F) DP Values and Hazard Ratios (H) for the Period 2008-2020

Year	**PD Value**	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Bank A	S	16	3.7	3.9	3.0	0.0	1.6	3.2	2.7	2.2	2.4	1.9	2.1	2.0
	F	8.0	2.6	0.3	0.0	0.0	0.1	1.2	1.6	1.0	1.2	1.3	1.6	1.4
	H	2.2	1.8	1.4	0.8	0.9	1.4	1.8	0.8	1.5	1.8	1.6	1.9	1.7
Bank B	S	24	2.7	2.4	2.7	2.9	2.8	2.2	1.9	1.6	1.8	2.1	2.5	1.8
	F	2.0	3.8	0.0	0.0	0.1	2.4	1.8	1.4	1.3	1.5	1.6	2.2	1.5
	H	1.2	1.6	1.5	1.2	1.0	0.8	1.2	0.7	1.4	0.9	1.0	0.5	0.7
Bank C	S	2.8	4.0	3.5	1.5	3.4	3.9	2.8	2.4	2.7	2.3	2.5	1.8	1.6
	F	0.0	0.0	0.0	0.0	0.0	0.0	1.2	1.4	16	1.4	1.5	1.2	1.1
	H	1.2	0.9	1.0	0.8	0.6	1.2	1.8	1.6	1.7	1.8	1.6	1.3	1.5
Bank D	S	0.8	0.4	0.0	0.0	0.0	1.8	1.2	1.6	2.0	1.8	1.4	1.5	1.7
	F	0.4	0.2	0.0	0.0	0.0	0.8	0.6	0.7	1.2	1.0	0.9	0.9	1.0
	H	1.6	0.9	0.7	1.1	0.8	1.6	1.3	1.5	1.6	1.6	1.0	0.8	0.9
Bank E	S	0.5	0.0	0.0	0.0	0.0	0.0	1.2	0.8	1.0	1.6	1.8	1.7	1.5
	F	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.4	0.4	0.8	0.7	0.9	0.6
	H	1.3	0.8	0.6	0.9	0.7	1.0	1.0	0.9	0.7	1.2	1.4	1.2	1.3
Bank F	S	2.5	4.0	0.0	3.1	3.8	4.9	0.8	2.2	1.5	0.4	0.6	0.3	0.5
	F	1.8	2.1	0.0	0.0	2.2	0.8	0.0	0.7	0.8	0.2	0.2	0.1	0.2
	H	1.8	1.4	1.0	0.8	1.2	1.5	1.3	1.1	0.6	0.4	0.6	0.4	0.5
Bank G	S	42	38	34	30	26	32	34	36	33	48	44	42	40
	F	18	16	12	14	15	17	20	22	24	19	21	23	25
	H	1.4	1.8	2.0	1.7	2.2	1.8	1.6	2.0	1.8	1.4	1.7	0.9	0.8
Bank H	S	56	48	55	47	56	44	54	43	52	48	53	48	56
	F	34	22	24	25	32	38	46	32	36	28	36	29	28

	H	1.3	0.9	1.1	1.6	1.8	1.7	1.4	1.9	1.6	1.5	0.9	0.7	0.6

The results of the study show that all independent variables namely market values and their volatilities, liabilities, return on equity and cost of equity influence the risk of default of a bank. The study also postulates that asset values and volatilities and return on equity are inversely related to the risk of default of the bank (Appendix IV). The results concur with those of the standard structural model research study by Nagel and Purnanandam (2019) that risk of default and equity values of banks are more sensitive to negative shocks than to asset values and volatilities. The inverse relationship between the risk of default of a bank and stock returns signifies the presence of inefficiencies in the financial markets which motivated the undertaking of this study.

The above research result is contrary to the finding of a similar research study by Chava and Purnanandam (2010) that realises a positive cross-sectional relationship between stock returns and the risk of default of a bank or similar financial firm. Therefore financial investors expect higher returns as compensation for bearing investment risks, and thus accords with the direct and fundamental principle of high risk, high return and low risk, low return. However it is not very clear why some rational investors do not exploit such arbitrage opportunities to improve their financial performance in these markets. The study also acknowledges that the liabilities of a bank are directly related to the risk of default on its obligations and this is in line with results of a similar research study by Zabai (2019). In other words, a bank has a higher default risk when it has a poor credit risk rating and limited cash flow base.

The main independent variable drawn into the model is the cost of capital (a form of market friction), which is a function of the market's risk-free rate of return plus a premium or compensation for the risk associated with the bank's investment. The study further discovers that the cost of capital like the bank’s liabilities has a positive relationship with its risk of default level (Appendix I). This result concurs with findings of a related research by Gleibner (2019) which argues that financial markets are not perfect and are always frictional. Therefore his study postulates that the cost of capital has a direct impact on risk of default of a bank and hence the need for it to be included in estimation of risk metrics. Uncertainty in the form of fuzziness is provided for in the variables of the proposed KMV-risk of the default model. It is observed that uncertainty is an economically important risk factor in the estimation of DPs of banks. This is because it captures variables such as corporate governance, economic booms, and recessions and expert judgments such as bullish and bearish market conditions (Switzer, Wang and Zhang, 2018). The study reveals that fuzziness is an uncertainty that increases the risk of default in banks as it lowers their overall financial performance and profitability.

According to Zhang, Li and Ortiz (2021) uncertainty is an economically significant risk with a direct bearing on banks' risks in that it increases the risk of default of borrowers. Effective governance of such uncertainty can alter the risk-increasing and profit-decreasing effects of bank performance. The results of the study also demonstrate that good and strong corporate governance and ethics mechanisms play a key role in weakening the hazardous consequences of economic uncertainty on banks and the promotion of sustainable growth and development of the banking sector. This research observation supports the findings of a related study by Switzer,

Wang and Zhang (2018) that ownership structure has a significant impact on default risk of financial firms and none on nonfinancial firms. The study further notes that the bigger the independent board of directors of financial institutions the higher the risk of default and the lower the risks facing nonfinancial institutions. This research study also finds that there is a direct linkage between the bank and market sizes and the risk of default faced. According to a study by Laeven, Ratnovski and Tong (2014) unconditional risk of default varies with the size of the bank, and financial market of operation. Although their study observes that large banks enjoy economies of scale in the form of capital or equity buffers and profits, optimal bank size can be associated with some degree of uncertain in most cases. Large banks can have a direct negative bearing on the whole economy, no wonder why their failure in the form of for instance insolvency or liquidity stresses can be more disruptive to the whole financial system of a country.

The major findings of the study based on the three fundamental objectives and results tabulated above (Table 4.1) are that the DPs from the KMV model extended for market friction and fuzziness are more smoothened than those calculated using the traditional structural PD model. However, a comparison between these two model results with those of the hazard function, which measures the conditional probability of an event, reveals that the latter gives very high-risk ratios stretching to well above 1.00, the upper limit of classical probability. The risk of default values are based on conditions of the classical probability theory which are bound between 0 and 1, making comparisons between the former and latter models very unique and suitable for different fields of study. The trends from the hazard model demonstrate that although it can be used to estimate the chance of an event happening, it suits very well the natural science disciplines as articulated in the Cox (1992) theory.

In other words hazard models feed into natural sciences or disciplines where risks for instance human and animal fractures can be highly magnified. The former models namely structural and proposed KMV approaches fit into economics, banking, and finance disciplines where default probabilities are usually marginal values. This research result is contrary to the findings of a research by Dirick, Claeskens and Baesens (2016) that Cox proportional hazards regression models with splines in hazard functions can perform very well in bank credit risk estimations or context. The results of their research also indicate that banks from fairly large economies with deeper and broadened markets are characterised by lower risks of default compared to those in shallow and smaller economies.

The study uses the ROE variable in place of the risk-free rate of return (pegged by monetary authorities) on which structural models are based because it is a fair and internal measure of return based on the actual market performance of a bank. Therefore overall the study demonstrates that transaction costs and fuzziness are indispensable factors to be drawn into contemporary models to improve the precision needed in the estimation of risks of default of banks in emerging financial markets. The result supports the results of a related research by Maffett, Owens and Srinivasan (2017) that restrictive pessimistic financial trading such as short-sale constraints lead to higher credit spreads and thus affects assessment of a firm’s risk of default to a greater extent.

5.5 Conclusions and Recommendations

The study proposes and validates a KMV risk of default model using financial data drawn from banks in emerging markets of countries in Southern Africa. Banks in Southern Africa operate in frictional and fuzzy financial markets contrary to the assumptions of the existence of efficient and frictionless markets on which structural credit risk models are founded. Based on the results of the study discussed above we conclude that banks in emerging economies need the risk of default models different from structural models which suit the financial circumstances of developed economies of the world. Structur/a//l models are founded on assumptions such as frictionless and efficient financial markets, and constant rates of return and asset volatilities which are far from being realistic in most emerging economies such as those in Southern Africa. In practice banks in emerging economies operate in frictional and fuzzy financial markets. Hence the need for researchers to develop asset valuation and risk metrics models that are divorced from structural models that capture variables such as market friction and uncertainty (or fuzziness) which significantly influence the estimation of DPs of banks.

Conclusions

The study examines the impact of several independent variables on the estimation of risks of default of banks in Southern Africa. From the results (in Appendices I-IV) we conclude that asset values and volatilities and ROE are inversely or negatively related to the risk of default level of a bank. On the other hand, the bank's liabilities, cost of equity or capital, and uncertainty (fuzziness) are directly or positively related to its risk of default. The study also concludes that banks in smaller economies of the region have higher risks of default compared to those in larger economies that are better capitalized, regulated, and managed under good and sound corporate governance and ethics systems or frameworks.

We also compare results estimated from the proposed KMV risk of the default model of banks with those generated from both hazard function and structural risk models (Table 4.1 above). The study concludes that the proposed KMV model is a better estimator of the risk of default of a bank than the traditional structural models. The model has results that are more stable, moderate, or smoothened (or less volatile) to signify that it does not over or under-estimate the banks' risks of default. From a comparison between these two models and the hazard semi-parametric approach, we conclude that hazard function models do not suit well into the risk metrics models whose values are marginal and range from 0 to 1.00. The results from the hazard model are not constrained by the limits of the classical probability theory by Kolmogorov (1933). In reality, hazard models are divorced from probability theory and thus fit very well into natural sciences as advanced by Cox (1992).

We also conclude that the proposed KMV model can contribute to the board of knowledge as it captures market friction and fuzziness in the estimation of the risk of default of a bank, which are not provided for in other models. Overall the study concludes that all structural credit risk models and hazard functions are limited and unsuitable for application to the valuation of banks in frictional and fuzzy financial markets or environments. Therefore we reject the null hypothesis and conclude that market friction and uncertainty (fuzziness) have effects on the DPs of banks in emerging markets such as those in Southern Africa. Hence all banks in emerging financial markets can adopt and implement risk models extended for friction and uncertainty to come up with fairly estimated risks of default, needed in the valuation of their financial performance.

Limitations

Although the above findings and conclusions of the study could be credible, we also acknowledge that it is based on selected input variables, which are asset values and volatilities, return on equity, cost of equity (market friction), and uncertainty (fuzziness). The model variables used are banks' market values and their volatilities, return on equity, and market friction, which could be far away from giving a true reflection of the diversity of variables that affect their DPs in the real world. Therefore a lot of other bank-specific and market variables such as expected losses and profitability, unemployment, inflation, and exchange and interest rates could have been factored into the model respectively to make it more diverse and realistic. In reality, the inclusion of the above variables in the proposed DP model could go a long way in influencing the findings and conclusions of the study as well as contributing to the banking and finance board of knowledge.

Recommendations

However based on the above conclusions the study recommends that banks should include transaction costs and uncertainty in existing structural models to make them more rigorous, reliable, consistent, and practical in the estimation of DPs of banks. Overall we recommend that banks in frictional and fuzzy markets can adopt and implement the proposed KMV-DP model for the estimation of fair values of their DPs. In this regard errors to do with and under- and overcasting of DPs are significantly reduced and banks' reported profitability levels will be fair and consistent reflections of their actual performance for given accounting periods. Financial regulators and supervisors of banks and similar institutions should require these supervisees to establish and maintain sound risk corporate governance and ethics systems to efficiently and effectively mitigate internal risks such as the risk of default and non-performing loans (NPLs) and systematic financial risks to safeguard their countries' overall financial stabilities.

CHAPTER VI

EXPECTED LOSS MODELLING IN BANKING CORPORATIONS IN THE PRESENCE OF MARKET FRICTION

6.1 Introduction

Credit risk models are considered to be direct applications of the frequency and severity of hazard rate models in financial organisations (Elizalde, 2005). In modern financial world, structural and reduced form models represent two main categories of credit risk models. Structural form models aim to provide an explicit relationship between default risk and the capital structure of a firm. According to reduced form models on the other hand are used for modelling credit defaults as exogenous events or variables. These models are driven by a stochastic process for example the Poisson jump process which is more realistic when it comes to returns to financial investments. Structural models by Merton (1974) and Black-Scholes (1973) use modern option-pricing theory in the valuation of corporate debt. The Merton model was the first structured model and serves as the cornerstone for all structural credit risk models. The chapter investigates the impact of market friction on risk metrics of banks in Southern Africa in their desire to accurately measure expected losses and improve financial performance for growth towards sustainable development.

6.2 Background to the Study

Most banks in Southern Africa are characterised by the accumulation of non-performing wholesale and retail loans (NPLs) which have rendered their credit risk modelling and risk management policies and strategies ineffective. The above developments have seen most banks in the region being unable to meet their minimum capital requirements (MCRs), grow asset bases and shareholders' wealth, face solvency and liquidity challenges and/or liquidation. It is therefore against the above background and challenges that this study is motivated to investigate the impact of the inclusion of market friction in credit risk models on bank financial performance in Southern Africa in their desire to grow and develop.

Shen (2014) defines the concept of market friction as financial costs comprising transaction costs and taxes on capital gains. He goes further to explain that market friction is not always a monetary cost as it includes incentives and commissions for agents and fees for brokers. In other words, the concept of market friction is taken to encompass costs of transacting financial operations such as capital and concentration charges, computers or machines, time-to-recovery and insider trading costs. The concept of market friction is extended to cover costs incurred by lending institutions from poor composition of banks' boards of directors (BODs), weak implementation of corporate governance, ethics and fraudulent transactions by loan officers and asset-liability committees (ALCOs).

Market friction is therefore anything that prevents a financial transaction from being executed smoothly and transparently (Fuchs and Uy, 2010). It can be taken to mean any reason which influences the process of decision-making of the investor in making financial transactions. Market friction can arise from misinformation about a financial product or the process of getting the exposure in the product, to the various legislative and legal hurdles and/or taxes levied on the transactions or any tedious and cumbersome activities that are likely to standing in a line to conduct a transaction, which might end up altering a preceding decision. Liao and Lu (2009) studied the effects of agency and information asymmetry issues on credit risk evaluation in American banks. The study by Liao and Lu (2009) discovered that both caused significant deviations in credit risk evaluations of structural form models from agency ratings.

Duffie (2003) argues that innovations in credit risk management and transfer were central if banks were to attain financial stability. The above view by Duffie was in sync with works by also reiterated by Fuchs and Uy (2010) which concluded that lack of financial innovation and financial deepening seriously retarded growth and development of financial institutions. Total overhead costs are intimately related to market friction which the study seeks to factor into existing financial models to make them suitable to conditions existent in banks in emerging economies. Friction costs are the direct and indirect costs that are associated with execution of financial transactions, for instance, the fees and commissions along with total investments by banks. Macroeconomic factors (or system-wide variables) were identified as factors that cause banks' credit risk to rise (Bliss and Kaufman, 2013).

Bliss and Kaufman, (2013) went further to note that the Central Bank's monetary policy can negatively impact on commercial banks' risk taking behavior. The study found out that monetary authorities' austerity policies were stronger than expansionary policies and damaged the financial sector and commercial banks in particularly those in emerging countries. Direct transaction costs are total costs that financial borrowers incurred in the process of applying for a loan from a banking corporation. The costs included, for example, processing, insurance and additional costs that accrue on the loan obligation in the event of the failure of the borrower to settle both the principal and interest amounts as and when they fall due (Fuchs and Uy, 2010).

Overheads, general and administration costs according to Fuchs and Uy (2010), are divided into two main categories of indirect costs that have a significant bearing on the financial growth and development of banking institutions in emerging economies. Hence this research study seeks to factor both direct and indirect transaction costs into existing credit risk models in its desire to accurately measure the financial performance of banks and similar financial institutions in developing countries. It has also been discovered that market friction is intimately related to corporate governance and economic rent particularly in developing countries. Williamson (1996) states that there are several theories that are concerned with financial sector problems, for instance the transaction costs and resource-based theories which focus on governance choice and economic rents respectively. The transaction theories cited by Okafor and Fadul (2019) and Williamson (1996) go further to analyse market friction with respect to convergence and diversion of human factor, decision making, uncertainty and organizational factors, such as insider lending and their impact on overall performance of firms. The term economic rent is defined as the amount of money that the owner of land, labour or capital must receive in order to let someone else use the factor of production (FOP) respectively.

Therefore, this research study was set to extend present-day financial credit risk models to include market friction which is not included in current models. Hence in the absence of market friction existing credit risk models become problematic to apply in estimation of risk metrics in most banks in developing countries. Market friction has been observed over time to be a real serious problem in most banks in developing countries that constrained their capacity to grow and develop. Most banks in Southern Africa are facing a multiplicity of challenges such as very high transaction costs that ranged from processing charges, commissions, interest obligations, low returns to savers, lending to connections and non-creditworthy borrowers to poor application of corporate governance and ethics.

6.3 Literature Review

According to Elizalde (2005) countries of the world use reduced and structural models in estimation of credit risk in banking institutions. Although these models are based on unrealistic assumptions such as constant risk free rates of return and constant asset volatilities they are acknowledged as the benchmark for valuation of assets and risk metrics of banks. Elizalde goes further to argue that reduced-form models do not consider the link between default and firm value in an explicit manner in the modelling of credit risk. Reduced form models go further to specify recovery rates (RRs) after credit events have happened in banking corporations. However structural models on the other hand do not determine the time to default using the value of the firm, but take this variable to be an exogenous jump process parameter governing default hazard rate inferred from observable market data. Structural models are appraised for providing linkages between the credit quality of a firm and the economic and financial conditions it faces. According to Crouhy, Galai and Mark (2009) structural models do not specify RRs but provide values of assets and liabilities that are at default to be used in estimation of the recovery rates. This flexibility in structural models suits very well the varied circumstances of banks in Southern Africa. Hence the expected loss model proposed by the study was derived from the structural models because of the robustness they contain in estimation of credit risk facing financial institutions.

The current credit risk models used by international banks are based on the stipulations of the Basel Committee on Banking Supervision (BCBS, 2009)'s Basel I, II and III Capital Accords. According to BCBS (2009) internal-based rating (IRB) models, credit risk modelling may indeed result in better risk management systems. The banks' IRB models can also be used in supervisory oversight frameworks of banking corporations including those in developing countries such as those in Southern Africa provided they are adjusted for market friction. Kurtz (2018) proposes a new model for the evaluation of capital charges for concentration of credit risk. Kurtz's model holds when economic capital measurements are conducted within a multifactor Merton (1974) framework. The concept of concentration charge is defined through the impact of a particular sector on a portfolio's credit loss curve. One of the study's main propositions is that the Monte Carlo simulation should be used in credit risk modelling in banks. This is because the simulation does not require the calibration of additional parameters and hence was easily applicable to banks that performed simulations. Secondly, the simulation method has a tractable analytical formula that provides an efficient approximation because it is a simple or initiative location of the resultant capital charge. The study by Kurtz (2018) concludes that the simulation model was suitable for use in modelling capital charges for sector concentration risk under pillar II of the Basel II Capital Accord.

