HEAT MASS TRANSFER WITH CHEMICAL REACTION OF AN EXPONENTIALLY STRETCHING STENOSED ARTERY

BY

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APPROVAL FORM

The undersigned certify that they have supervised, read and recommended the Midlands State University to accept a research project entitled, Heat transfer with chemical reaction through an exponentially stretching artery, submitted in partial fulfilment of the requirements of the Bachelor of Science Mathematics Honours Degree at the Midlands State University.

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Declaration

I hereby declare that the work incorporated in this dissertation is original and has not been submitted to any institution for the award of a degree, diploma or certificate. I further declare that the results in the research and considerations made contribute in general to the advancement of knowledge in education and in particular to Fluid Dynamics.

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Name

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Signature

Dedication

This research is dedicated to my parents, Mr and Mrs Majiri. I will always be your source of pride.

Acknowledgements

Before and above everything I give special thanks to the Lord God Almighty for making this project a successful endeavour. I will live to testify your mighty. I also extend my sincere gratitude to my supervisor, Mr C. Murewi whose contribution to this research is immeasurable, your assistance, sir, is highly appreciated. I must say it was a great honour working with you.

To all the Mathematics lecturers, I am very grateful for the unwavering support you offered. I am also heavily indebted to a number of people who, through their remarkable assistance have contributed significantly to make this research a success; R Majiri, R Murungweni, Mrs Mukwembi, Mr and Mrs Nyakudya, may the Lord God Almighty bless you abundantly.

Lastly I would like to say thanks to my fellow undergraduate students at the Department of Mathematics during my studies. Their cooperative spirit and contribution during the whole period of my study is appreciated.

Abstract

The laminar boundary layer flow in an exponentially stretching stenosed artery immersed in viscous and incompressible blood is investigated along with the effect of a chemical reaction. Many researchers have used mathematical modelling to predict blood flow through stenosed arteries. Pressure, shear stress and velocity are the parameters that have been analysed in the past. Not a lot of work has been done to highlight the effect of heat transfer on an exponentially stretching stenosed artery with a chemical reaction effect. In this research, stenosis is defined as a condition whereby arteries abnormally narrow. The ability to describe the flow of blood through a stenosed artery provides the possibility of diagnosing the related diseases even before they become clinically relevant. The governing partial differential boundary layer equations in Cartesian Co-ordinate form are first transformed into ordinary differential equations which are then solved by the Runge-Kutta and Shooting methods using the Matlab software package. Potential improvements to previous models have been made by the inco-operation of the effect of exponential stretching of the stenosed artery as blood flows and also by including the effect of a chemical reaction to blood flow. The researcher has come to conclude that the dimensionless temperature field depends on thermal diffusivity (α), the heat component (η) and the dimensionless $(U_0.D_0)$/ Re. U_0 is the stretching velocity, D_0 represents the length of the stenosed portion and Re is the Reynolds' number. Also, chemical reactions mostly caused by foreign substances to the body, generally lower the flow of blood in arteries. The effects of asymmetric stenosis in a realm of the arterial plaque may be useful for early detection of cardiovascular diseases. Hence the researcher recommends that people take low fat and cholesterol diets as these highly lead to stenotic conditions. Stenotic conditions lead to diseases such as Athesclerosis, Coronary heart disease and high blood pressure.

Table of Contents

CHAPTER ONE

1.1 Introduction

Stenosis is a condition whereby arteries abnormally narrow. The ability to describe the flow of blood through a stenosed artery provides the possibility of diagnosing the related diseases even before they become clinically relevant. Causes of stenosis include accumulation of fat and the commonly known fact, drinking cold water during meals. The latter causes coagulation of fat hence narrowing the arteries. This leads to high blood pressure, coronary heart diseases, brain failure, Athesclerosis. The worst cases are cardiac arrest and stroke.

The artery contains viscous incompressible blood. It is treated to be complaint. The shape of the stenosis in the arterial lumen is chosen to be symmetric as well as asymmetric about the middle cross section, perpendicular to the axis of the tube to improve the in vivo situation [1].