A recent study on credit risk by Chen (2018) proposes a new loss given default (LGD) model to address the missing and sample selectivity biases found in real life experiences. Chen (2018) proposes a time to recovery survival model for the estimation of the LGD model with varying performance windows. Using an existing LGD data set, Chen performed five specification tests to evaluate the new approach to LGD modelling. The study realized that a trade LGD model (one that omits time to recovery and ignores censoring) was biased when applied to non-defaulted performing loans in which the time to recovery was unknown. This problem was addressed by proposing yet another new modelling approach. The approach entailed predicting both existing work out LGD data set comprising both censored and uncensored recoveries (Chen, 2018). The model performed by Chen ensured that the new approximation model fitted data well resulting in a higher LGD prediction and marginal-sensitivity to triangles. It is important to note that a number of contemporary credit risk models such as Okafor and Fadul (2019), Wang, Zhao and Peng (2018), Zhang, Lu and Sang, (2014), and Merton (1976) and references therein, compete to explain the factors that impact on bank credit risk. The concept of bank credit losses is mainly influenced by three main traditional factors, namely PD, EAD and LGD.

Virginia (1988) developed a transaction cost model and realized that transaction costs were mainly influenced by the amount of loan applied for, real interest rates and land owned by the borrower. He further argues that dummy variables such as collateral security, delinquency of the loan, Central Bank policies, the borrower's distance from the bank and the year in which the loan was borrowed are also part of the transaction costs. According to Aymanns et al (2016) banks needed good understanding of the link between solvency and funding risks to be able to assess their fragility efficiently and effectively. According to Altman and Kuehne (2014) credit bubbles are becoming more common for several credit asset classes to which banks are exposed. They proceed to argue further that credit bubbles have been observed to increase sharply with increases in corporate bonds and default on loans. It has also been observed that crises in credit and equity markets have contributed to periods of unfavourable price movements and increases in volatility in the above asset classes (before bursting of bubbles) and hence the need to manage the risks for growth and development of banking corporations.

Most banks in Southern Africa have gone through a lot of changes and challenges in the 21st century whose impact on the financial sector cannot be quantified and compared with other emerging and stable economies of the world. For instance, Barclay's Bank Africa has recently been sold due to its failure to meet certain financial benchmarks that the shareholders were expecting over a long time period (Zhang, Lu and Sang,2014), Tang and Fang,2011). Therefore, a number of developing countries and those in Southern African countries in particular have financial sectors that have not adopted certain international banking standards for them to be globally recognized, efficient, stable, well-capitalized, competent and developmental. Hence wholesale credit risk models need to be developed that suit the regional countries' circumstances and capital bases in their quest to grow and develop.

6.4 Risk Metrics for the Proposed Expected Loss Model

This section discusses the approaches used in the estimation of risk metrics of banking corporations namely the PD, LGD, EAD using structural credit risk models. However structural credit risk models are based on assumptions of frictionless markets together with constant risk free rate of return and asset volatilities.(Black-Scholes, 1973 and Merton, 1974). These assumptions are far from explaining the reality found in financial markets in most emerging economies such as those in Southern Africa. Hence the research proposed and validated an EL model for implementation by banks in fuzzy financial markets characterised by market friction (Duffe, 2003). Fuzziness is defined by Zadeh (2008) and Zimmermann (2001) as a market condition in which returns to financial market investments are not precisely defined as in probability theory but expressed in linguistic terms such as high, average or low. This implies that the concept of fuzziness is intimately related to uncertainty as characterised by vagueness, generality and ambiguity (Zhang, 1998 and Zadeh, 1973).

Fuzziness is founded on the principle of continuous variables in the range (0;1) and not exactness or discrete variables as under structural credit risk models. The study calculated three market friction adjusted risk metrics namely PD, EAD and LGD required in the estimating of ELs of banks in fuzzy financial markets. Therefore fuzziness does not have a well-defined set of bounds and is not resolvable with specific reference to context as opposed to the other terms (Qin and Li, 2008, Zadeh, 2008). The other terms vagueness, generality and ambiguity can be contextually eliminated and conclusions that are closely linked to investors' language judgements can be made. It is a fact that integral applications that combine linguistic variables and pragmatism are more powerful and beneficial to individual investors and firms and hence the need for new credit risk models that suit in a given financial market conditions facing most banks in emerging economies..

6.4.1 Approach to Estimation of the PD of a firm

The fundamental accounting equation of a firm is given by

$$A = E+L, \tag{6.1}$$

where A is total assets, E is total equity and L is total liabilities (Elizalde, 2005 and Merton, 1974).

Merton (1974) was the first theorist to transform the Black-Scholes (1973) option pricing model into a valuation model for estimation of assets and risk metrics of firms. Although the Black-Scholes option pricing model was so flexible in application, it was founded on unrealistic assumptions that necessitated the need to extend structural models to capture market friction and uncertainty. The research therefore extended the structural PD model to the case for inclusion of market friction and uncertainty in valuation of PDs of banks in emerging economies. According to Elizalde (2005) a firm defaults on its obligations when its assets are less than its liabilities. This is because its Equity will be negative, which can be given away at zero cost. Structural form models are also known as firm-value models. Merton (1974) acknowledges that the liabilities of a firm consist of one zero coupon bond with notional value, L maturing at time, T and will have no payments until, T at which default decision is taken. The PD is defined as the probability that the value of a firm's assets, A< L, its liabilities, at time, T.

The probability distribution of a firm's assets at time, t is developed on the assumption that the firm's assets follow a lognormal distribution (Buhm, Overbeck and Wagner, 2003). The logarithm of the assets of a firm follows a normal distribution (ND) at T. In other words once the mean and variance of the credit exposures of a firm have been estimated, its risk metrics such as expected loss (EL) can then be calculated. Merton (1974) uses the Black-Scholes model to model the default behavior in a financial organization. The study combines structural and reduced form models in order to come up with a hybrid PD model for banks in emerging economies. Reduced form models are based on credit spreads on non-defaulted risky bonds or loans trading on markets currently. Spreads that lie above treasury bonds for instance are an indicator of risk premiums that are demanded by investors. According to Oksendal and Sulem (2009) spreads normally reflect ELs including PDs, LGDs and liquidity premiums. The study sought to extend the existing structural credit risk model for PD to include a market friction component. Therefore the famous Merton's asset valuation model (AVM) that was extended to the case for market friction in this study was premised on two simultaneous linear equations founded on the assumption that firms' asset values and volatilities, VA and σA are unknown. The two equations for estimation of firms' asset values and standard deviations if not known are as outlined below.

Market Value of Equity is given by,

$$\text{VE} = \text{VA} \times N(d_1) - Xte^{-rT} \times N(d_2) \tag{6.2}$$

The volatility of equity of a firm is given by,

The standard deviation of equity,

$$\sigma E = \frac{VA}{VE} \times N(d_1)\sigma A; \tag{6.3}$$

However for the research at hand firms' asset values were given in their financial statements and only asset volatilities were calculated. The research extended the Merton's structural PD model,

$$\text{PD} = \frac{N[ln\left(\frac{VA}{Xt}\right) + (r - \frac{\sigma_A^2}{2})T]}{\sigma_A\sqrt{T}}, \tag{6.4}$$

to the case for a PD model adjusted for market friction given by the general form,

$$\text{PD} = \frac{N[ln\left(\frac{VA}{Xt}\right)+(\mu RE-\mu CE+\frac{\sigma_A^2}{2})T]}{\sigma_A\sqrt{T}}. \tag{6.5}$$

Where; VA=Value of firm's Assets and VE=Value of the Firm's Equity, T=The tenure of the asset, μRE =The return on ordinary equity, μCE = The cost of ordinary equity (market friction).
On the other hand $N(d_1)$= The cumulative normal probability distribution of the Z-Score, d_1 and $N(d_2)$=The cumulative normal probability distribution of the Z-Score, d_2.
The extension of the structural PD model was reached in the desire to make the model suitable to the financial circumstances of banks in developing regions such as Southern Africa.

6.4.2 The Estimation of the EAD

This is the amount that a bank is expected to lose in the event that the obligor will default on a loan obligation. According to the Bank for International Settlements (BIS) and Basel Committee on Banking Supervision (BCBS, 2009), EAD must not be lower than the book value of the Statement of financial position (SFP or balance sheet) receivables and should be calculated at the facility level. The EAD of a firm can be based on lines of credit or derivatives that is vanilla and on the counter (OTC) instruments or depending on movements of certain asset classes. The methods to be used in the modelling of credit derivatives include current exposure methods (CEM), standardized methods (SM) and internal model method (IMM). However under the internal ratings based approach (IRB), EAD can be calculated using the Foundation approach (F-IRB) based on lines of credit and off-balance sheet (OBS) transactions (Zhang, Lu and Sang, 2014). The traditional EAD is calculated using credit conversion factors (CCF) that are provided for in the Basel guidelines excluding collaterals and guarantees or securities.

On the other hand the EAD of a firm can also be estimated using the advanced approach (A-IRB) which allow banks to use own models. In other words A-IRBs accord banks the flexibility to generate or select models for use in calculating their EADs. Under the CCFs, the amounts owed by borrowers to the bank at time T =EADs (Elizalde, 2005). These can either be fixed or variable exposures. Fixed exposures are exposures that banks have not made commitments to provide credit in the future and on-balance sheet (OBS) values such that EAD=Drawn Credit Lines that is EAD =The Current Amount Outstanding on a firm's balance sheet and hence no modelling is required for Basel II Requirements (Zhang, Lu and Sang, 2014; Tang and Fang, 2011). On the other hand variable exposures are exposures under which banks will provide future commitments on in addition to the current credits that is such exposures have both on and off BS values.

In other words the firm's EAD was estimated in this study using the formula,

$$\text{EAD} = [\text{Drawn Credit Lines} + \text{CCF} \times \text{Undrawn Credit Lines}](1\text{-MF}), \tag{6.6}$$

where

$$\text{CCF} = \frac{Increase\ in\ Exposure\ Until\ Default\ Day}{Maximum\ Possible\ Increas\ in\ Exposure\ Until\ Default\ Day}. \tag{6.7}$$

and MF is market friction or costs of issuing loans in this case.

Calculated CCFs must be checked for appropriateness for current macroeconomic scenarios before being used in the calculation of EADs of firms (See Zhang, Lu and Sang, 2014; Tang and Fang, 2011). The study at hand intends to adjust the above EAD model for market friction for instance corporate governance costs to enhance its robust in estimation of EADs for banks in Southern Africa where markets are highly frictional, unlike the case in developed countries.

6.4.3 The Formula for Calculation of LGD

A bank is said to have incurred a loss when a company to which it has lent out money defaults on its principal and interest obligations. According to the Bank for International Settlement (BIS, 2018), default on a credit exposure is said to have occurred when one or more of the following events have taken place.

.The obligor is past due more than 90 days on a credit obligation.

.The obligor has filed for bankruptcy or similar protection from creditors and

. The LGD is the percentage loss rate on the EAD given the obligor's defaults.

The actual loss incurred by the bank=LGD× EAD (Zhang, Lu and Sang, 2014; Tang and Fang, 2011). The components of loss to be incurred by the bank are the loss of the principal, carrying costs and workout expenses. It should however be noted that firms' LGD values are known for varying with economic cycles namely cyclical LGDs (Point in time LGDs), long run LGDs

(Throughout the cycle LGDs) and downturn LGDs. Cyclical LGDs are based on recent data and depend on economic cycles while long term LGDs are average long term LGDs corresponding to noncyclical variables that do not depend on the time at which the LGDs are calculated. Downturn LGDs represent the LGDs of firms at the worst time of the economic cycle, say at the lowest peak of a recession. The Basel II Framework (See BCBS, 2009) requires that LGDs of firms must reflect downturn conditions wherever it is necessary to capture relevant risks facing the organization. It is also recommended that banks should use downturn LGDs when credit losses for given asset classes are expected to be higher than the averages. Therefore under the F-IRB approach, senior claims on sovereigns, corporates and banks that are not secured by acceptable collaterals are given higher LGD values of 45% and subordinated claims are given LGD values of 75%. Under the A-IRB approaches, LGDs should be estimated using any of the following internal rating methods.

.The market LGD, based on market values of defaulted bonds or loans.

.Workout LGD, based on cash flows from a firm's workout processes.

.Implied LGD, based on the market prices of non-defaulted bonds or loans and

.Statistical LGD, based on regression techniques on LGDs and facility characteristics for example qualitative forms of market friction such as spreads and macroeconomic environment.

It can be argued that of the four LGD methods above only market and implied LGDs approaches are less computation intensive and normally work well for liquid financial market instruments. Banks are therefore advised to use market or implied LGD approaches to estimate their LGDs under the above conditions and employ workout LGD methods when they hold illiquid and non-marketable instruments, which is usually the case in most emerging economies (Zhang, Lu and Sang, 2014; Tang and Fang, 2011). However under conditions of large exposures, banks should apply techniques that make it possible to estimate more precise LGDs. For forecasting of LGDs statistical LGD methods should be used as long as it is possible to establish dependent and independent linear relationships. The LGD under the workout approach is estimated from the equation,

$$\text{LGD} = \frac{EAD_T - PV[\sum R_t) + PV(\sum C_t)}{EAD_T}, \quad (6.8)$$

where

PV (R_t) and PV (C_t) are recoveries and costs incurred during workout prices and processes respectively.

The implied LGD approaches are based on observed market information such as stock prices and hence the usefor instance of the Merton model, as specified in this study. On the other hand statistical LGD approaches stipulate that a firm's LGD lies between values of 0 and 1 (Buhm, Overbeck and Wagner, 2003). Hence the study estimated banks' LGDs according to Buhm, Overbeck and Wagner (2003) model after transforming banks' LGDs into a variable,

$$X_t = = \text{Log}\frac{LGD}{1-LGD}, \tag{6.9}$$

to suit into the current family of logistic models where,

$$X_t .= \alpha_0 + \alpha_1 y_1 + \alpha_2 y_2 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots + \alpha_n \, y_n. \tag{6.10}$$

The above logistic model for LGD estimation is applicable when,

.Only significant variables are incorporated into the model.

.The variables used have economic meaning in explaining the variability in firms' LGDs.

.Independent variables are able to explain the LGDs significantly and

.The financial data collected should be properly processed leaving out all outliers (Buhm, Overbeck and Wagner, 2003).

6.4.4 Proposed model for estimation of ELs of banks

After estimation of all the three risk metrics above, ELs of banks can be evaluated based on realistic market circumstances faced in developing countries such as those in Southern Africa (Oksendal and Sulem, 2009). The study therefore proposes an expected loss (EL) model for banks' credit exposures given by:

$$\text{EL} = \text{F(PD; EAD; LGD; MAF)} \tag{6.11}$$

where all the independent variables are market friction adjusted parameters. A model of this nature suits very well the fuzzy or uncertain nature of human behaviour when it comes to making planning and decision making processes in financial markets.

6.5 Model Validation, Results and Discussion

The proposed model above is validated using financial data conveniently drawn from ten listed banking corporations that is five foreign and five indigenous banks conveniently drawn from five emerging economies in Southern Africa, for the period 2008-2020. The study targeted both indigenous and foreign banks in order to compare their individual operating circumstances and contributions to the countries' financial sectors and economies respectively. Logit and logistic models were used to estimate expected loss (EL) values of the sampled banks. These linear regression models are mainly used for predicting dichotomous outcomes such as events that result in success and/or failure outcomes. Logistics models are frequently used in economic and financial modelling because they are so flexible that they can also be used to predict odds dependent variables from quantitative continuous random variables. The majority of dependent variables of interest to researchers such as expected loss suit well for dichotomous analyses and interpretations.

The logit model is a binary model in which the dependent variable, Y is a binary response to X is any type of covariate or independent variable, which is either dichotomous or continuous variable. In this case X assumes three independent variables, namely PD, EAD and LGD that impact on expected loss which are extended for market friction (MAF), with specific reference to the cost of capital. The exponential linear regression model, also called the logit model is a function of fuzzy variables above given by,

$$\text{Y}= e^{\sigma+\beta_1 X_1+\beta_2 X_2+\beta_3 X_3+\beta_n MF}, \tag{6.12}$$

where MF = market friction, represented by the cost of capital or equity of a firm. When the above logit model is transformed into a multiple linear model using logarithmic form it becomes a logistic regression model of the form,

$$\log\left(\frac{EL}{NEL}\right)= \sigma+\beta_1 X_1+\beta_2 X_2+\text{-------}+\beta_{n-1} X_{n-1} + \beta_n MF, \tag{6.13}$$

where EL= Probability of expected loss from a corporate borrower and NEL= Non-probability of expected loss (See, Chen, Zhang and Gupta, 2014). The Xs are covariates that is predictor variables, while σ and βs are logistic regression coefficients estimated from given odds and predictor values. The term in the brackets on the left hand side is called the odds, calculated as the probability of the success event divided by the event of no success.

The logarithmic term of the odds is a linear function of the covariates given on the right hand side of equation 6.13. The study employed a STATA package to analyse the banks' financial data because of its strength of having two commands namely the logit and logistic models. The two models are respectively used for analysing data where expected loss output could be used to generate coefficients and generate odds ratios. The study's logit and logistic models regressed log-odds of the banks based on four independent variables which are PD, EAD, LGD and MAF. The three risk metrics are expressed as percentages of total assets in order to harmonise financial data of indigenous and foreign banks in different currencies for ease comparability of the findings.

6.5.1 Distribution of foreign banks by expected losses' logit model results (2008-2020)

The five foreign-owned banks' logit regression model used was based on 180 observations and 16 replications. The following results were attained using jack-knife approach that is running logit on estimation for analysis and interpretation of research data:

The F statistic, $F(1, 15) = 4.54$ and Log likelihood = 0 and Prob > F = 0.05. .

Table 6.1 Showing Logit Model Statistical Measures for Foreign-Owned Banks (2008-2020)

Financial Ratio	Coeff	Std Error	t-Value	$P>(t)$	95% CI
PD	9.86e-06	8.68e-08	-0.98	0.684	-5.82e-09 9.78e-09
EAD/TA	8.24e-06	8.68e-08	-0.86	0.764	-4.46e-08 7.86e-09
LGD/TA	6.72e-06	6.69e-08	-0.74	0.867	-3.32e-09 5.37e-09
MAF	6.86e-06	5.71e-08	-0.63	0.658	-2.45e-08 4.28e-09
Cons	48.64				

Source: Author

Results and discussion

Based on the results in table 6.1 above the logit model connecting the variables is given by Y=48.64+9.86e-06PD+8.24e-06EAD/TA+6.72e-09LGD/TA+6.86e-09MF. The study thus banks' expected losses have a weak positive relationship with all independent variables that is PD variable, EAD, LGD and MAF. Only the constant has a strong positive contribution to banks' expected losses in the period under review. The betas of the logit model's 95% confidence intervals for all independent variables are in the range -5.82e-09 to 9.78e-09 which lies between the discrete bounds -1 and 1. The results imply that the expected loss for the bank is around 0.00 which means that banks' expected losses from loans issued during 2008-2020 period are very small. By concentrating on corporate loans foreign banks become efficient and effective in management of their credit exposures and hence their NPLs fell significantly.

6.5.2 Logit model statistical results for indigenous banks (2008-2020)

The logit regression model for indigenous banks also used 180 observations with 16 replications to analyse their expected losses over the period 2008-2020. The study came up with the following results, F statistic, F(1, 15) = 4.54 and Log likelihood = 0 and Prob > F =0.05.

Table 6.2 Showing Logit Model Statistical Measures for Indigenous Banks (2008-2020)

Financial Ratio	Coeff	Std Error	t-Value	P> (t)	95% CI	
PD	8.69e-08	8.64e-09	-0.76	0.756	-4.88e-09	2.48e-09
EAD/TA	7.86e-08	6.98e-09	0.95	0.678	-3.66e-09	3.82e-09
LGD/TA	7.39e-08	5.56e-09	0.63	0.654	-2.49e-09	5.54e-09
MAF	9.43e-08	3.78e-09	0.46	0.633	-1.62e-09	8.87e-09
Cons	28.75					

Source: Author

Results and discussion

The STATA package used drew up 0 failures and 180 completely determined successes. The results in table 6.2 above gives rise to the logit model Y = 28.75 +8.69e-08PD+ 7.86e-08EAD/TA + 7.39e-08LGD/TA+ 6.43e-08MF. The coefficients of the logit model reveal that the bank's expected losses increased as PD, EAD, LGD and MAF increased. Although all the independent variables increased expected losses of banks, their contribution represents a weak positive relationship with EAD/TA and LGD/TA ratios lower than those of PD and MAF. The 95% confidence intervals for the betas of the independent variables fall in the range, -4.88e-09 to 8.87e-

09, which lies in the discrete bounds of -1 and 1. The main finding of the study is that the banks' expected losses are concentrated around 0.00 for the period in question. Hence the indigenous banks' expected losses are overall smaller or less-dispersed compared with those of foreign banks.This could be attributed to less volumes of credit exposures issued by indigenous banks and mostly small-salary based consumer loans compared to corporate loans mostly issued by foreign banks.

6.5.3 Distribution of foreign banks by expected losses logistic model results (2008-2020)

Table 6.3 Showing Logistic Model Statistical Results for Foreign-Owned Banks (2008-2020)

Financial Ratio	Odds Ratio	Std Error	t-Value	P> (t)	95% CI
PD	1	8.67e-10	-0.97	0.665	1 1
EAD/TA	1	8.86e-10	-0.78	0.763	1 1
LGD/TA	1	6.78e-10	-0.63	0.896	1 1
MAF	1	5.93e-10	0.35	0.937	1 1

Source: Author

Results and discussion

Table 6.3 above shows that 95% confidence intervals for the betas of all variables in the logistic model contain the bound 1. This means that the banks' expected losses have no significant association with all the four independent variables drawn into the model. The t-statistic values for all variable betas are less than 0.50 which means that the model could be very reliable in predicting the expected losses of banks in emerging economies. However odds ratios of value 1 as shown in the table imply that the three risk metrics and market friction resulted in expected losses in banks the period 2008-2020.

6.5.4 Distribution of indigenous banks by logistic model results (2008-2020)

Table 6.4 Showing Logistic Model Statistical Measures (Results) for Bank S (2008-2020)

Financial Ratio	Odds Ratio	Std Error	t-Value	P> (t)	95% CI
PD	1	8.98e-09	-0.56	0.672	1 1
EAD/TA	1	7.84e-09	0.60	0.568	1 1
LGD/TA	1	6.48e-09	0.53	0.586	1 1
MAF	1	4.58e-09	0.48	0.646	1 1

Source: Author

Resutls and discussion

Table 6.4 above demonstrates that 95% confidence intervals for input betas drawn from banks' financial data contained the bound 1.00 represented by both lower and upper bounds 1.00. However the intervals imply that the bank's expected losses have no significant association with all independent variables drawn into the model. The t-distribution values for all variable incorporated in the model aee less than 0.50 suggesting that the model could be very consistent,valid and reliable in predicting the expected losses of banks in fuzzy financial environments. The model's odds values of 1.00 across the four independent variables reflect that expected losses occurred in all indigenous banks' credit exposures in the period under consideration.

However since the percentages of expected loss to total assets for PD, EAD, LGD and market friction are relatively low, the study notes that this coiuld be a result of use of improved credit risk policies and strategies in the management of credit exposures and NPLs. The 95% confidence intervals for the betas of the covariates in the model include the bound 1 which means that indigenous banks' expected losses do no have a significant association with all independent variables included in the model. However the t-distribution values for the independent variables have values less than 0.50 which depicts that the model may be fairly reliable for implementation in estimation of the expected losses of banks situated in frictional and fuzzy financial markets.

6.6 Conclusions and Recommendations

The study proposed and validated a market friction adjusted expected loss model in order give a true redflection of the characteristics of financial markets in emerging economies. The study used logit and logistic valuation models to analyse financial data drawn from audited financial statements of banks for the period 2008-13. Market friction and uncertainty were added to the model in order to increase the precision or exactness required in valuation of firms' exposures in emerging financial markets. The study therefore concluded that banks in emerging economies operated in uncertain financial markets characterised by friction, volatile interest and asset values. It was also concluded that efficient estimation of ELs was achieved through inclusion of mathematical languages or human behaviours and non-quantitative variables contrary to notions of the classical probability theory used in structural models. The adjustment of existing structural credit risk models for market friction created the rigour needed in the estimation of both asset and EL values of firms in emerging economies.

Expected loss models that were adjusted for market friction fairly reflected the firms' actual market values and risk metrics. This development was likely to go a long way in assisting banks in their planning and management of corporate loans, loan loss provisioning and decision-making processes. The study also concluded that expected losses of both firms had an indirect relationship with their PD, EAD and LGD variables. Furthermore, the study concluded that banks drawn from the region were poorly capitalized, measured in terms of the ration of ordinary equity capital compared to debt financing. The banks over-depended on borrowed capital in their capitalization which rendered them vulnerable to hostile takeovers by the providers of such capital in the foreseeable future. However since banks were able to turn both equity and debt finance into assets it implies that they were somehow hedged against hostile takeovers by bondholders. Shareholders of banks in the region needed to inject adequate equity capital into their banks to be able to grab their ownership and improve financial performance, asset accumulation and shareholders' wealth from the debt-equity funders.

Alternatively they could negotiate with existing lenders for converting their over-borrowed statuses into equity to reduce the banks' indebtedness and increase issued share capital. This strategy had the ability to reduce their exposure to interest and principal loan obligations. The study finally concluded that banks in emerging and frictional financial markets need new look expected loss models in order to be efficient in valuation of risk metrics and EL in particular. Expected loss models adjusted for market friction, uncertainty and human perceptions or uncertainty were more realistic, precise and practical compared to the structural and reduced-form models premised on assumptions of frictionless markets. New models to be used in estimating risk metrics must shift from estimation of incurred to expected losses. The expected loss models to be adopted and implemented by banks in emerging markets must be motivated by human perceptions that reporting only incurred credit losses will not provide investors with sufficient information about their true credit risk levels and metrics.

Based on the above conclusions premised on frictional financial markets, the study recommends that banks in emerging economies need to urgently come up with well formulated, coordinated and prudentially implemented financial policies and strategies in order to effectively manage their capital challenges, credit exposures, expected losses and finance their development processes through mainly equity capital. These policies and strategies were to go a long way in the betterment of banks' fortunes in terms of their regulation and supervision, capital injection and let alone effective issuing and management of their credit exposures in frictional financial markets. The study also recommends that banks must improve their assets' income generation power to enhance effectiveness in management of their loan exposures, financial obligations, both long and short term liabilities and accumulation of assets and generation of wealth for the shareholders.

The study went further to recommend that banks' boards of directors (BODs) and senior management should be re-oriented so that they shift from estimation of firm values and risk metrics using structural models. Extending current structural models to inclusion of market friction in firm valuation was not only critical but improved their precision and rigour. Factors such as perceptions of investors or shareholders, market friction and uncertainty should be included in estimation of firms' equities, assets and expected losses valuation models to improve practicability and reliability of their overall financial performance, growth and development processes. Therefore overall the study recommends that banks in emerging frictional markets can adopt and implement the proposed expected loss model as a more realistic and reflective models compared to structural models that suited frictionless markets that existed mostly in developed countries of the world. The model at hand suits very well the financial circumstances of banks such as those in Southern Africa because it was robust and sensitive to the impact of both market friction and uncertainty on the fair valuation of banks' asset, equity and expected losses.

Comments

Equation 6.1 says EL = PD x LGD x EAD suggesting that EL is a product of these three. Regression would say EL is a function of (not a product of). These are two different things. Align this.

The objective of the chapter is pasted below

This paper was set out to investigate the impact of market friction on risk metrics of banks in Southern Africa in their desire to accurately measure expected losses and financial performance in their desire to grow and develop.

Was this achieved?

Why sample two banks? Can a researcher rightly conclude something based on two banks?

The p-values suggest statistically insignificant relationsips – where is the discussion of the results obtained because it looks the candidate went straight to conclusions.

CHAPTER VII

MARKET FRICTION AND BANK FINANCIAL PERFORMANCE IN EMERGING ECONOMIES

7.1 Introduction

It has been noted that both structural and reduced form credit risk models (CRMs) are direct applications of the severity and frequency of hazard rate models in financial management of banks and other similar organisations. The research intends to extend the vector auto-regression (VAR) widely used in evaluation of profitability of banks based on both micro- and macroeconomic variables to the case for market friction. Thus the research postulates the use of dynamic profitability assessment models based on short-term asset growth rates of banks in markets that are characterised by friction and uncertainty or fuzziness. In economic and financial theory a frictionless market is defined as a financial market without transaction costs. Friction on the other hand is a type of market incompleteness or inefficiency. This means that every complete market is frictionless, but the opposite does not hold. DeGennaro and Robotti (2007) define market friction as anything that interferes with trading of firms or investors and can exist even in efficient markets. They go on to acknowledge that financial market frictions are pertinent because they generate business opportunities, costs to investors and change over time.

By the concept of fuzziness we mean a situation where ambiguity and vagueness exist in real life financial situations (Zadeh, 1980 and Zebda, 1989). Fuzziness is thus a condition in which human psychology or language is used to explain risks and returns to investment, as high, moderate or low in nature, which terms must be mathematicised before the profitability of a bank is estimated. Therefore the research proposes a fuzzy-market-friction VAR model for assessment of bank financial performance in emerging countries. The inclusion of market friction in the proposed VAR model is meant to improve precision and accuracy in the estimation of profitability of banks. The profitability of growth rates of banks are described by a system of stochastic differential equations (SDEs) which considers their asset values as evolving processes from time to time. Contemporary structural bank performance models or valuations based on both industry (firm-specific) and macroeconomic variables are known for suffering serious estimation errors. Hence SDE adjusted models are used because they have the ability to improve estimation accuracy and precision since they are based on practical financial circumstances facing banks particularly in emerging economies.

In light of the above developments in emerging markets, the VAR model was adopted because it employs an extra advantage of the lagged first difference of the dependent variable to enhance the efficiency of the estimator and curtail the problem of weak instruments in difference models. Taking first differences in VAR model eliminates the firm specific effects on the dependent variable. The proposed use of VAR model specification is motivated by its ability to control correlation of errors over time, measurement errors, heteroskedasticity across firms and simultaneity due to utilisation of orthogonal conditions on the variance- covariance matrices (Antoniou et al, 2008). This paper presentation is organised as follows; section two covers a brief background to the study, section three presents literature review, section four the research methodology and sections five and six are centred on validation of the model and conclusions and recommendations respectively.

Background to the study

Most banks in Southern Africa are characterized by the accumulation of non-performing wholesale and consumer loans (NPLs) over time. The existence of NPLs in most banks and similar financial institutions is an indicator that there are poor and weak credit policies of banks that are a major source of their perennial liquidity and profitability challenges. VAR approach by developed by Blundell and Bond (1998) has been applauded for its robustness and ability to fairly value bank profitability based on quantitative variables such as return on equity (ROE), return on assets (ROA) and lagged profit level (profit from preceding period). It is argued that the lagged profit level of a bank is a formidable variable for measurement of its current profit level. The presence of market friction and fuzziness of markets in most emerging economies have seen profitability of most banks dwindling and making them fail to meet their minimum capital requirements (MCRs) by Central Banks, grow asset bases, wealth for shareholders, solvency and liquidity requirements.

According to Shen (2014) market friction refers to financial costs faced by banks in their construction of investment portfolios and comprises capital and transaction costs and taxes levied on capital gains. Shen also explains that market friction is not always a monetary cost or variable because it can also include incentives and fees for brokers or commissions for agents. The concept of market friction is extended to cover all costs incurred by lending and investment banks such as poor composition of banks' boards of directors (BODs), weak corporate governance, ethics and fraudulent activities by loan officers, treasury and investment managers and asset-liability committees (ALCOs). From the definitions above, market friction can be taken to be anything that prevents financial and investment transactions from being executed smoothly, economically and transparently (Bliss and Kaufman, 2013). It can be taken to mean any reasons which influence the processes of planning and decision-making of banks in making their financial performances grow and develop over time. Because of the voluminous nature of market friction in financial markets, we see it fit to extend the current VAR model for this variable in measurement of bank profitability to increase accuracy and account for contribution of human psychology in this endeavour.

Duffie (2003) argues that banks in emerging economies need to incorporate innovations in their credit risk and profitability models if they are to attain financial stability and growth. Lack of financial innovation, widening and deepening in emerging economies such as Sub-Saharan Africa have seriously crippled their capitalization, profitability, solvency and liquidity positions over time. Friction costs are therefore divided into direct and indirect costs both of which are associated with execution of financial transactions, fees and commissions along with total investment costs by banks. Direct transaction costs are total costs (which are assumed to be zero in efficient markets) that borrowers and banks incur in the process of applying for loans from financial corporations (Fuchs and Uy, 2010). These costs could include processing, insurance and additional costs that accrue on the loan obligations in the event of the failure of the borrower or bank to meet both the principal and interest amounts as and when they fall due. They define indirect costs as costs that do not have direct effect on loaning and investment decisions of a bank as is the case for cost of capital, interest and exchange rates. Indirect costs can include bank overheads, general and administration costs which have a significant bearing on the financial growth and development of banking institutions in emerging economies and hence need to be included in the VAR model to come up with precise and fairly reflective bank profitability results.

Economy or system-wide factors commonly known as macroeconomic variables are identified as factors that are outside the scope or control of banks which cause banks' credit risks to rise and make their profitability abilities fall (Bliss and Kaufman, 2013). Central Banks' monetary policies must be made autonomous, transparent and efficient because if they are weak they can negatively impact on commercial banks' risk taking behavior in lending and investment decisions. The study above realized that monetary authorities' austerity policies such as those used in Zimbabwe (such as IMMT) were stronger than expansionary policies and could damage the financial sector as a whole and commercial banks in particularly in emerging countries. The study thus incorporates both controllable (firm-specific) and market or economy-wide (uncontrollable) variables in order to efficiently and effectively measure bank profitability in the quest to grow and develop their businesses.

Statement of the problem

Market friction has been observed over time to be a real serious problem in most banks in developing countries that constrains their capacity to grow and develop. Most banks in Southern Africa are facing a multiplicity of challenges as alluded to above which have a direct impact on bank performance and cannot be swept under the carpet. Banks in fuzzy financial markets are characterised by high transaction costs which include transaction processing charges, commissions, fees, costs of capital, interest obligations, low returns to savers, lending to connections and non-creditworthy borrowers to poor application of corporate governance and ethics (Duffie, 2003). It is against the above background and challenges that this study is motivated to investigate the impact of extending the VAR approach for market friction in measurement of profitability of banks in their desire to grow and develop.

7.2 Literature Review

There are three central issues facing companies or banking corporations, namely financing, investment and liquidity concerns in their quest to boost their profitability levels. These three variables always generate a lot of debate in financial performance of banks in both developed and emerging markets and economies (Fuchs and Uy, 2010). Banks' financing and liquidity decisions are known for impacting significantly on their market asset values and investments and the whole corporation value at large. It is believed that by effectively managing their investments and credit risks which have a direct bearing on their operations, banks' capital bases, leverages, solvency, profitability and ability to grow and develop can be significantly enhanced. Literature on performance of banks reflects that both firm- specific and macroeconomic factors impact on their profitability, liquidity and cash flows which are their life blood for growth and development. In light of these observations we extend the VAR technique for market friction which characterizes the real costs faced by banks, in their quest to improve financial performance needed in accumulating assets and growing wealth for their shareholders.

The research adopted the VAR approach to measurement of bank profitability as a fairly efficient model compared to structural and reduced firm valuation models which are founded on assumptions such as availability of efficient and frictionless markets. Structural firm or asset valuation models, such as Merton (1974) and Black-Scholes (1973) assume that there are no transaction costs incurred in market trading, as is the case in developed markets and economies, the converse holds in the majority of emerging economies. These unrealistic assumptions on which structural and reduced form asset valuation models (AVMs) are built, make their credit, bank and profitability estimates very questionable in terms of rigour and accuracy in prediction. Hence contemporary credit and asset valuation models used in examining the impact of both specific and market variables on bank profitability must be extended to the case for market friction and fuzziness of financial markets to make them credible and fairly reflective of practical market conditions (Fuchs and Uy, 2010). Most structural models are also founded on the other assumption that financial markets are efficient but in reality most of these markets, particularly in emerging economies are far from being perfect as they are characterised by asymmetric information and fuzziness.

Hence it is on the basis of the above practical orientations or observations that the VAR technique is to be extended to include market friction and fuzziness of markets so as to improve its precision, robustness and ability in estimation of bank profitability. The research proposes the use of dynamic profitability measurement models that are based on short-term growth rates of asset of banks which are described by a system of stochastic differential equations (SDEs). The SDEs consider the short-term asset values of banks as evolving processes from time to time. Previous models that use both firm-specific and macroeconomic variables in measurement of bank profitability are known for suffering from serious estimation errors and hence adoption of SDE would improve accuracy and precision in estimations based on practical financial circumstances facing banks in emerging economies (Fuchs and Uy, 2010). Interest has emerged in bank financial management that we examine the relationship that exists between bank profitability and market friction under fuzzy firm-specific and market-wide factors that are characteristic of most banks in developing economies.

Studies based on market friction and fuzziness have not gained much attention since most literature available reveals that estimation of bank profitability has been carried out mostly in developed countries under critical assumptions such as existing of perfect financial markets and no transaction costs incurred by investors. The proposed VAR model, extended for market friction is motivated by its ability to control correlation of errors of variables over time, measurement errors, heteroskedasticity across firms and simultaneity due to utilisation of orthogonal conditions on the variance- covariance matrices (Antoniou et al, 2008). This study therefore seeks to provide novel and substantial evidence on the impact of market friction and fuzziness (using GMM technique) on investment and profitability in the context of banks in emerging or developing markets with specific reference to those in Southern Africa which have not been explored before.

7.3 Research Methodology

The research proposes a panel data econometric model for assessment of bank financial performance extended for transaction costs in fuzzy financial markets such as those in Southern Africa. This section specifically presents sources of data, the VAR model adopted by the study and its independent variables (bank specific and market-wide factors), tests performed and techniques used in estimation of financial performance of banks in Southern Africa.

7.3.1 Sources of data

The study used audited financial statements of sixteen banks conveniently drawn from six Southern African countries namely South Africa, Namibia, Botswana, Malawi, Tanzania and Zambia (World Development Index, WDI, 2022). These sources of data are credible and useful and can be downloaded directly from the Website for one's research purposes.

7.3.2 Independent and Dependent Variables of the Proposed Multiple VAR Model

The research caught up with a number of bank and market factors that impact on bank performance as outlined below.

Firm specific factors affecting bank performance

The firm-specific factors that were drawn into the multiple VAR model for assessing bank performance in the presence of market friction are bank profitability for the preceding year, bank size, bank expected losses, and boards of directors (BODs).

Macroeconomic factors affecting performance of banks

The macroeconomic variables that the study used to measure bank performance are economic growth or gross domestic product (GDP), unemployment, inflation, interest and tax rates of countries drawn into the study. The GDP was measured separately from the other system-wide variables above that were lumped together into the market friction factor (MAF).

Measures of bank financial performance

The financial performance of the banks were measured using four proxies namely return on assets (ROA), return on equity (ROE) and return on investment (ROT). The impact of firm-specific and macroeconomic factors on bank financial performance is most frequently analysed through panel data regression models. Panel data have the advantage of reducing the co-linearity among explanatory variables, hence improving the efficiency of econometric estimates. The study employed unbalanced panel data of sixteen banks, after checking and screening them for apparent coding of errors and missing variables.