During blood flow, a constricted tube is transformed to a straight tube and the resultant governing equations are solved by the numerical methods namely Runge-Kutta and Shooting methods [8]. The governing equations are transformed to Ordinary Differential Equations by the Similarity Method to get numerical solutions which can be interpreted easily. Researchers in the past have analysed influences of hemodynamic parameters which include wall shear stress, pressure and velocity. However not a lot of work has been done to highlight the effect of heat transfer with exponential stretching of the stenosed artery during blood flow. Hence the need for study.

Mathematical modelling provides an economic and non-invansive method of studying blood flow through arteries. Two approaches, analytical and computational are used. Analytical methods are best studied to explore the underlying physics of the situation and provide real time results and simplified solutions. Computational fluid dynamics modelling is one of the most powerful means to analysize the blood flow because we can in-cooperate the complex nature of blood flow and the blood vessel interactions into the study. Mathematical modelling to predict flow through Athesclerosis arteries augment the percipience and experiment of cardiologists and assist in understanding of the genesis and progression of stenosis development. Such techniques allow predicting the hemodynamic characteristics such as pressure, shear stress, velocity and reduction in flow.

Similarity solutions are defined mathematically as a solution where a change of variables allows for a reduction in the number of independent variables. The researcher obtained similarity solutions of the two dimensional third order boundary value problem. Blood flow is modelled by the commonly known Navier stokes equations.

The details of the flow within the boundary layer are very important for many problems on aerodynamics, including wing stall, the skin friction drag on an object, and heat transfer that occurs in high speed flight [27]. We will only present some of the effects of the boundary layer.

1.2 Problem statement

Many researchers have used mathematical modelling to predict blood flow through stenosed arteries. This has helped doctors and cardiologists to understand the genesis of stenosis and its development. According to past researchers blood tends to follow different patterns in the presence of stenosis at different stages. While pressure, shear stress and velocity are the parameters that have been analysed in the past, the resultant effect of heat transfer with exponential stretching of the stenosed artery during blood flow has not been fully highlighted. The researcher has also included the effect of a chemical reaction to heat mass transfer. Current researchers have raised concern on this topic. These include [21], [25], [29], [31].

1.3 Aim

To create a heat transfer model of an exponentially stretching stenosed artery with a chemical reaction during blood flow.

1.4 Objectives

- To construct a mathematical model of heat transfer with exponential stretching for a stenosed artery during the flow of blood.
- To introduce a chemical reaction into the model.
- To use the Similarity Theory to reduce the governing PDEs to ODEs.
- To find numerical solutions to the ODEs.
- To draw physical interpretation of the solutions from the model.

1.5 Methodology

- Concept of Model Development was applied to modify governing Navier-Stokes equations using theoretical and physiological considerations.

- The Similarity Theory was applied to convert the PDEs to ODEs.

- The Runge-Kutta and Shooting Methods were applied to get numerical solutions to the ODEs.

- Matlab was used to carry out the calculations.

1.6 Significance of the study

The research gives a clear understanding of the genesis of a stenosis. It also highlights the hemodynamic behaviour of blood flow with the mechanical properties of the arterial wall material under physiological conditions. The ability to describe the flow in stenosed arteries provides the possibility of diagnosing related diseases in earlier stages before they become clinically relevant. This becomes the basis for surgical intervention. Appropriate and timely interventions by cardiologists greatly reduce the risk of death caused by stenosis. The research also incorperates major parameters that influence the flow of blood. Hence this research gives more information on this area. The graph below shows the numbers of individuals susceptible to stenotic diseases.