Panel data models are recommended ahead of other forms of data because they allow multiple phenomena obtained over multiple time periods to be observed simultaneously, increases the degrees of freedom from error and reduces co-linearity among variables, leading to improved efficiency and consistency. Both firm specific and macroeconomic variables faced in financial markets are the bedrock of bank financial performance and hence should be modelled using dynamic panel data models to assist in dealing with endogeneity problems found in the real world of financial investment. The panel data used are obtained from Audited Financial Statements of sixteen listed commercial banks based on homogenous items on the statements. The proposed model's independent firm-specific and macroeconomic variables are as tabulated below.

Table 7.1: Showing Measurements of Firm Specific Factors Affecting Bank Performance

Variable	Measurement	Formulae / Proxy
Preceding Year's Bank Profitability	BAP_{it-1}	$\frac{Operating\ Profit\ for\ the\ Past\ Year}{Total\ Sales}$
Size of Bank	SOB	Natural logarithm of total assets of bank
Total Expected Loss of Bank	BEL	$\frac{Total\ Expected\ Loss\ of\ a\ Bank}{Total\ Assets}$
Board of Directors	BOD	BOD size and Composition

Source: Author

Table 7.2: Showing Macroeconomic Factors for Bank Performance Measurement

Variable	Measurement	Formulae / Proxy
Government Tax Rates	GVT	Country's Taxes (%)
Inflation Rates	IFL	Country's Consumer Price
Interest Rates	INT	Prime Interest Rates
Unemployment Rates	UER	Annual Unemployment Rates

Source: Author

Table 7.3: Showing Common Measures of Bank Financial Performance

Bank Performance	Measurement	Formula / Proxy
Return on Investment	ROT	$\frac{Earnings\ After\ Tax}{Total\ Investment}$
Return on Equity	ROE	$\frac{Profit\ After\ Tax}{Total\ Market\ Value\ of\ Equity}$
Return on Assets	ROA	$\frac{Earnings\ After\ Tax}{Total\ Assets}$

Source: Author

7.4 Proposed Econometric VAR Market Friction and Bank Performance Model

The values of the dependent variables (various bank returns) can be used as regressors (X factors) in order to account for their impact on bank profitability. Under such a scenario the general form of the multiple regression model proposed for bank profitability (BP) can be specified as:

$$BAP_{it} = \propto \ + \rho \ BAP_{it-1} + \beta X_{it} + \mathcal{E}_{it} \qquad (7.1)$$

The proposed model for assessing of the factors affecting the financial performance of banks is a transformation of model adapted from previous empirical studies by Fosu (2013) and Chadha and Sharma (2015). The model captures firm-specific and macroeconomic factors together with control variables as other factors that can also influence bank financial performance. Thus the model proposed by the research can then be given by:

$$BAP_{it} = \propto_0 + \rho BAP_{it-1} + \ \beta X_{it} + \sum_{k=1}^{N} \theta_k \, Z_{kit} + \mathcal{E}_{it} \qquad (7.2)$$

where BAP_{it} is a measure of financial performance for a bank in year, t, X_{it} stands for the controlled bank variables which include bank expected loss (BEL), size of bank (SOB) and board of directors (BOD) (size and structure). The variable Z_{kit} is a measure of uncontrollable variables, namely tax, interest, inflation and unemployment rates bunched into a single systematic factor (market friction, MAF). The lagged profitability variable (BP_{it-1}) is included in the regression model because it significantly influences the bank's current year's capital expenditures and profitability level. The term, BP_{it} is current year's bank profitability level measured by return on investment (ROT), return on equity (ROE) and return on assets (ROA) for bank, i, in year t, ρ and β are multiple linear regression parameters to be estimated and $\mathcal{E} =$ The error term. Hence the X_{it} and Z_{kit} sets of variables for the research study are firm-specific variables (controllable variables) and macroeconomic (uncontrollable factors) as listed on Tables 7.1, 7.2 and 7.3 above.

7.4.1 Specific VAR Panel Data Model for the Study

The analysis of secondary data drawn from financial statements of banks was done using *Panel Data Linear Regression Analysis* that involves various analytical techniques. Panel data are said to be repeated observations on the same cross section, typically of individual variables that are observed for several time periods (Baum, 2006). They also provide a major means to longitudinal analysis of data especially data that are from various sources and the time series that are rather short for separate time series analysis. The Multiple VAR model used for measurement of bank performance (BAP) is given by:

$$\text{BAPit}=\beta_0 + \beta_1 BAP_{it-1} + \beta_2\text{SOB}+\beta_3\text{BEL}+\beta_4\text{MAF}+\beta_5\text{GDP}+\beta_5\text{BOD} \quad (7.3)$$

where and $\beta_0 =$ Autonomous bank profitability, $BAP_{it-1} =$ Profitability of the bank from the preceding period, SOB = Size of bank, BEL= Bank expected loss, GDP =Gross domestic product, MAF = Market factor or friction (average of tax, interest, inflation and unemployment rates), BOD = Board of directors and betas (β_{is}) are the sensitivities of various bank performance measures to all model independent variables.

7.4.2 Assumptions of the proposed multiple VAR model

There are four principal assumptions which justify the model for the purpose of prediction of bank profitability namely:

.Linearity of the relationship between dependent and independent variables

It is the relationship between two variables that can be connected using a straight line on a Cartesian plane. Thus it is a measure of the degree to which one variable depends on the other.

.Independence of the error terms

It is the assumption that all error term variables have no serial correlation between or among them.

.Homoscedasticity assumption

It is assumed that all errors in the model have constant variance (homoscedasticity) either versus time or predictions (or versus any independent variable).

.Normality of the error distribution

Errors used in the model are assumed to follow or constitute a normal distribution.

7.4.3 VAR estimation techniques

The research postulates the use of dynamic profitability measurement models based on short-term asset growth rates of banks. These growth rates are described by a system of Stochastic Differential equations (SDEs) which considers the asset values of banks as evolving processes from time to time. Previous studies concerning measurement of bank performance based on both firm-specific and macroeconomic variables are known for suffering serious estimation errors and hence adoption of SDE to improve accuracy and precision in estimation based on practical financial circumstances facing banks in emerging economies. In light of this the VAR technique developed by VAR (1980) was employed to estimate the model. The VAR model system employs an extra advantage of the lagged first difference of the dependent variable to enhance the efficiency of the estimator and curtail the problem of weak instruments in difference VAR technique. Taking first differences in VAR eliminates the firm specific effects on the dependent variable.

The proposed VAR model is also appraised for its capacity to address problems of endogeneity from the relationship between dependent and independent variables. In the presence of the above considerations, the study employed an E-Views 8 program to conduct the main regression procedure connecting market friction and financial performance of banks in Southern Africa. Where multiple regression techniques are used, Pearson's product-moment correlations and coefficients of determination were conducted using ANOVA and Chi-Square tests. We used panel financial data of 16 banks conveniently drawn from Southern Africa over a period of 24 years (1997-2020) to validate the proposed a log-VAR model for assessing bank performance in the presence of market friction. The discussion below details the major findings of the research on all countries drawn into the study after their financial data under different currencies were harmonised through the log-VAR model above.

7.5 Validation of the Proposed Model, Results and Discussion

The VAR model is flexible because it allows us to use logarithms to analyse financial data of banks drawn from different countries in Southern Africa that use different currencies. The tables below presents the descriptive statistics generated from financial data of banks for the period 1997-2020.

7.5.1 Unit root tests for the variables of the models

By unit root we mean the constant (0) used to represent data that are normally distributed. Unit root tests are performed so as to understand the nature of the research data at hand. Unit root test is carried out to understand the nature of data used in the research. In other words the unit root test must give a constant man of zero (0) if data are normally distributed. Data are said to be at a stationary level or integral order (0) provided they do not have unit roots. This therefore means that non-stationary data results cannot be used in planning and decision making processes of banks. We performed four tests for stationarity of research data using the unit root technique under the null hypothesis that the variables are stationary and of order (1). The results of the four unit root tests performed are tabulated below **(**Appendix I).

Table 7.4 Showing Results of the Four Unit Root Tests Performed on Model Variable Data

Variable	**Levin, Lin and Chu (t^*)**	**Im, Pesaran and Shin (W)**	**ADF-Fisher-Chi-Square**	**PP-Fisher Chi-Square**
STA	-2.28738 (0.0111)	-2.21228 (0.0135)	46.8668 (0.1061)	4.5318 (0.2104)
SOB	0.27034 (0.6066)	0.42882 (0.3340)	48.3787 (0.0814)	100.01 (0.0000)
MAF	-4.8531 (0.0000)	-5.97807 (0.0000)	115.705 (0.0000)	110.064 (0.0000)
BEL	-0.21120 (0.4164)	-2.86268 (0.021)	63.7272 (0.0015)	118.492 (0.0000)
ROA	-4.84697 (.0000)	-5.52094 (0.0000)	112.12 (0.0000)	135.378 (0.0000)
ROE	1.54506 (0.9388)	-5.81859 (0.0000)	113.926 (0.0000)	150.341 (0.0000)
ROT	-4.91852 (0.0000)	-7.66035 (0.0000)	123.463 (0.0000)	147.231 (0.0000)

Source: Author

The unit root test results are Pedroni-based and they represent a summary of four unit root testing techniques that is LLC, IPS, ADF and the PP tests. The model testing results indicate that all the variables are integrated of order zero (I(0)). However SA (economic growth) and SOB are mixed between (I(0) and 1(1). Therefore, these variables suggest that we should estimate a VAR in levels. In theory SA (GDP) should have I(0) values although variation can be I(1) because of its normal behavior in practice. The study realized that SOB was not stationary across all for tests and hence we accept H0. A result of 0.0000 is significant at 5%, and 0.008 is significant at 10% level, I (0). BEL was found significant under tests two, three and four but insignificant for test 1. We also note that ROA and ROI are significant at 5% level for all four unit root tests. However ROE show significant results for all four tests at 5% level except the LLC model.

7.5.2 AR Characteristics of the inverse roots

The AR roots polynomial test results suggest that the variables employed by the study are level as per the unit root tests (See Appendix 1).

7.5.3 Lag Structure: Study performed VAR lag order variable selection criteria test (Appendix 2).

The appendix is characterised by the following variables:

. * indicates lag order selected by the criterion
. LR: sequential modified LR test statistic (each test at 5% level)
. FPE: Final prediction error
. AIC: Akaike information criterion
.SC: Schwarz information criterion
.HQ: Hannan-Quinn information criterion.

The lag structure selected for estimation of the VAR in levels is based on the Schwarz (SC) information criterion, hence lag one was selected for this purpose.

The best variables for measurement of bank performance namely ROA, ROE and ROT were selected from a family of dependent variables using the principal component analysis (PCA) technique.

7.5.4 Vector-auto regression (VAR) estimations

The VAR models estimated with respect to the variables employed in the study were performed using lagged variables dependent variables. The negative logarithms of the variables were treated as missing values for the purposes of our VAR estimations. In this regard, three econometric models were estimated and the results obtained are as tabulated below:

Table 7.5 Showing Results of the Three Econometric Models Used by the Study

Variable	ROA	BEL	MAF	SOB	GDP
ROA (-1)	1.6964	0.92270	0.8736	0.8214	0.3997
	ROE	**BEL**	**MAF**	**SOB**	**GDP**
ROE (-1)	1.0481	1.1649	0.8355	0.0560	0.3603
	ROT	**BEL**	**MAF**	**SOB**	**GDP**
ROT (-1)	2.2684	1.3951	1.0875	2.0010	0.1381

Source: Author

The results above show that the lagged ROA contributes the highest to the current year's ROA, more than 80% to BEL, MAF and SOB and the least to the country's GDP. On the other hand the lagged ROE has a strong positive relationship with BEL, and ROE for the current year and MAF and weak positive covariance with SOB and GDP respectively. The VAR system of equations go further to show that ROT is directly related to current ROT then SOB, BEL and MAF and weakly connected to GDP. Overall the results of the VAR denote that ROA, ROE and ROT are strongly related to all other independent variables and weakly related to the country's GDP.

VAR ROA Results Lagged Variables

The study performed a VAR ROA test under BEL MAF, SOB and SA variables. The coefficients of the regression model results obtained are outlined as in the table above (See Appendix 3).

VAR Probabilities

Since the estimated VAR model did not present the probabilities, thus c(1) upto c(6) should be considered (Appendix 3): The specific VAR equation connecting ROA to the independent variables of the model is given by:

Equation: LROA = C(1)*LROA(-1) + C(2)*LBEL(-1) + C(3)*LMAF(-1) + C(4) *LSOB(-1) + C(5)*LSA(-1) + C(6).

Observations: 326

R-squared	0.954195	Mean dependent var	-1.919280
Adjusted R-squared	0.953479	S.D. dependent var	3.628067
S.E. of regression	0.782525	Sum squared resid	195.9505

Durbin-Watson stat 1.794702

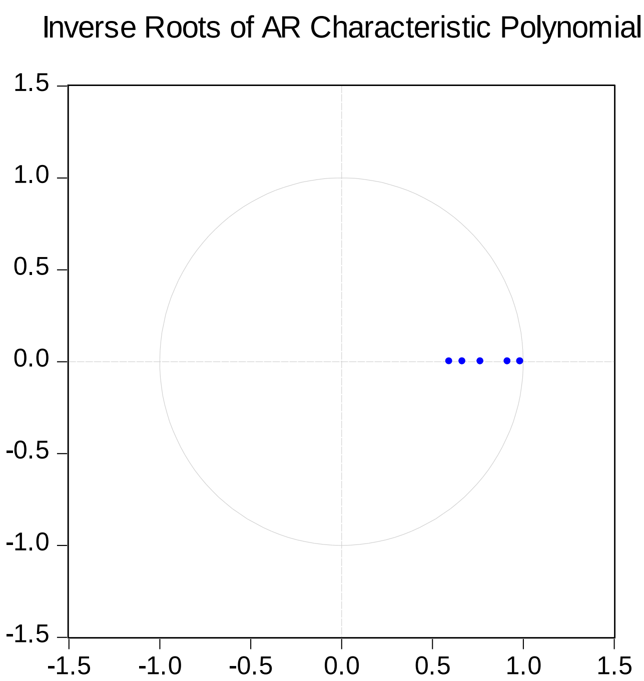
Source: Author

Diagnosis

The above equation's results indicate that from lags 1 to 5, the residuals are auto-correlated, however, from lags 6 to12, the residuals are not correlated. The study also notes that all the independent variables drawn into the model account for about 95.42% of the ROA measure of bank performance while the adjusted measure is around 95.35%. This means that the variables of the model are so robust, relevant and significant in term of their ability to contribution to measurement of bank financial performance. On the other hand a Durbin-Watson statistic of 1.795 is less than 2.00 ad therefore we accept the Ho that the model variables ae not auto-correlated at 5% level of significance.

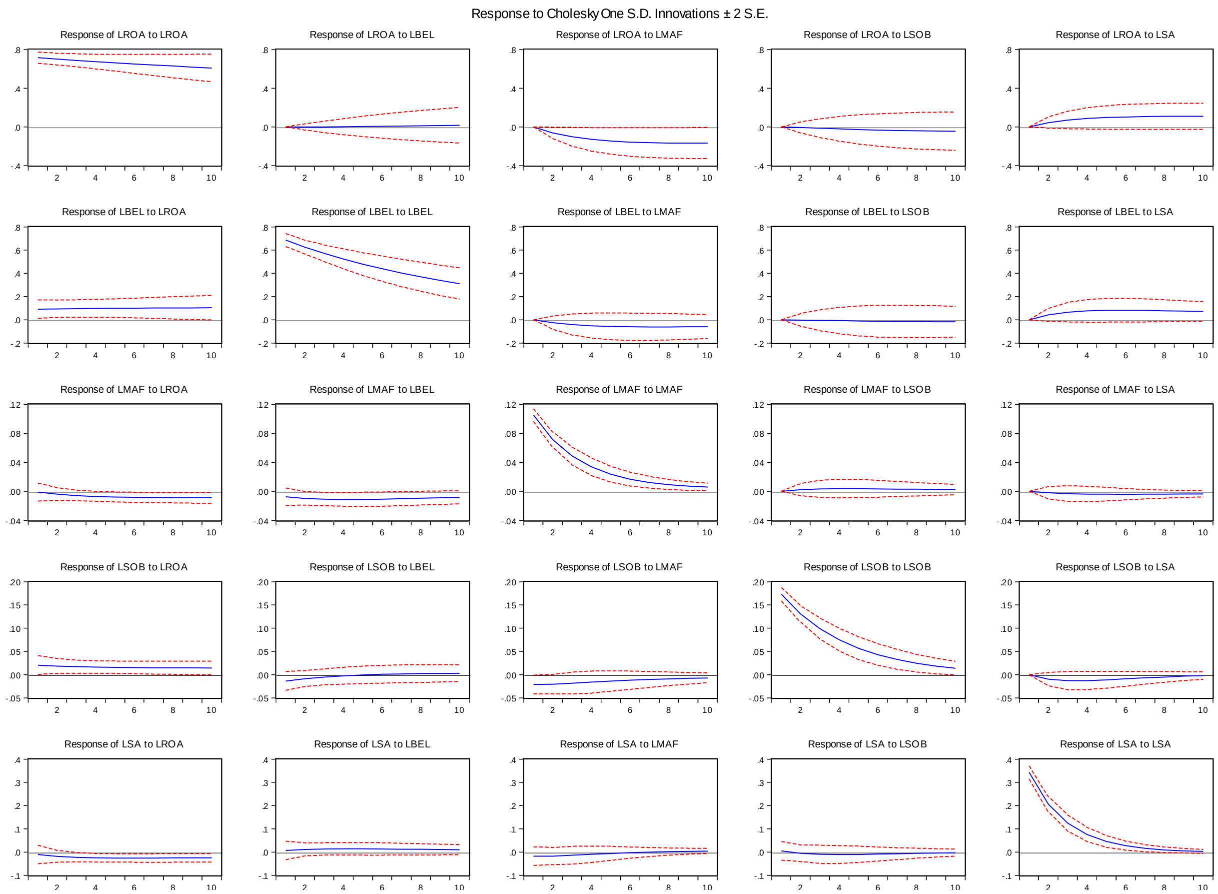
AR Roots

The AR roots of the study reveal that the economic systems of the countries investigated are stationary and will remain stationary into the long run.



Source: Author

Impulse responses



Source: Author

The impulse responses represented above indicate that the shocks in the countries' economic systems will gradually phase out in the long run.

VAR ROE Results

The study also carried out a VAR ROE test under BEL MAF, SOB and SA variables. The coefficients of the VAR regression model results are outlined as Appendix 5 attached at the end of the paper.

VAR Probabilities:

Since the estimated VAR does not present the probabilities, thus c(1) until c(6) should be considered: The specific VAR equation connecting ROE to the independent variables of the model is given by (Appendix 5):

Equation: LROE = C(1)*LROE(-1) + C(2)*LBEL(-1)+C(3)*LMAF(-1) + C(4) *LSOB(-1) + C(5)*LSA(-1) + C(6)

Observations: 322

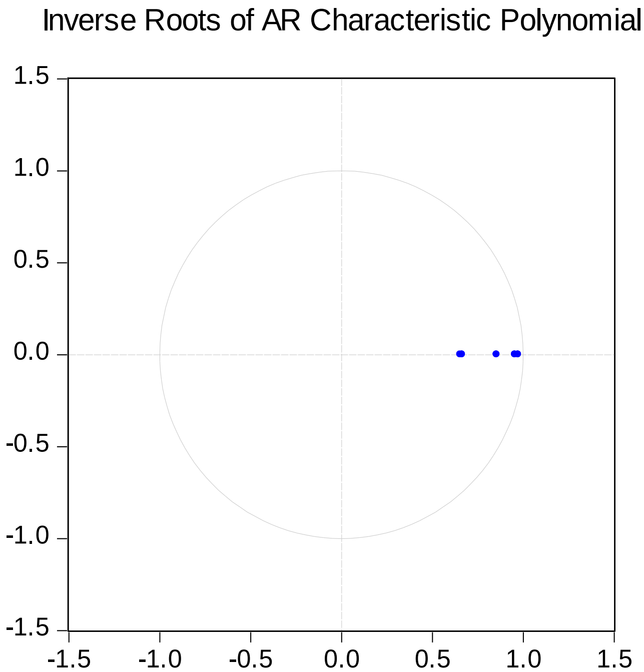
R-squared	0.869818	Mean dependent var	0.306388
Adjusted R-squared	0.867758	S.D. dependent var	3.334341
S.E. of regression	1.212536	Sum squared resid	464.5970
Durbin-Watson stat	1.897531		

Source: Author

The study discovers that all the independent variables drawn into the model account for about 86.98% of the ROA measure of bank performance while the adjusted measure is approximately 86.78%. This means that the variables of the model are so robust, relevant and significant in term of their ability to contribution to measurement of bank financial performance. On the other hand a Durbin-Watson statistic of 1.898 is less than 2.00 and therefore we accept the Ho that the model variables ae not auto-correlated at 5% level of significance. These results indicate that from lags 1 to 5, the residuals are auto-correlated, however, from lags 6 to12, the residuals are not correlated.

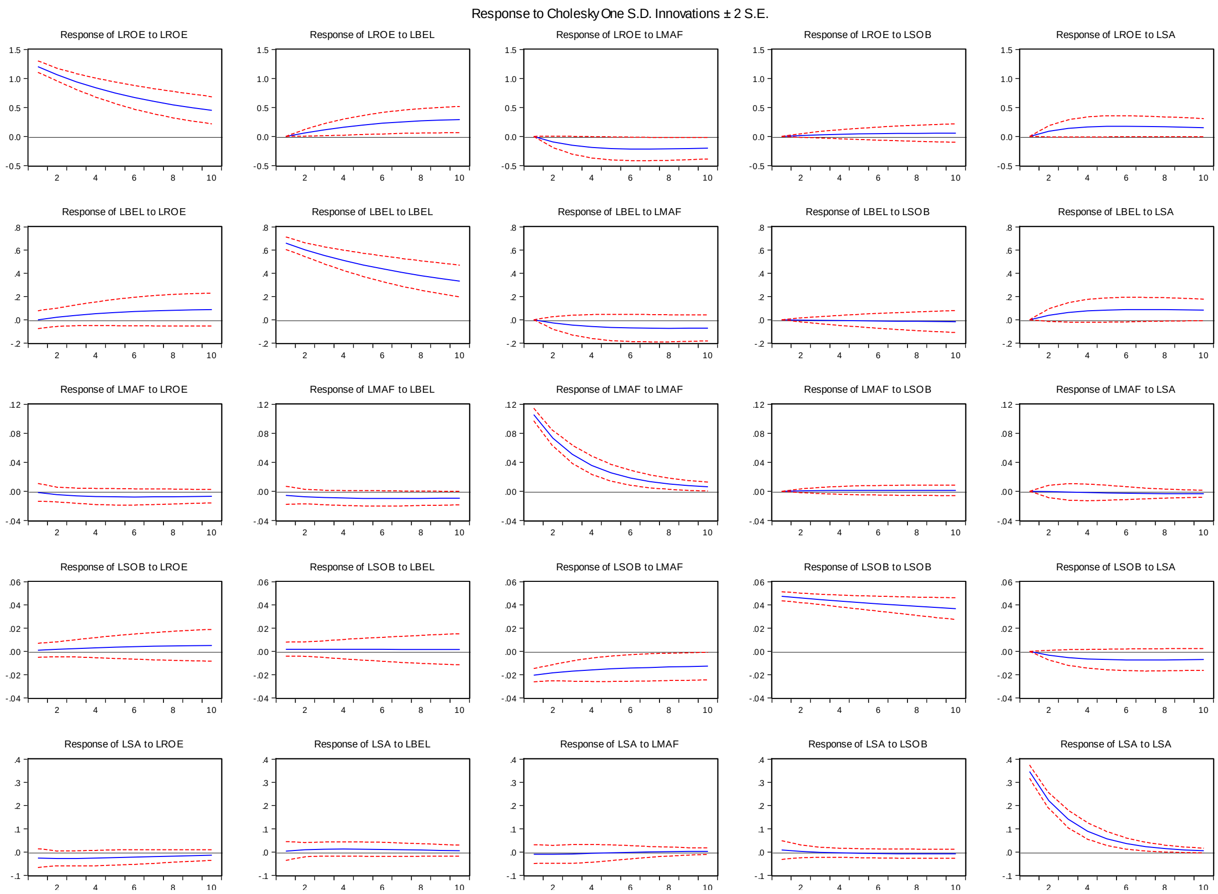
AR Roots:

Indeed the economic systems of countries drawn into the model are stationary and expected to remain stationary into the long run.



Source: Author

Impulse responses



Source: Author

The impulse responses above demonstrate that the shocks in the economic systems investigated system will gradually fall out in the long run.

ROT Results

The study further performed a VAR ROT test under BEL MAF, SOB and SA as independent variables. The comprehensive VAR return on investment are summarised under Appendix 6 attached below.

VAR Probabilities:

Since the estimated VAR did not present the probabilities, thus c(1) until c(6) should be considered: The specific VAR equation connecting ROT to the independent variables of the model is given by (Appendix 6):

Durbin-Watson stat 2.094998

Equation: LROT = C(1)*LROT(-1) + C(2)*LBEL(-1) + C(3)*LMAF(-1) + C(4)

*LSOB(-1) + C(5)*LSA(-1) + C(6)

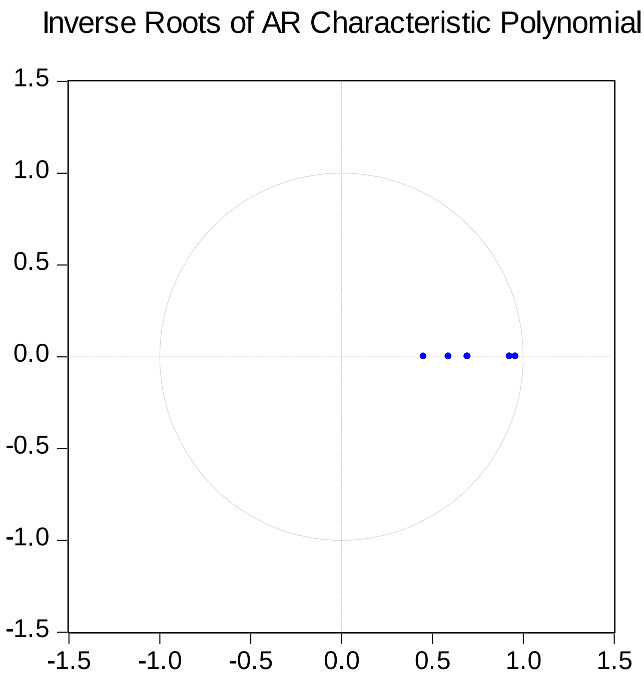
Observations: 267			0.999791
R-squared	0.918824	Mean dependent var	0.999791
Adjusted R-squared	0.917269	S.D. dependent var	3.967567
S.E. of regression	1.141189	Sum squared resid	339.9035

Source: Author

The study finds that all the independent variables drawn into the model explain about 91.88% of the ROT measure of bank performance while the adjusted measure is close to 91.73%. This means that the variables of the model are so robust, relevant and significant in term of their ability to contribution to measurement of bank financial performance. On the other hand a Durbin-Watson statistic of 2.095 is just around 2.00 and therefore we accept the Ho that the model variables ae not auto-correlated at 5% level of significance. The impulse responses above indicate that the shocks in our system will die out in the long run.

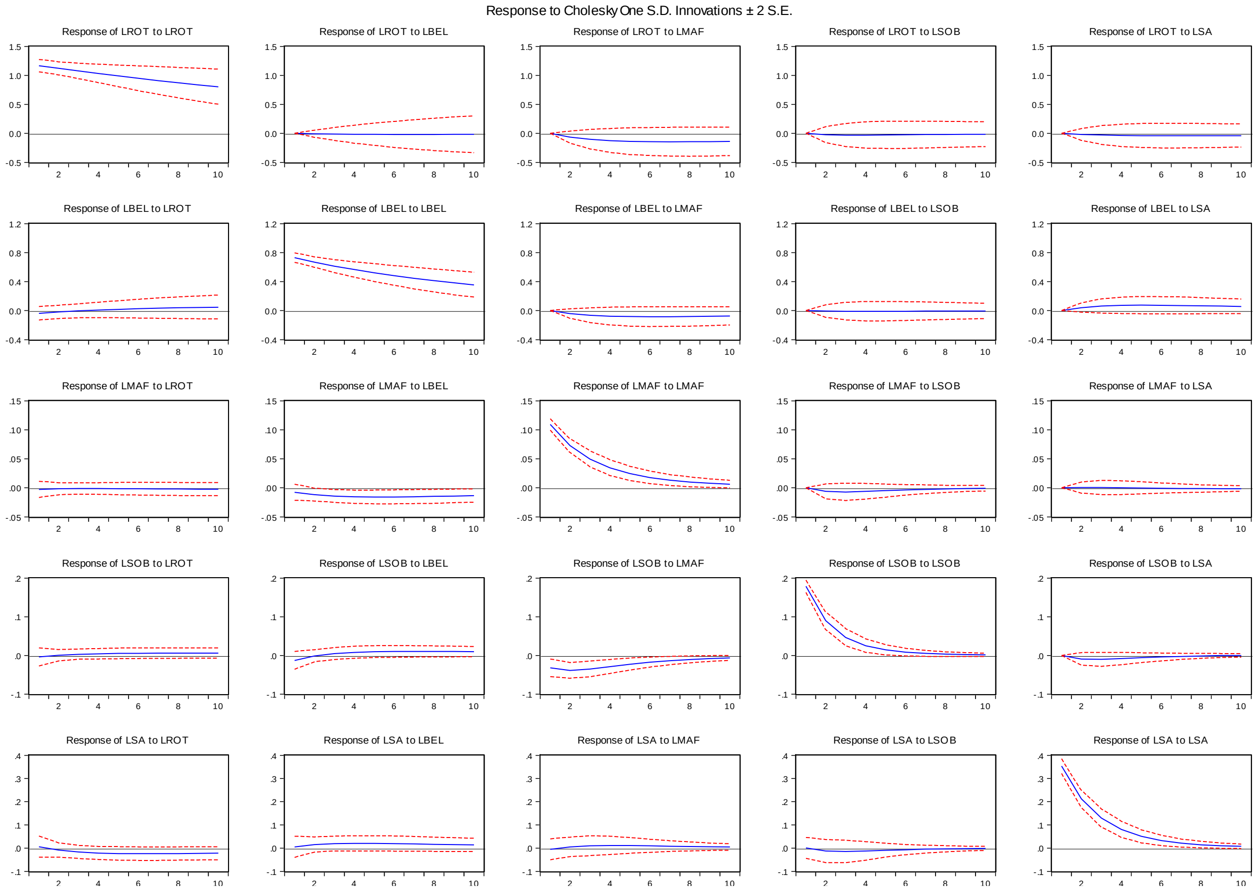
AR Roots

Indeed the economic systems investigated are stationary and are expected to remain stationary into the long run.



Source: Author

Impulse responses



Source: Author

The impulse responses demonstrated above indicate that the shocks in the countries' economic systems will be phased out in the long run.

7.5.5 System Residuals Tests

The study also carried out two residuals tests below under the null hypothesis test that there are no residual autocorrelations among model variables (Model is normally distributed) (Appendix 4).

System Residual Normality Tests

Orthogonalization: Cholesky (Lutkepohl)

Null Hypothesis: residuals are multivariate normal

Date: 05/12/22 Time: 11:53

Sample: 1997 2020

Included observations: 333

Component	Skewness	Chi-sq	Df	Prob.
1	1.251357	86.90713	1	0.0000
2	0.179727	1.792750	1	0.1806
3	-0.322344	5.766782	1	0.0163
4	-3.995529	886.0161	1	0.0000
5	-1.752146	170.3858	1	0.0000
Joint		1150.869	5	0.0000

Source:Author

The results of the normality test on the data are that the null hypothesis is rejected at lag 1, normally distributed at lag 2 and not normally distributed at lags 3-5. However the F (ANOVA) statistic is significant at 5% level of importance. At lag 2, the residuals become normally distributed and hence we accept the null hypothesis above.

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: No residual autocorrelations up to lag h

Date: 05/12/22 Time: 11:55

Sample: 1997 2020

Included observations: 333

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	72.37860	0.0000	72.62148	0.0000	25
2	88.08529	0.0007	88.43394	0.0007	50
3	106.6225	0.0096	107.1590	0.0088	75
4	129.7432	0.0244	130.5933	0.0216	100
5	154.9931	0.0355	156.2725	0.0304	125
6	168.9187	0.1384	170.4833	0.1208	150

7	188.2597	0.2336	190.2879	0.2033	175
8	218.3182	0.1781	221.1728	0.1454	200
9	229.5887	0.4027	232.7931	0.3466	225
10	237.3389	0.7075	240.8115	0.6503	250
11	244.9357	0.9037	248.6984	0.8709	275
12	281.8204	0.7674	287.1254	0.6936	300

Source: Author

*The test is valid only for lags larger than the System lag order.

df is degrees of freedom for (approximate) Chi-square distribution

*df and Prob. may not be valid for models with lagged endogenous variables

These results indicate that from data lags 1 to 5, the residuals are auto-correlated, however, from lags 6 to12, the residuals show no signs of auto-correlation. We also carried out an auto-correlation test on the residuals of the research data. Results obtained reveal that the residuals of the data fall together, are correlated at lag 5 but do not show any auto-correlation at lag 6 and beyond. These results indicate that from lags 1 to 5, the residuals are auto-correlated, however, from lags 6 to12, the residuals are not correlated.

7.6 Summarised VAR Equations and Ordinary Least Squares (OLS) Results

ROA, ROE and ROI are the best bank performance variables used under principal component analysis. The results obtained are as tabulated below:

Table 7.6 Showing Summarised VAR Equations and Ordinary Least Squares (OLS) Results

Bank Performance	**LAG**	**BEL**	**MAF**	**SOB**	**SA/GDP**	**Constant**
Measure	C1	C2	C3	C4	C5	C6

ROA	+Sig	-Insig	-Sig	-Insig	+Insig	+I nsig
ROE	+Sig at 5%	+Sig at 5%	-Insig	+Insig	+Sig at 5%	-Insig
ROI	+Sig	-Insig	-Insig	-Insig	-Sig	+Insig

Source: Author

The results of the study are that ROA has a negative insignificant relationship with MAF and BEF and strong positive relationship with the lag variable. On the other hand ROA a weak positive relationship with the constant and SA (GDP). We also discovered that ROE has a weak positive relationship with SOB, a strong positive correlation with BEL, SA, and the lag variable at 5% level of significance. Conversely the model constant and MAF have a weak inverse relationship with ROE. Last but not least we found that ROI has a strong positive relationship with the lag variable, and weak negative correlation with BEL, MAF and SOB. We also realised that ROI has a strong negative correlation with SA (GDP) and a weak positive relationship with the constant of the model.

7.7 Conclusions and Recommendations

The study was carried out to assess the impact of market friction (MAF) on bank financial performance in fuzzy emerging markets such as those in countries in Southern Africa. The VAR model used is robust as it allowed us to use logarithms to analyse financial data across various countries that use different currencies. The dependent and independent variables of the study used are ROA, ROE and ROI (bank performance measures) and SOB, BEL, SA (GDP) and MAF respectively. The relationships among model variables are summarised on the Granger causality test results table below.

Table 7.7 Granger Causality Test Results

Pairwise Granger Causality Tests
Date: 05/25/22 Time: 16:19
Sample: 1996 2020
Lags: 8

Null Hypothesis:	Obs	F-Statistic	Prob.
ROE does not Granger Cause ROA	306	0.07875	0.9997
ROA does not Granger Cause ROE		0.98333	0.4491
ROT does not Granger Cause ROA	306	0.00299	1.0000
ROA does not Granger Cause ROT		0.34134	0.9493
BEL does not Granger Cause ROA	306	0.15495	0.9961
ROA does not Granger Cause BEL		0.27374	0.9741
MAF does not Granger Cause ROA	303	0.44705	0.8920
ROA does not Granger Cause MAF		0.33097	0.9537
SOB does not Granger Cause ROA	306	0.23724	0.9836
ROA does not Granger Cause SOB		0.63376	0.7492
SA does not Granger Cause ROA	306	4.88522	1.E-05
ROA does not Granger Cause SA		2.30820	0.0206
ROT does not Granger Cause ROE	306	1.34607	0.2204
ROE does not Granger Cause ROT		0.74235	0.6540
BEL does not Granger Cause ROE	306	2.68444	0.0073
ROE does not Granger Cause BEL		0.11496	0.9987
MAF does not Granger Cause ROE	303	1.26345	0.2625
ROE does not Granger Cause MAF		0.47278	0.8750
SOB does not Granger Cause ROE	306	1.46448	0.1698
ROE does not Granger Cause SOB		0.55179	0.8168
SA does not Granger Cause ROE	306	1.07488	0.3807
ROE does not Granger Cause SA		1.62539	0.1171

BEL does not Granger Cause ROT	306	0.10618	0.9990
ROT does not Granger Cause BEL		0.01468	1.0000
MAF does not Granger Cause ROT	303	0.76488	0.6340
ROT does not Granger Cause MAF		1.74202	0.0885
SOB does not Granger Cause ROT	306	15.9430	2.E-19
ROT does not Granger Cause SOB		23.2328	2.E-27
SA does not Granger Cause ROT	306	0.75317	0.6444
ROT does not Granger Cause SA		1.24975	0.2699
MAF does not Granger Cause BEL	303	0.11704	0.9986
BEL does not Granger Cause MAF		0.18344	0.9930
SOB does not Granger Cause BEL	306	0.18325	0.9931
BEL does not Granger Cause SOB		0.38678	0.9273
SA does not Granger Cause BEL	306	0.62925	0.7530
BEL does not Granger Cause SA		0.80107	0.6021
SOB does not Granger Cause MAF	303	0.48902	0.8637
MAF does not Granger Cause SOB		3.88936	0.0002
SA does not Granger Cause MAF	303	0.67601	0.7126
MAF does not Granger Cause SA		0.57834	0.7955
SA does not Granger Cause SOB	306	1.43726	0.1805
SOB does not Granger Cause SA		0.68329	0.7062

Source: Author

The study concludes that ROA is positively influenced by MAF and BEF, and strongly by the ROA from the preceding period. It is also concluded that there is a weak direct relationship between ROA and the model constant and the SA (GDP) of the country. The study concludes that ROE is positively varies with SOB, BEL, SA, and the lag variable at 5% level of significance.

Conversely the bank's ROE depicts a weak positive relationship with the model constant and MAF. Last but not least the study concludes there is a fair direct relationship between the ROI and the lag variable, the constant and negative relationship with BEL, MAF and SOB. We also conclude that there is a strong negative connection between ROI and SA (GDP). We carried out a Pairwise Granger causality test on all dependent and independent variables of the VAR model whose results are tabulated above.