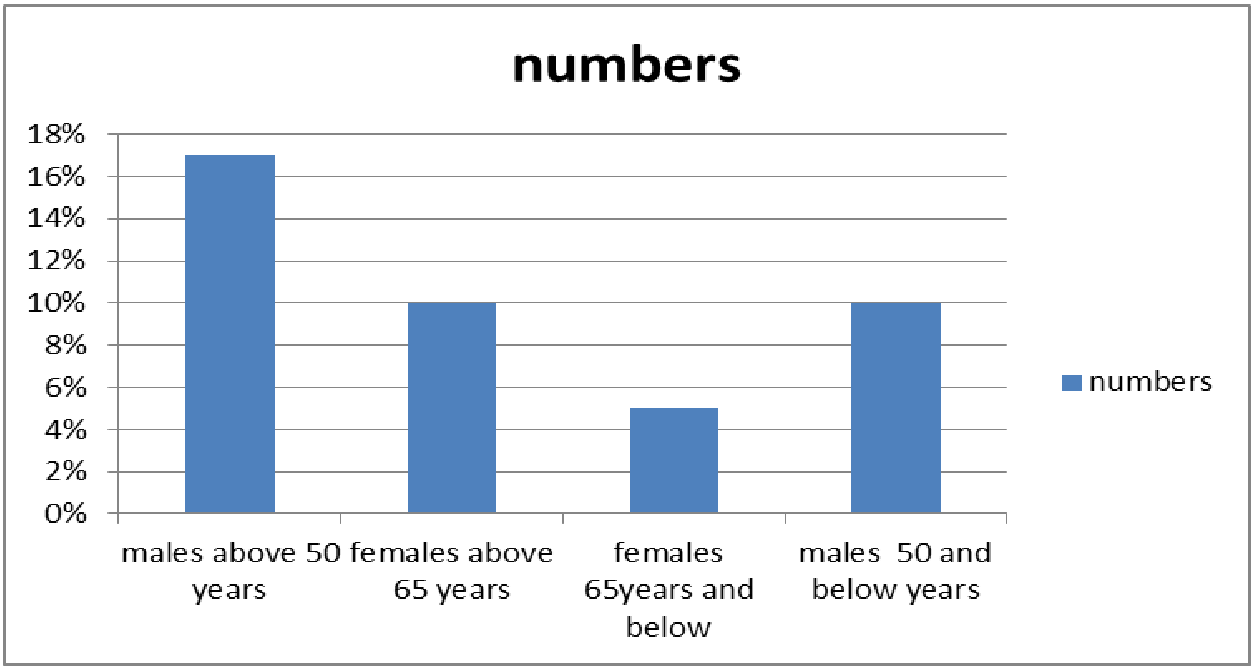


Figure 1: Susceptible population

From the graph, males above 50 years have the highest risk of stenotic diseases mostly because statistics has it that males at this age tend to smoke and take in alcohol more than any other age group [8]. This comes with foreign substances like tar which blocks the arteries hence causing a constriction and this is what is being referred to as a stenosis. The other age groups are affected by stenotic diseases because of factors like the diet. Too much fat and cholesterol can accumulate inside the lumen of the artery hence causing a stenosis and eventually diseases related to it like high blood pressure, Athesclerosis, brain damage [25].

1.7 Assumptions

The researcher has made the following assumptions in trying to model blood flow through a stenosed artery.

- We make use of the Cartesian Co-ordinate system.
- The artery stretches exponentially as the blood flows through the stenosed artery.
- The shape of the stenosis in the arterial lumen is chosen to be symmetric as well as asymmetric about the middle cross section, perpendicular to the axis of the tube to improve the in-vivo situation [29].
- The blood flow is incompressible
- There is steady flow
- Two –dimensional flow with $w = 0, \frac{\partial}{\partial z} = 0$
- Gravity was ignored in the study.

CHAPTER TWO

This chapter is a review of both supporting and critiquing literature concerning the flow of blood through a stenosed artery. It aims to expose the opinions of different authors with regards to fluid dynamics and the issues embraced under this subject. Definitions of fluid dynamics, the factors prompting the implementation of different fluid flow constants for example the Reynolds' number. The chapter also highlights the models used by other researchers in trying to model blood flow.

2.0 Blood Environment

Blood is a complex mixture of proteins, lipoproteins, and ions by which nutrients and waste are transported. Red blood cells typically comprise approximately 40% of blood by volume [17]. Because red blood cells are small semisolid particles they increase viscosity of blood and affect the behaviour of the fluid. The diagram below shows a complex mixture of red blood cells mixture of proteins, lipoproteins, and ions in an artery [15].