One of the major recommendations of the study for policy formulation is that GDP causes ROA and hence Monetary Authorities and Ministry of Finance have a role to play to influence banking policies. ROE and ROA influence each other and the same is true for GDP and ROA, and SOB and ROI and hence there are no policies to be formulated since the variables are bi-dimensional. We also recommend that the BODs and senior managers of banks must come up with solid policies after realising that BEL influences ROE but the converse does not hold. We end by finally calling on government authorities and BODs and senior managers of banks to regularly engage to strengthen the operations of banks as engines of growth and development of nations. This recommendation is made after noting that MAF is a serious factor that erodes SOB and bank financial performance and let alone scares potential bank investors.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 Introduction

The study was set to interrogate the practicability of the use of structural and reduced-form models currently used in valuations of banks in both frictionless and frictional and fuzzy financial markets. In other words, the study aimed to compare and contrast bank valuations based on structural and reduced-form credit risk models with those extended for market friction and uncertainty. The central objective of the study was to assess the impact of transaction costs on the valuation of banks in emerging markets characterized by market friction and uncertainty with specific reference to fuzziness. After validation of the four proposed models in the preceding chapters 4 to 7, presentation, and discussion of results, this chapter draws conclusions and recommendations on the overall impact of transaction costs on CRM and the performance of banks located in frictional and fuzzy financial environments. The results of the proposed models are presented in the order of the objectives of the study outlined in the introduction chapter.

The chapter starts by presenting conclusions and recommendations of the proposed fuzzy Merton AVM followed by those on the fuzzy probability of default (PD) hereby called the risk of the default model. The rigour of the proposed risk of default or PD model, extended for market friction and fuzzy variables was tested against the reliability and validity of results generated using structural CRMs and AVMs. It goes further to draw conclusions and recommendations on the impact of market friction and fuzzy PDs, EADs, and LGDs on bank expected losses (ELs) estimated using logit and logistic models. Finally, the study uses a vector auto-regression (VAR) model to estimate the effects of both firm-specific and market factors on bank performance measured by three specific variables namely ROA, ROI, and ROE. The data used for validation of the four proposed Merton equity, KMV-PD, EL, and VAR models were drawn from Stock

Exchange listed banks in emerging countries of Southern Africa (Source: World Development Index, 2020).

8.2 Impact of Transaction Costs on Bank Equity in Fuzzy Financial Markets

The study proposed and analysed the valuation of equity of banks based on a Merton model combining transaction costs and uncertainty arising from the random evolution of asset prices, imprecision, and vagueness. The model was proposed after noting that financial markets in which banks operate are far from being efficient and perfect as assumed under the structural models such as Merton (1974) and Black-Scholes (1973).

Conclusions

Based on the research results in chapter four above, the study concludes that banks in emerging economies are heavily geared and characterised by high debt-equity ratios that need efficient and effective management. The study also concludes that these banks are poorly capitalised, regulated and supervised, over-borrowed or foreign-owned, and have large volumes of non-performing loans (NPLs). The future of these banks lies in their ability to negotiate for turning their high debt levels into ordinary equity.

Based on the specific independent variables of the equity model proposed in chapter 4, the study concludes that both structural and proposed fuzzy equity model values have a direct relationship with banks' ROE values. The study also concludes that banks' equity values are inversely related to market friction that is costs of capital. Hence the above model variables are pertinent for inclusion in bank valuations of banks because they fairly reflect the practical conditions faced by investors in their investment planning and decision-making processes in financial markets. Investors in emerging economies often use human psychology or language to express levels of risk or return to their investments. Hence implicit 'fuzziness' significantly impacts on equity values of banks, particularly those situated in emerging financial markets. The study further concludes that the inclusion of fuzziness in the proposed equity model estimates the expected values of equity of banks fairly consistent and reflective of their actual market values.

The expected market values of equity of banking corporations are precisely estimated and this assists in accurate investment planning and decision-making processes. The study further concludes that all banks' ordinary equity values are directly and indirectly related to their asset market values and liability exposures respectively. The study further concludes that most banks in Southern Africa depend mainly on debt or borrowed capital to finance their activities because their debt-to-equity ratios are very high. The banks' over-dependence on borrowed capital renders them vulnerable to hostile takeovers by creditors in the foreseeable future. However, despite the banks being over-borrowed, they used such debt capital to acquire assets whose market values outweigh the values of the accumulated liability values**.** Therefore, by comparison, the equity values generated from the proposed equity model are more precise, consistent, and stable compared to those estimated using structural models such as Merton (1974) and Black-Scholes (1973).

Recommendations

Based on the above conclusions the study makes some recommendations to bank managers, boards of directors (BODs), and their shareholders. The study starts by recommending that banks in emerging markets such as those in Southern Africa can employ equity valuation models extended for friction and fuzziness to come up with precise asset and equity values. On the other hand, banks in emerging markets should issue more ordinary shares to increase their ordinary capital holding, which can act as a buffer against the takeover by debt equity funders. Alternatively, bank ordinary shareholders can negotiate with their creditors for converting debt holdings into ordinary equity or shares. It is also recommended that banks must come up with aggressive investment strategies that can improve their asset utilisation ability. In this respect, they can generate more income to act as a buffer against higher liability levels currently observed in their capital structures. By retaining substantial portions of residual income from one year to another banks can improve their ordinary capital through internally generated resources that carry no costs but build protection of shareholders against potential threats from external financiers.

The study also recommends that banks' boards of directors (BOD) and managers should factor investors' perceptions, market friction, and uncertainty into the estimation of firms' equity values and capital bases to increase precision, practicability, and reliability of their overall performance. The study also recommends that banks' managers should include the above variables in their valuation models to increase the precision and reliability of their equity values and capital bases. Such consideration can go a long way in justifying the significance and prudence to be drawn from the inclusion of both human language and market friction in the estimation of equity values of firms in developing countries. Finally, based on the above conclusions, the study recommends that transaction costs and uncertainty must be adjusted for in existing financial models to make them more rigorous, reliable, and precise in the estimation of bank values.

The study overall recommends that the proposed model can be adopted by banks in frictional and fuzzy emerging markets to come up with a fair valuation of their equities and financial performances. The study notes that an equity estimation model extended for market friction and uncertainty or human perceptions is more precise, realistic, and practical in the estimation of equity values of banking institutions than the traditional structural and reduced-form models. This is because the model is sensitive to both market friction and uncertainty factors faced by investors in the valuation of their banks' equity values and overall financial performance. Hence banks in frictional and fuzzy financial markets can improve their equity and risk metric estimations by adopting such a model which fairly reflects conditions these investors face in their day-to-day investment decisions.

8.3 Market Friction and the KMV-Risk of Default Model for Banks in Emerging Financial Markets

The study proposes and validates a KMV risk of default model using financial data drawn from banks in emerging economies in Southern Africa. Banks in Southern Africa operate in frictional and fuzzy financial environments contrary to the assumptions of efficient and frictionless markets underlying structural credit risk models. Based on the results of the study discussed in chapter 5 above it is concluded that banks in emerging economies need the risk of default models different from structural models which suit their financial circumstances and practical market conditions. Structural models are premised on assumptions such as frictionless and efficient financial markets, and constant rates of return and asset volatilities which are far from being realistic, particularly for emerging markets such as those in Southern Africa. In practice banks in emerging economies operate in very frictional and fuzzy financial markets or environments. Hence the need for current researchers to develop asset valuation and risk metrics models divorced from structural models, extended for market friction and fuzziness which significantly impact the fair estimation of risks of default or DPs of banks.

Conclusions

The study examines the impact of several independent variables on the estimation of risks of default of banks located in Southern Africa. From the results in chapter 5 above the study concludes that asset values and volatilities and ROE are indirectly related to the risk of default of a bank. On the other hand, the bank's liabilities, cost of ordinary equity, and fuzziness are directly related to its risk of default of a bank. The study also concludes that banks in smaller economies of Southern Africa have higher risks of default compared to those in larger economies. This is because the latter are better capitalised, regulated, supervised, and managed under progressive or good and sound corporate governance and ethical frameworks.

Proposed KMV model results of banks are compared results with those generated from both hazard function and structural risk models. The study concludes that the proposed KMV model is a good estimator of the risk of default of a bank compared to the latter models. The proposed model gives more moderate results, less variant, stable, or smoothened, which implies that it does not over or under-estimate the banks' risks of default. From a comparison of the proposed and traditional models and the hazard semi-parametric approach, it can be concluded that hazard function models are not suitable for use in risk metrics models whose values are marginal and range from 0 to 1.00. The hazard model results are not constrained by the boundaries of the classical probability theory propounded by Kolmogorov (1933). In practice, hazard models are not directly related to classical probability theory and hence fit very well into models in natural sciences as advanced by Cox (1992).

The study also concludes that the proposed KMV model can go a long way to contribute to the financial board of knowledge as it draws market friction and fuzziness into the estimation of the risk of default of a bank, which are not captured in contemporary structural models. Overall the study concludes that all structural credit risk models, reduced-form models, and hazard function models are constrained and unsuitable for the valuation of banks situated in frictional and fuzzy financial markets or environments. Hence all banks in emerging economies can adopt and implement risk models extended for market friction and uncertainty to achieve fair estimates of risks of default, requiring an efficient valuation of bank values and their financial performance.

Recommendations

Well based on the above conclusions the study recommends that banks should extend existing structural and reduced-form models for transaction costs and uncertainty to make them more consistent, reliable, rigorous, and practical in the estimation of DPs of banks. Overall the study recommends banks in frictional and fuzzy financial environments adopt and implement the proposed KMV-DP model in the estimation of fair values of their DPs. In this respect errors associated with and under- and overcasting of DPs become significantly reduced. On the other hand banks' reported profitability levels could be fairly and consistently reflected on their actual performance for given accounting periods. Financial regulators and supervisors of banks and similar institutions must reinforce the establishment and maintenance of sound risk corporate governance and ethics systems in banks. This would go a long way in efficient and effective mitigation of internal bank risks such as the risk of default, LGD and non-performing loans (NPLs), and market-wide financial risks in the quest to safeguard overall financial system stabilities in emerging markets.

8.4 Effects of Market Friction on Expected Loss Modelling in Banks

The study used logit and logistic valuation models to regress financial data for 2008-2020 drawn from the audited financial statements of foreign and indigenous-owned banks situated in Southern Africa. In other words, the study proposed and employed a structural expected loss (EL) model based on PD, EAD and LGD extended for market friction and fuzziness.

Conclusions

Based on the results in chapter 6 above, the study concludes that logistic models extended for market friction are more stable than those estimated using logit models. The study also concludes that structural logistic and logit models extended for the two independent variables improved the estimation rigour and accuracy of the expected loss values of banks. Furthermore, the study concludes that foreign-owned banks are more stable and have lower ELs than indigenous banks. This could be attributable to higher capital formation and less dependence on debt capital relative to domestic banks.

Results drawn from ELs estimated using fuzzy PDs, EADs and LGDs are less variable and fairly reflect on actual market values of banks compared to those obtained from structural models. The study also concludes that mathematical languages prevalently used in bank financial investments and transactions involve both fuzziness and non-quantitative variables contrary to notions of the classical probability theory used in structural CRMs. The extension of existing structural models for fuzziness and market friction creates the rigour and precision required in the estimation of bank market values and exposures or expected losses in emerging economies in Sothern Africa. This development could assist banks in accurate planning, loan loss provisioning, and investment and credit exposure decision-making processes.

The study also concludes that both banks' expected loss values have a direct relationship with their PD, EAD, and LGD variables. Furthermore, the research concludes that indigenous banks used in the research are poorly capitalised in terms of ordinary equity compared to debt financing. The study thus postulates that these banks over-depended on borrowed capital rendering them vulnerable to hostile takeovers by debt-equity holders. The study also concludes that shareholders of domestic banks need to inject adequate equity to prevent them from being takeovers by debt-equity financiers. This can be achieved through negotiating with current lenders to convert debt into equity or increasing retained earnings to improve equity capital to levels above debt financing. The study finally concludes that an expected loss model adjusted for market friction and human perceptions is the way to go for banks in frictional and fuzzy financial environments in their quest to be precise and practical in the estimation of their asset values and risk metrics. Contemporary asset and structural and reduced-form risk metrics models are not suitable for the valuation of banks in fuzzy financial environments, because of the unrealistic assumptions on which they are founded.

Recommendations

Based on the above conclusions the study makes some recommendations for the betterment of the performance of banks measured in terms of capital injection, regulation, and supervision, and let alone effective management of loans and credit exposures. The study recommends that banks' boards of directors (BOD) and managers should be highly sensitive to the perceptions of investors, market friction, and uncertainty when it comes to capital formation, estimation of equity and asset values, and expected losses. Such sensitivity could go a long way in increasing valuation precision, model practicability, and reliability of the overall financial performance of banks in financial markets.

The extension of new ELs models for market friction and fuzziness has the potential to improve the estimation precision of expected losses of banks, their lending and borrowing activities, and let alone magnitudes of NPLs. Therefore the study also recommends that banks in emerging markets such as those in Southern Africa can adopt and implement the proposed EL model as it fairly captures practical market conditions experienced by investors when they make financing and investment decisions. Overall the study recommends that banks in emerging economies need to urgently come up with financial policies that are well-formulated, coordinated, and prudentially implemented to effectively manage their expected losses and finance their banks in line with the dictates of the new millennium strategic development goals (SDGs).

8.5 Market Friction and Bank Financial Performance

The study used the VAR approach to assess the impact of firm and market-wide variables on bank performance measured by ROA, ROE, and ROI. The study concludes that both firm-specific and macroeconomic independent variables drawn into the model contribute significantly to bank performance. Hence MAF, BOD, MKF, and BEL are impediments that have a significant impact on bank performance which must be factored into bank financial models to improve estimation precision and rigour. We used pooled Ordinary Least Squares (OLS) method to determine the regression results tabulated above. We measured the bank's performance by ROA, ROE, PMA, and EPS and input variables by MAF, SOB, BEL, MKF, BOD and We conclude that BAP from previous periods and SOB impact all bank performance measures directly. Conversely, BOD, BEL, MAF, and MKF indirectly impacted bank performance, results that are in tandem with the findings of Hogue et al (2013).

Based on the above conclusions the study recommends that banks must streamline their BODs, reduce liabilities and apply corporate governance and ethics to improve financial performance for their growth and development. Central banks of emerging economies must adopt and implement Basel I and II Capital Accords, and efficiently regulate and supervise banks for their growth and development. The study also recommends that banks in emerging economies characterised by friction and fuzzy financial markets need to adopt models adjusted for these variables to be able to estimate their performances with precision and rigour. Further studies in this area can examine the impact of variables such as capitalization, ownership structure, liquidity challenges, corporate governance and ethics, national policy inconsistencies, corruption, and country and political risks on bank performance in emerging financial markets.

8.6 Limitations of the Study

Although the above research results and conclusions could be credible and consistent, the researcher acknowledges that the financial data of banks used in model validations were in different currencies. Unlike for example studies that involve the European Union where the Euro is used as a common currency. The researcher had to use asset-equity ratios of banks in the valuation of DPs of banks to circumvent the variances in currencies used by countries drawn into the research to make the results comparable. The research on the impact of market friction on bank performance adopted the VAR model because it can do away with the difference in currencies used by banks in different countries. The study also acknowledges that the proposed models are based on selected input variables, namely asset values, volatilities, return on equity, cost of equity (market friction), and uncertainty (fuzziness). The selected model variables could be far from giving a fair or true reflection of the diversity of bank-specific variables that influence the valuation of equity, risk of default, expected losses, and bank performance in the real financial market world.

The study thus admits that bank-specific and market variables must be included in contemporary CRMs to make them more diverse, realistic, and reflective of conditions faced by investors in emerging markets. In other words, bank-specific and market factors such as board size, remuneration, expected losses and profitability for preceding periods, and unemployment, inflation, exchange, and interest rates respectively are critical input variables that must be used in the estimation of risk metrics and other valuations to improve precision and consistency. Adoption of bank structural valuation models such as the one on the impact of market friction on bank performance, extended for both firm-specific and market-wide variables can go a long in achieving efficiency and effectiveness needed in estimation accuracy and contributing to the board of knowledge in economics, banking, and finance.

8.7 Further Study Recommendations

The study was mainly motivated by the need to compare and contrast results from structural CRMs and AVMs with those from similar models extended for human psychology and market friction. The extension of existing structural models for the above input variables was aimed at making them more realistic, accurate, and applicable to the valuation of banks in financial markets characterised by friction and fuzziness. The study thus recommends that similar researches need to extend structural models for diverse uncertainty, market friction, and firm-specific and market variables to improve robustness and precision in asset, equity, risk metrics, and financial performance valuations of banks, in their quest to grow towards sustainable development.

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APPENDICES

APPENDIX I: Distribution of Banks by Their Transaction Costs (Costs of Ordinary Equity, K_{es}) for 2008-2020 (%)

Year	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	12.7	12.8	16.1	16.0	12.3	8.7	9.6	10.5	10.2	10.8	11.3	12.7	12.6
B	12.5	13.9	14.4	13.8	14.2	11.6	12.8	13.3	10.6	14.8	12.5	10.9	12.4
C	12.1	10.5	18.0	15.2	16.4	11.9	13.6	15.5	12.3	15.9	11.7	13.6	14.2
D	12.8	13.6	16.8	16.1	15.5	14.3	15.4	12.7	14.1	16.3	13.2	10.6	10.8
E	19.9	13.5	17.8	17.6	16.4	12.4	12.7	11.9	13.8	12.6	13.2	14.5	12.9
F	17.0	11.6	13.4	13.8	14.3	14.1	13.8	12.7	11.5	10.8	12.2	13.7	13.4
G	96.0	12.4	12.8	12.6	10.3	13.8	14.2	14.9	18.6	22.1	24.7	26.3	32.6
H	92.8	14.2	14.6	16.2	13.5	13.7	14.8	15.7	16.8	19.6	23.3	24.6	33.4

APPENDIX II: Showing Banks' Traditional and Fuzzy Asset Standard Deviations for 2008-2020 (%)

	08	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A-T	16.8	21.6	53.1	0.6	12.6	16.5	11.2	18.3	9.5	16.8	22.7	15.6	36.4
F	18.4	20.8	34.4	10.3	16.2	17.6	13.8	21.6	11.7	18.4	25.2	17.3	28.8
B – T	11.5	4.3	15.8	34.0	23.2	20.0	18.9	21.6	24.2	26.8	19.5	22.7	20.4
F	15.8	12.2	17.9	27.0	21.6	20.0	19.3	17.5	19.6	17.8	20.2	18.4	18.6
C-T	9.8	15.4	8.2	27.4	10.0	24.3	18.6	16.5	20.8	17.2	16.8	13.6	22.4
F	14.9	17.7	14.1	23.7	15.0	22.2	19.8	17.3	17.5	15.4	18.5	16.8	17.4
D-T	8.7	5.2	12.8	7.7	14.8	12.6	10.2	9.4	11.6	9.8	10.7	11.3	12.8
F	14.4	12.6	16.4	13.9	17.4	15.6	13.7	12.9	12.6	11.7	12.9	12.3	15.2
E-T	6.8	8.4	1.7	7.6	8.0	11.2	9.5	8.8	7.6	4.9	6.7	7.8	9.6
F	13.4	14.2	12.5	13.8	14.0	5.60	15.9	15.6	13.8	12.5	14.9	13.7	15.8
F-T	24.6	35.02	33.8	11.0	54.0	32.6	44.4	28.9	48.2	33.7	28.8	44.5	50.8
F	23.0	27.5	26.9	15.5	37.0	27.8	32.6	26.4	36.3	27.1	25.8	32.4	37.6

G-T	72.6	10.5	12.5	14.6	11.2	12.3	14.9	15.7	16.5	18.6	20.4	25.7	27.5
F	68.5	11.6	15.8	16.5	13.3	14.8	15.3	16.7	17.4	19.6	21.9	26.2	28.7
H-T	74.4	12.6	14.3	18.6	8.7	5.4	6.3	9.2	14.8	18.7	20.7	23.5	26.8
F	71.6	14.8	16.8	20.4	8.8	5.6	7.8	10.8	15.1	18.7	22.6	25.5	27.8