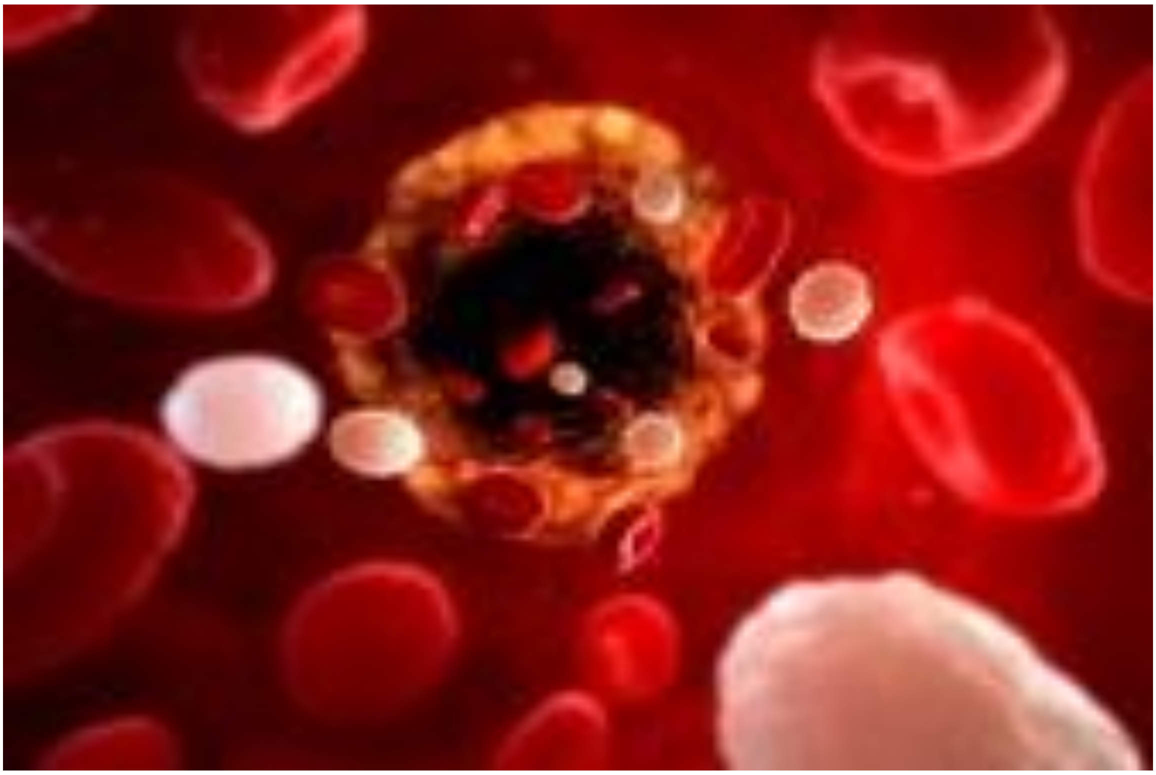


Figure 2: Artery and its components.

Blood is approximately four times more viscous than water [9]. Moreover blood does not exhibit a constant viscosity at all flow rates and is especially non-Newtonian in the microcirculatory system. The non-Newtonian behaviour is most evident at low shear rates when the red blood cells clump together into larger particles. However, in most arteries blood behaves in a Newtonian fashion and the viscosity can be taken as a constant, 4 centipoises [8].

The heat exchange between the living tissues and the blood network that passes through it depends on the geometry of the blood vessels, the blood flow through it, and the properties of the blood and surrounding tissue [16]. In the past, heat transfers in blood vessels have been studied by many researchers. [17] investigated the effect of large blood vessel in heated tissues and showed that the dissipation of heat from heated tissues was carried out by convection through blood flow and also by conduction process. [18] demonstrated that blood flow through large blood vessels play an important role in determining temperature profiles of heated tissue even when the treatment time is within 3-20s. Blood circulation is considered to play an important role in heat transfer between living tissues, particularly, in peripheral vessels where the temperature is, generally, closely related with blood flow rate.

Blood flow characteristics through an artery in the presence of multi-stenosis was developed by [20]. They considered unsteady flow characteristics of blood flow as Newtonian fluid in a multi-stenosed distensible artery constrained with pulsatile pressure gradient and variable viscosity when subjected to body acceleration. A study of blood flow through mild stenosis arterial segment has been investigated by [28]. Based upon the considerations of laminar and fully thermally developed flow in large blood vessels, [22] analysed temperature distribution in the entrance region around the vessels during hyperthermia. [29] calculated heat transfer in the entrance region considering the rheological properties of the blood stream and a cell free peripheral plasma layer at the vessel wall.

Many other authors investigated the heat and mass transfer phenomena in arteries and biological tissues. [19, 20] developed a mathematical model describing the dynamic response of heat and mass transfer in blood through bifurcated arteries under stenotic condition. [23] considered the flow equations and energy equations together with the boundary conditions in dimensionless form by formulation the problem in term of vorticity- stream function model, applying analytical axis transformation. [21] studied the influence of pulsatile laminar flow and heating protocol on temperature distribution in a single blood vessel and tumour tissue receiving hyperthermia treatment. They concluded that large vessel have profound effect on the heat transfer characteristics in tissues receiving hyperthermia treatment. Whilst this area is broad and is of paramount interest to several researchers, not much has been done to highlight heat transfer considering exponential stretching of the stenosed artery during blood flow.

2.1 Stenosis

When arteries become severely diseased, the arterial lumen becomes locally restricted over a 1cm distance. This constriction is commonly referred to as stenosis. In clinical medicine, stenosis is defined as percent occlusion by diameter as illustrated by the diagram below.

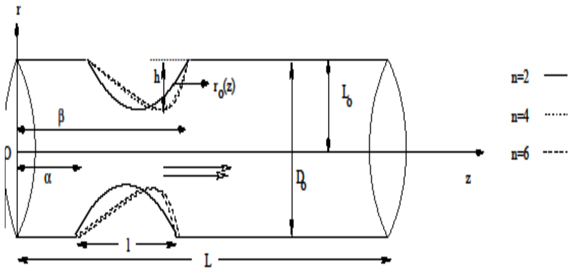


Figure 3: Occlusion by diameter in a stenosed artery.

As disease advances so does the percent stenosis. For stenosis greater than 75 %, several mechanisms severely limit the flow. Turbulence downstream the stenosis is very large and creates significant resistance. The critical Reynolds number for turbulence rapidly falls for stenosis greater than 25%. At 50% stenosis, turbulence is generated throughout the pulsatile cycle [11].The figure below shows streamline constrictions in a stenosed artery with 75% area reduction at Re=600.This is illustrated by the following diagram adapted from [11].

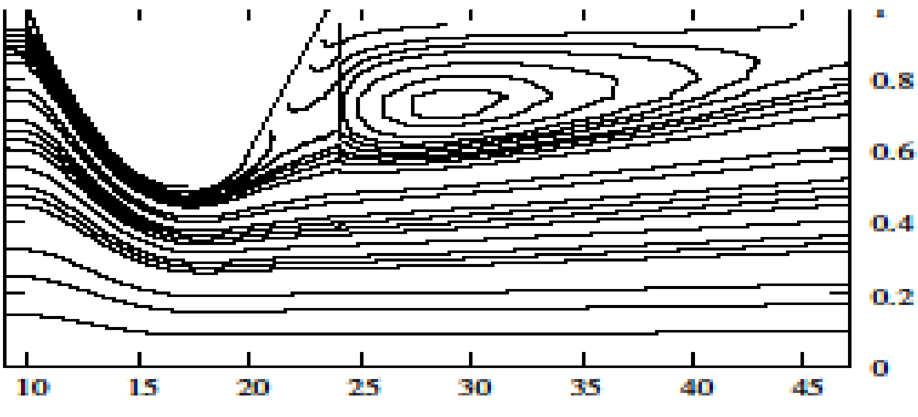


Fig 4: Streamline Constrictions in a Stenosed Artery with 75% Area Reduction at Re=600.

Stenosis severely remains the dominant factor for collapse under physiological conditions. When stenosis begins to collapse, an unstable cycle begins is setup. As the collapse increases, the resistance to flow increases and the flow rate drops [12].

Stenosis can also cause turbulence and reduce flow by means of viscous head losses and flow choking. Very high shear stresses near the throat of the stenosis can activate platelets and thereby induce thrombosis, which can totally block blood flow to the heart or brain. Detection and quantification of stenosis serve as the basis for surgical intervention. In future, the study of arterial blood will lead to the prediction of individual hemodynamic flows in a patient, the development of diagnostic tools to quantify disease, and the design of devices that mimic or alter blood flow [30].

The formation of stenosis is the direst biological response. Hence a complete understanding between pressures, flow, heat transfer, exponential stretching on a stenosed artery during blood flow remains critical problem hence the need for study. New devices for repairing stenotic arteries are now being developed cellular and tissue-engineered arteries will need bioreactor chambers that mimic the in vivo hemodynamic environment. Thus fluid mechanics will continue to play an important role in the future diagnosis, understanding and treatment of cardiovascular diseases.