APPENDIX III: Showing Distribution of Banks by Their Traditional and Fuzzy ROEs for 2008-2020 (%)

Year	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A-T	13.4	11.0	124.2	42.3	5.6	8.7	10.5	12.8	11.7	13.4	10.8	12.3	14.8
F	-9.6	53.0	258.8	78.0	47.9	56.6	70.5	82.8	75.6	88.4	71.3	74.7	86.6
B – T	16.1	20.1	23.5	20.7	21.9	20.0	18.6	16.4	17.7	18.9	15.8	16.5	17.6
F	7.2	92.3	115.4	83.1	110.7	53.5	86.3	63.6	72.2	68.7	72.8	58.3	81.6
C-T	81.4	87.3	63.1	35.0	36.4	29.0	32.7	36.8	28.6	37.5	32.6	28.8	36.4
F	253.4	346.8	162.3	138.2	166.4	165.0	140.4	172.0	148.5	126.7	143.8	124.5	136.8
D-T	25.1	20.8	21.3	13.7	22.6	18.6	22.7	16.9	19.5	23.4	17.2	22.6	18.8
F	35.6	90.8	105.9	56.4	113.4	78.2	96.4	68.5.	86.8	125.2	72.9	94.4	81.3
E-T	51.7	44.2	37.6	50.0	37.1	36.3	32.8	37.6	42.4	48.7	36.9	43.2	36.5
F	137.9	180.8	169.6	191.9	169.3	193.3	182.0	171.9	164.1	160.3	174.5	187.6	168.4
F-T	12.7	112.2	10.7	3.6	12.5	13.8	9.8	7.6	8.7	10.4	12.5	11.8	10.6

F	-15.0	442.3	457.0	17.5	74.6	79.8	57.2	43.3	51.6	59.9	73.4	65.8	61.2
G-T	232.0	9.6	12.8	15.0	16.5	15.8	16.8	14.6	16.2	18.7	20.5	22.5	24.8
F	324.6	12.8	16.4	18.7	21.2	20.8	21.6	18.8	20.5	22.4	23.8	25.2	27.4
H-T	216	8.8	10.7	13.6	15.6	16.7	17.8	16.4	14.6	17.3	19.3	21.8	23.5
F	228.0	10.6	14.4	17.5	20.8	22.4	22.8	20.2	19.6	21.8	22.9	24.6	26.5

APPENDIX IV: Showing Distribution of Banks by Their Traditional and Equity –Asset Ratios for 2008-2020 (%)

Bank	**Ratio**	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A-T	$(\frac{TE}{TA})$%	9.0	11.6	5.3	5.3	11.2	15.5	16.7	20.7	23.4	23.6	25.0	28.9	31.7
F	$(\frac{FE}{TA})$%	6.8	4.8	0.5	3.7	7.2	6.8	6.4	7.8	7.5	6.6	7.6	8.5	7.8
B –T	$(\frac{TE}{TA})$%	10.0	9.7	11.8	14.8	12.5	11.9	11.4	12.7	12.8	15.1	15.4	17.3	17.3
F	$(\frac{FE}{TA})$%	4.2	5.2	4.3	5.4	5.0	7.3	4.6	5.4	4.9	6.7	5.8	6.4	6.3
C-T	$(\frac{TE}{TA})$%	2.3	5.4	0.9	9.1	12.8	7.7	9.2	10.3	7.8	9.8	7.1	9.5	10.3
F	$(\frac{FE}{TA})$%	0.3	0.1	0.9	1.6	1.5	2.0	2.8	2.2	3.5	4.6	3.2	4.3	4.8
D-T	$(\frac{TE}{TA})$%	9.5	10.1	10.9	17.2	10.6	13.3	12.4	9.6	9.5	10.2	10.1	11.4	12.3
F	$(\frac{FE}{TA})$%	8.6	5.0	4.7	11.2	3.9	4.2	4.8	7.6	6.4	4.1	4.3	7.2	5.7
E-T	$(\frac{TE}{TA})$%	3.8	4.8	6.5	5.0	7.4	8.5	11.8	7.4	11.0	9.9	9.4	10.8	10.3
F	$(\frac{FE}{TA})$%	1.6	0.9	1.7	1.3	1.2	1.8	2.5	1.4	2.3	1.9	2.4	3.2	2.8
F-T	$(\frac{TE}{TA})$%	12.8	2.5	14.4	7.1	23.6	11.3	12.5	10.6	13.4	13.9	11.6	12.2	15.3
F	$(\frac{FE}{TA})$%	8.0	0.10	0.10	8.8	26.9	9.3	8.6	12.4	0.80	0.72	8.8	7.5	10.7

G-T	$(\frac{TE}{TA})\%$	0.38	0.64	0.74	0.63	0.40	0.66	0.61	0.65	0.63	0.61	0.61	0.62	0.58
F	$(\frac{FE}{TA})\%$	0.32	0.60	0.68	0.58	0.36	0.62	0.55	0.58	0.58	0.55	0.55	0.56	0.52
H-T	$(\frac{TE}{TA})\%$	0.46	0.60	0.37	0.23	0.21	0.20	0.17	0.17	0.17	0.15	0.15	0.13	0.12
F	$(\frac{FE}{TA})\%$	0.42	0.56	0.33	0.18	0.17	0.16	0.16	0.16	0.16	0.14	0.14	0.12	0.11

APPENDIX V Showing Market Friction and Bank Performance Results

Panel unit root test: Summary

Series: SA

Date: 05/12/22 Time: 11:02

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-Sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-2.28738	0.0111	18	414
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-2.21228	0.0135	18	414
ADF - Fisher Chi-square	46.8668	0.1061	18	414
PP - Fisher Chi-square	42.5318	0.2104	18	432

** Probabilities for Fisher tests are computed using an asymptotic Chi

-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: SOB

Date: 05/12/22 Time: 11:06

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	0.27034	0.6066	18	414
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-0.42882	0.3340	18	414
ADF - Fisher Chi-square	48.3787	0.0814	18	414
PP - Fisher Chi-square	100.001	0.0000	18	432

** Probabilities for Fisher tests are computed using an asymptotic Chi

-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: MAF

Date: 05/12/22 Time: 11:07

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.89531	0.0000	18	411
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-5.97807	0.0000	18	411
ADF - Fisher Chi-square	115.705	0.0000	18	411
PP - Fisher Chi-square	110.064	0.0000	18	430

** Probabilities for Fisher tests are computed using an asymptotic Chi

-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: BEL

Date: 05/12/22 Time: 11:09

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-0.21120	0.4164	17	391
Null: Unit root (assumes individual unit root process)				

Im, Pesaran and Shin W-stat	-2.86268	0.0021	17	391
ADF - Fisher Chi-square	63.7272	0.0015	17	391
PP - Fisher Chi-square	118.492	0.0000	17	408

** Probabilities for Fisher tests are computed using an asymptotic Chi
-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: ROA

Date: 05/12/22 Time: 11:09

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.84697	0.0000	18	414
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-5.52094	0.0000	18	414
ADF - Fisher Chi-square	112.102	0.0000	18	414
PP - Fisher Chi-square	135.378	0.0000	18	432

** Probabilities for Fisher tests are computed using an asymptotic Chi
-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: ROE

Date: 05/12/22 Time: 11:10

Sample: 1996 2020

Exogenous variables: Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	1.54506	0.9388	18	414
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-5.81859	0.0000	18	414
ADF - Fisher Chi-square	113.926	0.0000	18	414
PP - Fisher Chi-square	150.341	0.0000	18	432

** Probabilities for Fisher tests are computed using an asymptotic Chi

-square distribution. All other tests assume asymptotic normality.

Panel unit root test: Summary

Series: ROT

Date: 05/12/22 Time: 11:11

Sample: 1996 2020

Exogenous variables:

Individual effects

User-specified lags: 1

Newey-West automatic bandwidth selection and Bartlett kernel

Balanced observations for each test

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-4.91852	0.0000	16	368
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-7.66035	0.0000	16	368
ADF - Fisher Chi-square	123.463	0.0000	16	368
PP - Fisher Chi-square	147.231	0.0000	16	384

** Probabilities for Fisher tests are computed using an asymptotic Chi- Square

The lag structure selected for estimating the VAR in levels was based on the SC Information criterion, hence lag one was selected for this purpose.

ROA Results and Diagnosis thereof:

Vector Autoregression Estimates

Date: 05/12/22 Time: 11:40

Sample (adjusted): 1997 2020

Included observations: 299 after adjustments

Standard errors in ( ) & t-statistics in [ ]

	LROA	LBEL	LMAF	LSOB	LSA
LROA(-1)	0.983509	0.017875	-0.004076	0.003196	-0.016697
	(0.01314)	(0.01273)	(0.00194)	(0.00324)	(0.00633)
	[74.8296]	[1.40377]	[-2.09922]	[0.98597]	[-2.63931]

LBEL(-1)	-0.010949	0.907886	-0.006368	0.002221	0.008626
	(0.02281)	(0.02210)	(0.00337)	(0.00563)	(0.01098)
	[-0.47995]	[41.0753]	[-1.88956]	[0.39477]	[0.78551]
LMAF(-1)	-0.574435	-0.219471	0.679858	-0.045127	-0.075776
	(0.27950)	(0.27079)	(0.04129)	(0.06893)	(0.13453)
	[-2.05521]	[-0.81048]	[16.4664]	[-0.65467]	[-0.56326]
LSOB(-1)	-0.043626	-0.013194	0.012986	0.756411	-0.049504
	(0.16069)	(0.15568)	(0.02374)	(0.03963)	(0.07734)
	[-0.27149]	[-0.08475]	[0.54706]	[19.0870]	[-0.64004]
LSA(-1)	0.127075	0.122391	-0.005825	-0.028421	0.601650
	(0.08414)	(0.08152)	(0.01243)	(0.02075)	(0.04050)
	[1.51027]	[1.50139]	[-0.46869]	[-1.36963]	[14.8559]
C	1.696437	0.922697	0.873590	0.821388	0.399684
	(0.97115)	(0.94088)	(0.14346)	(0.23951)	(0.46744)
	[1.74684]	[0.98068]	[6.08956]	[3.42952]	[0.85505]
R-squared	0.961625	0.880532	0.582618	0.592481	0.485944
Adj. R-squared	0.960970	0.878493	0.575495	0.585527	0.477171
Sum sq. resids	149.4461	140.2751	3.261046	9.089566	34.62290
S.E. equation	0.714181	0.691921	0.105498	0.176132	0.343754
F-statistic	1468.433	431.9083	81.79883	85.19701	55.39529
Log likelihood	-320.5832	-311.1153	251.2375	97.98819	-101.9513
Akaike AIC	2.184503	2.121173	-1.640385	-0.615306	0.722082
Schwarz SC	2.258759	2.195429	-1.566128	-0.541049	0.796339
Mean dependent	-1.933063	2.576989	2.812271	2.810421	0.267258
S.D. dependent	3.615010	1.984982	0.161921	0.273583	0.475410

Determinant resid covariance (dof adj.)	9.37E-06
Determinant resid covariance	8.47E-06
Log likelihood	-375.2182
Akaike information criterion	2.710490
Schwarz criterion	3.081771

System: UNTITLED

Estimation Method: Least Squares

Date: 05/12/22 Time: 11:49

Sample: 1997 2020

Included observations: 333

Total system (unbalanced) observations 1628

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.987105	0.013849	71.27797	0.0000
C(2)	-0.025294	0.023783	-1.063552	0.2877
C(3)	-0.524523	0.283628	-1.849334	0.0646
C(4)	-0.023199	0.166034	-0.139723	0.8889
C(5)	0.097801	0.081302	1.202927	0.2292
C(6)	1.571465	0.983415	1.597967	0.1102
C(7)	0.014544	0.012567	1.157284	0.2473
C(8)	0.912985	0.021432	42.59876	0.0000
C(9)	-0.148057	0.257945	-0.573987	0.5661
C(10)	-0.010632	0.153173	-0.069410	0.9447
C(11)	0.100014	0.073750	1.356130	0.1752
C(12)	0.707520	0.892396	0.792832	0.4280
C(13)	-0.003873	0.001898	-2.040277	0.0415
C(14)	-0.009818	0.003254	-3.016746	0.0026
C(15)	0.611268	0.039130	15.62158	0.0000

C(16)	-0.003231	0.022875	-0.141245	0.8877
C(17)	-0.005635	0.011174	-0.504341	0.6141
C(18)	1.124145	0.135642	8.287583	0.0000
C(19)	0.003746	0.003020	1.240365	0.2150
C(20)	0.001701	0.005267	0.323025	0.7467
C(21)	-0.070303	0.064816	-1.084653	0.2782
C(22)	0.748223	0.036309	20.60695	0.0000
C(23)	-0.029208	0.017798	-1.641076	0.1010
C(24)	0.916960	0.222523	4.120744	0.0000
C(25)	-0.015487	0.006184	-2.504398	0.0124
C(26)	0.006837	0.010539	0.648699	0.5166
C(27)	-0.112774	0.127096	-0.887313	0.3750
C(28)	-0.037352	0.074944	-0.498394	0.6183
C(29)	0.611517	0.039618	15.43515	0.0000
C(30)	0.476046	0.447027	1.064915	0.2871

Determinant residual covariance 1.09E-05

Equation: LROA = C(1)*LROA(-1) + C(2)*LBEL(-1) + C(3)*LMAF(-1) + C(4)
*LSOB(-1) + C(5)*LSA(-1) + C(6)

Observations: 326

R-squared	0.954195	Mean dependent var	-1.919280
Adjusted R-squared	0.953479	S.D. dependent var	3.628067
S.E. of regression	0.782525	Sum squared resid	195.9505
Durbin-Watson stat	1.794702		

APPENDIX VI Showing VAR Model Residual Tests

System Residual Normality Tests

Orthogonalization: Cholesky (Lutkepohl)

Null Hypothesis: residuals are multivariate normal

Date: 05/12/22 Time: 11:53

Sample: 1997 2020

Included observations: 333

Component	Skewness	Chi-sq	Df	Prob.
1	1.251357	86.90713	1	0.0000
2	0.179727	1.792750	1	0.1806
3	-0.322344	5.766782	1	0.0163
4	-3.995529	886.0161	1	0.0000
5	-1.752146	170.3858	1	0.0000
Joint		1150.869	5	0.0000

At lag 2, the residuals become normally distributed as seen by the acceptance of the null hypothesis

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: no residual autocorrelations up to lag h

Date: 05/12/22 Time: 11:55

Sample: 1997 2020

Included observations: 333

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	72.37860	0.0000	72.62148	0.0000	25
2	88.08529	0.0007	88.43394	0.0007	50
3	106.6225	0.0096	107.1590	0.0088	75

4	129.7432	0.0244	130.5933	0.0216	100
5	154.9931	0.0355	156.2725	0.0304	125
6	168.9187	0.1384	170.4833	0.1208	150
7	188.2597	0.2336	190.2879	0.2033	175
8	218.3182	0.1781	221.1728	0.1454	200
9	229.5887	0.4027	232.7931	0.3466	225
10	237.3389	0.7075	240.8115	0.6503	250
11	244.9357	0.9037	248.6984	0.8709	275
12	281.8204	0.7674	287.1254	0.6936	300

*The test is valid only for lags larger than the System lag order.
df is degrees of freedom for (approximate) chi-square distribution
*df and Prob. may not be valid for models with lagged endogenous variables

These results indicate that from lags 1 to 5, the residuals are auto-correlated, however, from lags 6 to12, the residuals are not correlated.

NOTE: As reported for the ROA model, present these results throughout:

APPENDIX VII Showing VAR ROE Results Estimates

Vector Auto-regression Estimates

Date: 05/12/22 Time: 11:59

Sample (adjusted): 1997 2020

Included observations: 294 after adjustments

Standard errors in ( ) & t-statistics in [ ]

	LROE	LBEL	LMAF	LSOB	LSA
LROE(-1)	0.888275	0.019699	-0.002866	0.000505	-0.009485
	(0.02609)	(0.01429)	(0.00230)	(0.00112)	(0.00754)
	[34.0409]	[1.37863]	[-1.24673]	[0.44980]	[-1.25710]
LBEL(-1)	0.085869	0.911644	-0.005390	0.000307	0.010449
	(0.04403)	(0.02411)	(0.00388)	(0.00189)	(0.01273)
	[1.95033]	[37.8137]	[-1.38996]	[0.16237]	[0.82082]
LMAF(-1)	-0.797435	-0.264710	0.694317	0.012129	-0.047800
	(0.46727)	(0.25587)	(0.04116)	(0.02009)	(0.13511)
	[-1.70657]	[-1.03455]	[16.8691]	[0.60365]	[-0.35379]
LSOB(-1)	0.307862	-0.061241	0.013572	0.970270	-0.056680
	(0.30250)	(0.16564)	(0.02664)	(0.01301)	(0.08746)
	[1.01774]	[-0.36972]	[0.50936]	[74.5935]	[-0.64804]
LSA(-1)	0.257211	0.112692	-0.000929	-0.009660	0.639287
	(0.14057)	(0.07697)	(0.01238)	(0.00604)	(0.04064)
	[1.82981]	[1.46407]	[-0.07503]	[-1.59818]	[15.7290]
C	1.048125	1.164906	0.835468	0.055970	0.360303
	(1.70582)	(0.93407)	(0.15025)	(0.07335)	(0.49322)
	[0.61444]	[1.24713]	[5.56035]	[0.76305]	[0.73051]

R-squared	0.871107	0.892550	0.567104	0.954385	0.485756
Adj. R-squared	0.868869	0.890685	0.559588	0.953593	0.476828
Sum sq. resids	417.1870	125.0896	3.236814	0.771390	34.87799
S.E. equation	1.203564	0.659044	0.106014	0.051754	0.348000
F-statistic	389.2807	478.4641	75.45722	1205.138	54.40900
Log likelihood	-468.6113	-291.5492	245.6537	456.4739	-103.8045
Akaike AIC	3.228648	2.024144	-1.630297	-3.064448	0.746970
Schwarz SC	3.303823	2.099319	-1.555122	-2.989273	0.822145
Mean dependent	0.325126	2.636437	2.807010	2.815364	0.274431
S.D. dependent	3.323656	1.993306	0.159747	0.240242	0.481124

Determinant resid covariance (dof adj.)	1.91E-06
Determinant resid covariance	1.72E-06
Log likelihood	-135.1012
Akaike information criterion	1.123138
Schwarz criterion	1.499013

System: UNTITLED

Estimation Method: Least Squares

Date: 05/12/22 Time: 12:03

Sample: 1997 2020

Included observations: 330

Total system (unbalanced) observations 1612

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.887125	0.025036	35.43429	0.0000
C(2)	0.110303	0.042000	2.626293	0.0087
C(3)	-0.419557	0.435437	-0.963531	0.3354

C(4)	0.313689	0.288689	1.086598	0.2774
C(5)	0.245032	0.124895	1.961904	0.0499
C(6)	-0.082524	1.583239	-0.052124	0.9584
C(7)	0.018356	0.013322	1.377815	0.1685
C(8)	0.916836	0.022192	41.31395	0.0000
C(9)	-0.126719	0.231615	-0.547111	0.5844
C(10)	-0.030699	0.149757	-0.204993	0.8376
C(11)	0.106495	0.066343	1.605215	0.1086
C(12)	0.685495	0.841958	0.814167	0.4157
C(13)	-0.001775	0.002190	-0.810426	0.4178
C(14)	-0.009487	0.003694	-2.568301	0.0103
C(15)	0.620002	0.038763	15.99460	0.0000
C(16)	-0.013456	0.024568	-0.547698	0.5840
C(17)	-0.001660	0.011101	-0.149547	0.8811
C(18)	1.133379	0.141241	8.024444	0.0000
C(19)	-0.000244	0.001296	-0.188411	0.8506
C(20)	0.001487	0.002213	0.671961	0.5017
C(21)	-0.025177	0.023651	-1.064517	0.2873
C(22)	0.931124	0.014359	64.84710	0.0000
C(23)	-0.018117	0.006512	-2.781956	0.0055
C(24)	0.270871	0.085068	3.184179	0.0015
C(25)	-0.006216	0.007084	-0.877481	0.3804
C(26)	0.005858	0.011945	0.490400	0.6239
C(27)	-0.048855	0.126549	-0.386058	0.6995
C(28)	-0.064355	0.081342	-0.791167	0.4290
C(29)	0.633302	0.039321	16.10596	0.0000
C(30)	0.400885	0.468776	0.855173	0.3926

Determinant residual covariance3.05E-06

Equation: LROE = C(1)*LROE(-1) + C(2)*LBEL(-1) + C(3)*LMAF(-1) + C(4)

*LSOB(-1) + C(5)*LSA(-1) + C(6)

Observations: 322

R-squared	0.869818	Mean dependent var	0.306388
Adjusted R-squared	0.867758	S.D. dependent var	3.334341
S.E. of regression	1.212536	Sum squared resid	464.5970
Durbin-Watson stat	1.897531		

System Residual Normality Tests

Orthogonalization: Cholesky (Lutkepohl)

Null Hypothesis: residuals are multivariate normal

Date: 05/12/22 Time: 12:05

Sample: 1997 2020

Included observations: 330

Component	Skewness	Chi-sq	df	Prob.
1	-1.773673	173.0254	1	0.0000
2	1.401520	108.0341	1	0.0000
3	-0.264927	3.860238	1	0.0494
4	4.146673	945.7194	1	0.0000
5	-1.625025	145.2388	1	0.0000
Joint		1375.878	5	0.0000

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: no residual autocorrelations up to lag h

Date: 05/12/22 Time: 12:05

Sample: 1997 2020

Included observations: 330

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	46.97327	0.0049	47.13358	0.0047	25
2	68.76183	0.0403	69.07138	0.0382	50
3	101.4682	0.0226	102.1149	0.0204	75
4	129.1120	0.0266	130.1400	0.0231	100
5	165.7956	0.0086	167.4583	0.0067	125
6	184.4076	0.0294	186.4581	0.0231	150
7	206.8266	0.0502	209.4239	0.0386	175
8	236.7689	0.0384	240.2036	0.0273	200
9	247.7475	0.1425	251.5290	0.1083	225
10	271.0431	0.1722	275.6449	0.1273	250
11	281.6943	0.3777	286.7101	0.3013	275
12	321.4005	0.1892	328.1059	0.1271	300

*The test is valid only for lags larger than the System lag order.

df is degrees of freedom for (approximate) chi-square distribution

*df and Prob. may not be valid for models with lagged endogenous variables

APPENDIX VIII Showing VAR ROT Results

Vector Auto-regression Estimates

Date: 05/12/22 Time: 12:09

Sample (adjusted): 1997 2020

Included observations: 246 after adjustments

Standard errors in ( ) & t-statistics in [ ]

	LROT	LBEL	LMAF	LSOB	LSA
LROT(-1)	0.957942	0.012193	-0.000318	0.001959	-0.010105
	(0.01985)	(0.01245)	(0.00186)	(0.00310)	(0.00602)
	[48.2557]	[0.97971]	[-0.17042]	[0.63217]	[-1.67833]
LBEL(-1)	-0.017942	0.909507	-0.009699	0.005061	0.015927
	(0.04215)	(0.02642)	(0.00396)	(0.00658)	(0.01278)
	[-0.42567]	[34.4196]	[-2.45144]	[0.76920]	[1.24585]
LMAF(-1)	-0.634060	-0.383826	0.658066	-0.210191	0.053831
	(0.50385)	(0.31588)	(0.04729)	(0.07866)	(0.15282)
	[-1.25843]	[-1.21511]	[13.9143]	[-2.67225]	[0.35225]
LSOB(-1)	-0.133617	-0.035576	-0.034420	0.501349	-0.075363
	(0.38605)	(0.24203)	(0.03624)	(0.06027)	(0.11709)
	[-0.34611]	[-0.14699]	[-0.94986]	[8.31875]	[-0.64362]
LSA(-1)	-0.053303	0.117213	0.001028	-0.026683	0.601602
	(0.14641)	(0.09179)	(0.01374)	(0.02286)	(0.04441)
	[-0.36407]	[1.27703]	[0.07479]	[-1.16746]	[13.5477]
C	2.268368	1.395094	1.087481	2.001035	0.138088
	(2.10140)	(1.31742)	(0.19725)	(0.32805)	(0.63737)
	[1.07945]	[1.05896]	[5.51323]	[6.09973]	[0.21665]

R-squared	0.914606	0.868689	0.560691	0.335031	0.471534
Adj. R-squared	0.912827	0.865954	0.551538	0.321178	0.460524
Sum sq. resids	326.6809	128.3973	2.878296	7.961463	30.05315
S.E. equation	1.166692	0.731429	0.109512	0.182134	0.353866
F-statistic	514.1027	317.5457	61.26243	24.18385	42.82890
Log likelihood	-383.9481	-269.0840	198.0615	72.91952	-90.46809
Akaike AIC	3.170310	2.236455	-1.561475	-0.544061	0.784293
Schwarz SC	3.255806	2.321951	-1.475980	-0.458565	0.869789
Mean dependent	0.909587	2.520715	2.822130	2.837053	0.277292
S.D. dependent	3.951537	1.997768	0.163531	0.221061	0.481785

Determinant resid covariance (dof adj.)	3.46E-05
Determinant resid covariance	3.06E-05
Log likelihood	-466.7726
Akaike information criterion	4.038802
Schwarz criterion	4.466281

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.965808	0.018480	52.26154	0.0000
C(2)	-0.015436	0.038711	-0.398738	0.6902
C(3)	-0.418049	0.450225	-0.928534	0.3533
C(4)	-0.115212	0.365624	-0.315109	0.7527
C(5)	-0.042238	0.126328	-0.334350	0.7382
C(6)	1.613446	1.907871	0.845679	0.3979
C(7)	0.014949	0.011547	1.294609	0.1957
C(8)	0.907199	0.023831	38.06856	0.0000
C(9)	-0.290197	0.284974	-1.018330	0.3087

C(10)	0.040156	0.221842	0.181011	0.8564
C(11)	0.104490	0.079955	1.306858	0.1915
C(12)	0.929092	1.190668	0.780311	0.4353
C(13)	0.000966	0.001755	0.550532	0.5820
C(14)	-0.012893	0.003622	-3.559619	0.0004
C(15)	0.577728	0.043347	13.32809	0.0000
C(16)	-0.072668	0.033719	-2.155091	0.0313
C(17)	0.002568	0.012122	0.211863	0.8322
C(18)	1.430527	0.181228	7.893532	0.0000
C(19)	0.001580	0.002850	0.554414	0.5794
C(20)	0.003257	0.005897	0.552245	0.5809
C(21)	-0.199068	0.072654	-2.739932	0.0062
C(22)	0.566806	0.053688	10.55748	0.0000
C(23)	-0.018594	0.019326	-0.962101	0.3362
C(24)	1.783598	0.298624	5.972729	0.0000
C(25)	-0.011386	0.005672	-2.007458	0.0449
C(26)	0.013140	0.011680	1.124980	0.2608
C(27)	-0.008587	0.139859	-0.061395	0.9511
C(28)	-0.081748	0.108170	-0.755741	0.4499
C(29)	0.613758	0.042610	14.40404	0.0000
C(30)	0.342007	0.585876	0.583753	0.5595

Determinant residual covariance 2.76E-05

Equation: LROT = C(1)*LROT(-1) + C(2)*LBEL(-1) + C(3)*LMAF(-1) + C(4)

*LSOB(-1) + C(5)*LSA(-1) + C(6)

Observations: 267

R-squared	0.918824	Mean dependent var	0.999791

Adjusted R-squared	0.917269	S.D. dependent var	3.967567
S.E. of regression	1.141189	Sum squared resid	339.9035
Durbin-Watson stat	2.094998		

System Residual Normality Tests

Orthogonalization: Cholesky (Lutkepohl)

Null Hypothesis: residuals are multivariate normal

Date: 05/12/22 Time: 12:13

Sample: 1997 2020

Included observations: 280

Component	Skewness	Chi-sq	df	Prob.
1	-1.586194	117.4139	1	0.0000
2	0.107122	0.535508	1	0.4643
3	-0.618877	17.87371	1	0.0000
4	-2.717210	344.5508	1	0.0000
5	-1.730655	139.7745	1	0.0000
Joint		620.1484	5	0.0000

System Residual Portmanteau Tests for Autocorrelations

Null Hypothesis: no residual autocorrelations up to lag h

Date: 05/12/22 Time: 12:13

Sample: 1997 2020

Included observations: 280

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	Df
1	39.70185	0.0313	39.86390	0.0301	25
2	57.08133	0.2287	57.38583	0.2204	50
3	80.63709	0.3074	81.23241	0.2913	75
4	100.5286	0.4663	101.4527	0.4406	100
5	118.5947	0.6443	119.8936	0.6122	125
6	127.6893	0.9064	129.2156	0.8890	150
7	146.0099	0.9461	148.0727	0.9312	175
8	177.0990	0.8766	180.2069	0.8391	200
9	189.7323	0.9579	193.3199	0.9380	225
10	196.7388	0.9945	200.6234	0.9904	250
11	200.5356	0.9998	204.5979	0.9995	275
12	230.6459	0.9989	236.2523	0.9973	300

*The test is valid only for lags larger than the System lag order.

df is degrees of freedom for (approximate) chi-square distribution

*df and Prob. may not be valid for models with lagged endogenous variables

SADC BANKS FINANCIAL DATA (WORLD DEVELOPMENT INDEX, 2020)

	2020	2019	2018	2017	2016	2015	2014
FINANCIAL STATEMENTS REPORT							
ABSA GROUP LTD (ABG)							
031 Investments & Loans	9.53E+08	9.39E+08	8.59E+08	7.7E+08	7.4E+08	7.01E+08	5.64E+08
032 Investment at Cost/Market Value	19861000	20868000	16932000	20171000	19881000	20517000	39044000
033 Long Term Loans	9.33E+08	9.18E+08	8.42E+08	7.5E+08	7.2E+08	6.81E+08	5.25E+08
050 Total Assets (Excluding Intangible Assets)	1.52E+09	1.39E+09	1.28E+09	1.16E+09	1.1E+09	1.14E+09	9.88E+08
051 Total Assets (Including Intangible Assets)	1.53E+09	1.4E+09	1.29E+09	1.17E+09	1.1E+09	1.14E+09	9.91E+08
013 Total Equity	1.32E+08	1.29E+08	1.22E+08	1.19E+08	1.02E+08	98647000	90945000
022 Total Liabilities	1.4E+09	1.27E+09	1.17E+09	1.05E+09	9.99E+08	1.05E+09	9E+08
058 Total Equity and Liabilities	1.53E+09	1.4E+09	1.29E+09	1.17E+09	1.1E+09	1.14E+09	9.91E+08
201 Shares in Issue Y/E Ordinary	828.789	828.628	827.477	832.838	846.675	845.725	846.871
100 Profit After Interest and Tax	7213000	15980000	15259000	15022000	15847000	15404000	14144000

FINANCIAL STATEMENTS REPORT							
CAPITEC BANK HOLDINGS LTD (CPI)							
050 Total Assets (Excluding Intangible Assets)	1.33E+08	1E+08	84674223	73077951	62702854	53677600	45989647
051 Total Assets (Including Intangible Assets)	1.34E+08	1E+08	84957234	73357897	62945502	53916475	46190966
013 Total Equity	25580840	21675796	18891678	16118013	13659065	11563740	9982111
022 Total Liabilities	1.09E+08	78751953	66065556	57239884	49286437	42352735	36208855
058 Total Equity and Liabilities	1.34E+08	1E+08	84957234	73357897	62945502	53916475	46190966
201 Shares in Issue Y/E Ordinary	115.627	115.627	115.627	115.627	115.627	115.627	115.298
100 Profit After Interest and Tax	6220588	5295411	4470717	3806930	3228237	2563599	2037554
FINANCIAL STATEMENTS REPORT							
FINBOND GROUP LTD (FGL)							
031 Investments & Loans	162.219	171.928	190.894	423.946	397.561	0	0
032 Investment at Cost/Market Value	0	0	0	0	0	0	0
033 Long Term Loans	162.219	171.928	190.894	423.946	397.561	0	0

050 Total Assets (Excluding Intangible Assets)	3562473	2317514	2371174	2309839	1278281	1229047	1023251
051 Total Assets (Including Intangible Assets)	4673527	3422224	3309286	3177602	1431428	1349252	1085847
013 Total Equity	1699754	1652105	1170732	1137408	387.989	345.904	329.602
022 Total Liabilities	2973773	1770119	2138554	2040194	1043439	1003348	756.245
058 Total Equity and Liabilities	4673527	3422224	3309286	3177602	1431428	1349252	1085847
201 Shares in Issue Y/E Ordinary	877.255	923.727	748.547	746.712	590.981	589.614	605.025
100 Profit After Interest and Tax	214.6	152.988	334.961	180.446	57.254	50.867	36.917
FINANCIAL STATEMENTS REPORT							
FIRSTRAND LTD (FSR)							
031 Investments & Loans	1.57E+09	1.46E+09	1.34E+09	1.02E+09	1E+09	9.24E+08	5.86E+08
032 Investment at Cost/Market Value	3.06E+08	2.5E+08	2.16E+08	1.75E+08	1.92E+08	1.72E+08	1.26E+08
033 Long Term Loans	1.26E+09	1.21E+09	1.12E+09	8.49E+08	8.09E+08	7.52E+08	4.6E+08
050 Total Assets (Excluding Intangible Assets)	1.91E+09	1.66E+09	1.52E+09	1.22E+09	1.15E+09	1.06E+09	9.44E+08
051 Total Assets (Including Intangible Assets)	1.93E+09	1.67E+09	1.53E+09	1.22E+09	1.15E+09	1.06E+09	9.46E+08
013 Total Equity	1.52E+08	1.45E+08	1.31E+08	1.17E+08	1.08E+08	98604000	88217000
022 Total Liabilities	1.77E+09	1.52E+09	1.4E+09	1.1E+09	1.04E+09	9.61E+08	8.57E+08

058 Total Equity and Liabilities	1.93E+09	1.67E+09	1.53E+09	1.22E+09	1.15E+09	1.06E+09	9.46E+08
201 Shares in Issue Y/E Ordinary	5606248	5609102	5608442	5609176	5607287	5606532	5485118
100 Profit After Interest and Tax	19680000	31760000	28144000	26139000	24075000	23124000	19786000

FINANCIAL STATEMENTS REPORT

INVESTEC LTD (INL)

031 Investments & Loans	27368785	27829075	27883661	25451280	20222467	20603775	24147611
032 Investment at Cost/Market Value	2188722	2556956	2255368	2239931	1621740	2059439	1720149
033 Long Term Loans	25180063	25272119	25628293	23211349	18600727	18544336	22427462
050 Total Assets (Excluding Intangible Assets)	50299391	57250105	57122652	53023992	44835462	43844648	46549167
051 Total Assets (Including Intangible Assets)	50656316	57724212	57616844	53534832	45351781	44353402	47141907
013 Total Equity	4602039	4947286	5124158	4775831	3833276	4009896	4013041
022 Total Liabilities	46054277	52776926	52492686	48759001	41518505	40343506	43128866
058 Total Equity and Liabilities	50656316	57724212	57616844	53534832	45351781	44353402	47141907
201 Shares in Issue Y/E Ordinary	1014988	1001026	980.562	958.271	908.783	584.122	579.095
100 Profit After Interest and Tax	277.483	614.974	581.653	522.996	420.186	275.394	353.546

FINANCIAL STATEMENTS REPORT							
INVESTEC PLC (INP)							
031 Investments & Loans	27368785	27829075	27883661	25451280	20222467	20603775	24147611
032 Investment at Cost/Market Value	2188722	2556956	2255368	2239931	1621740	2059439	1720149
033 Long Term Loans	25180063	25272119	25628293	23211349	18600727	18544336	22427462
050 Total Assets (Excluding Intangible Assets)	50299391	57250105	57122652	53023992	44835462	43844648	46549167
051 Total Assets (Including Intangible Assets)	50656316	57724212	57616844	53534832	45351781	44353402	47141907
013 Total Equity	4602039	4947286	5124158	4775831	3833276	4009896	4013041
022 Total Liabilities	46054277	52776926	52492686	48759001	41518505	40343506	43128866
058 Total Equity and Liabilities	50656316	57724212	57616844	53534832	45351781	44353402	47141907
201 Shares in Issue Y/E Ordinary	1014988	1001026	980.562	958.271	908.783	584.122	579.095
100 Profit After Interest and Tax	277.483	614.974	581.653	522.996	420.186	275.394	353.546
FINANCIAL STATEMENTS REPORT							
NEDBANK GROUP LTD (NED)							
031 Investments & Loans	1.62E+08	1.61E+08	1.23E+08	72597000	71841000	4.22E+08	1.25E+08
032 Investment at Cost/Market Value	1.62E+08	1.61E+08	1.23E+08	72597000	71840000	65794000	54876000

033 Long Term Loans	0	0	0	0	1	3.56E+08	69750000
050 Total Assets (Excluding Intangible Assets)	1.21E+09	1.13E+09	1.03E+09	9.72E+08	9.56E+08	9.17E+08	8.01E+08
051 Total Assets (Including Intangible Assets)	1.23E+09	1.14E+09	1.04E+09	9.83E+08	9.66E+08	9.26E+08	8.09E+08
013 Total Equity	1E+08	98449000	91271000	88539000	81711000	78751000	70911000
022 Total Liabilities	1.13E+09	1.04E+09	9.53E+08	8.95E+08	8.84E+08	8.47E+08	7.38E+08
058 Total Equity and Liabilities	1.23E+09	1.14E+09	1.04E+09	9.83E+08	9.66E+08	9.26E+08	8.09E+08
201 Shares in Issue Y/E Ordinary	483.893	481.174	477.129	481.569	478.389	476.556	465.643
100 Profit After Interest and Tax	4454000	12810000	14135000	12299000	10659000	11162000	10188000
FINANCIAL STATEMENTS REPORT							
STANDARD BANK GROUP LTD (SBK)							
031 Investments & Loans	1.81E+09	1.68E+09	1.7E+09	1.53E+09	1.45E+09	1.39E+09	1.11E+09
032 Investment at Cost/Market Value	1.37E+08	1.46E+08	3.67E+08	3.57E+08	3.32E+08	3.86E+08	4.78E+08
033 Long Term Loans	1.67E+09	1.54E+09	1.33E+09	1.18E+09	1.12E+09	1E+09	6.32E+08
050 Total Assets (Excluding Intangible Assets)	2.51E+09	2.25E+09	2.1E+09	2E+09	1.93E+09	1.96E+09	1.88E+09
051 Total Assets (Including Intangible Assets)	2.53E+09	2.28E+09	2.13E+09	2.03E+09	1.95E+09	1.98E+09	1.9E+09
013 Total Equity	2.15E+08	2.09E+08	1.99E+08	1.9E+08	1.79E+08	1.79E+08	1.62E+08

022 Total Liabilities	2.32E+09	2.07E+09	1.93E+09	1.84E+09	1.77E+09	1.8E+09	1.74E+09
058 Total Equity and Liabilities	2.53E+09	2.28E+09	2.13E+09	2.03E+09	1.95E+09	1.98E+09	1.9E+09
201 Shares in Issue Y/E Ordinary	1619941	1619710	1618514	1619268	1618421	1618252	1577828
100 Profit After Interest and Tax	14513000	30696000	32643000	30715000	25794000	25360000	26213000