Hemodynamic studies of stenosis have many clinical implications. One area of investigation revolves around the diagnosis of severe stenosis. The most acceptable clinical predictors of impending heart attack, stroke and lowerlimb ischemia are based on the presence of hemodynamically significant stenosis. The arts of cardiovascular disease are based on the severity and location of stenosis. For carotid artery disease, surgery is recommended to patients with stenosis greater than 75%. For coronary heart disease, the type of treatment is often based on whether the artery is more than 75% stenotic, in the left main coronary artery the percentage is 50%.

Athesclerosis, also a result of stenosis, is a disease which severely influences human health. It is characterised by the hardening and thickening of the arterial wall during the formation of plaque. With the progress of disease, the formation of plaque reduces the arterial passage area creating uncharacteristic blood flow patterns. As a result this restriction, if severe enough, can cause individuals to suffer cardiac arrest or stroke. Stenosis has a complex influence the movement of blood through and beyond the narrowed arterial segment. Athesclerosis disease tends to be localised in regions of geometrical irregularity such as vessel branch, curved and tapered arteries and stenotic sites [9]. Currently the best indicator for surgical treatment of this disease is the degree of stenosis. One can determine the anatomy of the lesion using X-ray contrast angiography. This technique yields a percent stenosis but says little about the flow rate, flow reserve or nature of plaque. Although angiography is currently the standard diagnostic procedure, cost and morbidity are distinct disadvantages.

Coronary heart disease which is one of the largest causes of mortality in developed nations is also a result of stenosis. It occurs when the coronary arteries narrow as a result they are unable to transport blood sufficiently to the heart muscle for it to function efficiently [9]. The two main causes of death from coronary artery disease are rapture of the plaque causing sudden occlusion of the artery and the slow build up of a stenosis in the artery due to Athesclerosis. The reduction in blood flow caused by stenosis build up also causes debilitation which is serious weakening and loss of energy [12].

In order to have a full understanding of the stenosis from a physiological point of view one needs to fully relate to hemodynamic behaviour of the streaming blood together with the mechanical properties of the vascular wall material under physiological conditions. The ability to describe the flow of blood in the presence of a stenosis with exponential stretching of the artery would provide the possibility of diagnosing the disease in its early stages which greatly reduces the death rate due to stenosis.

2.2 Heat transfer

Heat transfer is the flow of thermal energy driven by thermal non-equilibrium (i.e. the effect of a non-uniform temperature field), commonly measured as a heat flux (vector), i.e. the heat flow per unit time (and usually unit normal area) at a control surface. The aim here is to understand heat transfer modelling, but the actual goal of most heat transfer (modelling) problems is to find the temperature field and heat fluxes in a material domain, given a previous knowledge of the subject (the general Partial Differential Equation), and a set of particular constraints, boundary conditions.

There are three basic ways in which heat is transferred; in fluids heat is often transferred by convection, in which the motion of the fluid itself carries heat from one place to another. Another way to transfer heat is by conduction, which does not involve any motion of a substance, but rather is a transfer of energy within a substance (or between substances in contact).The third way is radiation, which involves absorbing or giving off electromagnetic waves.

In the simplest terms, the aspect of heat transfer is concerned with only two things, temperature and the flow of heat. Temperature represents the amount of thermal energy available whereas heat flow represents the movement of thermal energy form place to place.

On a microscopic scale, thermal energy is related to the kinetic energy of molecules. The greater the a material's temperature, the greater the thermal agitation of its constituent molecules. It is natural for regions containing greater molecular kinetic energy to pass this energy to regions with less kinetic energy.

Several material properties serve to modulate the heat transferred between two regions at different temperatures. Examples include thermal conductivities, specific heats, material densities, fluid velocities, fluid viscosities, surface emissivities and more. Taken together these properties serve to make the solution of many heat transfer problems an involved process.

2.2.1 Convection

Heat transfer in fluids generally takes place via convection. Convection currents are setup in the fluid because the hotter part of the fluid is not as dense as the cooler. So there is an upward buoyancy force on the hotter fluid, making it rise while the cooler, dense fluid sinks as illustrated in the diagram below [15].

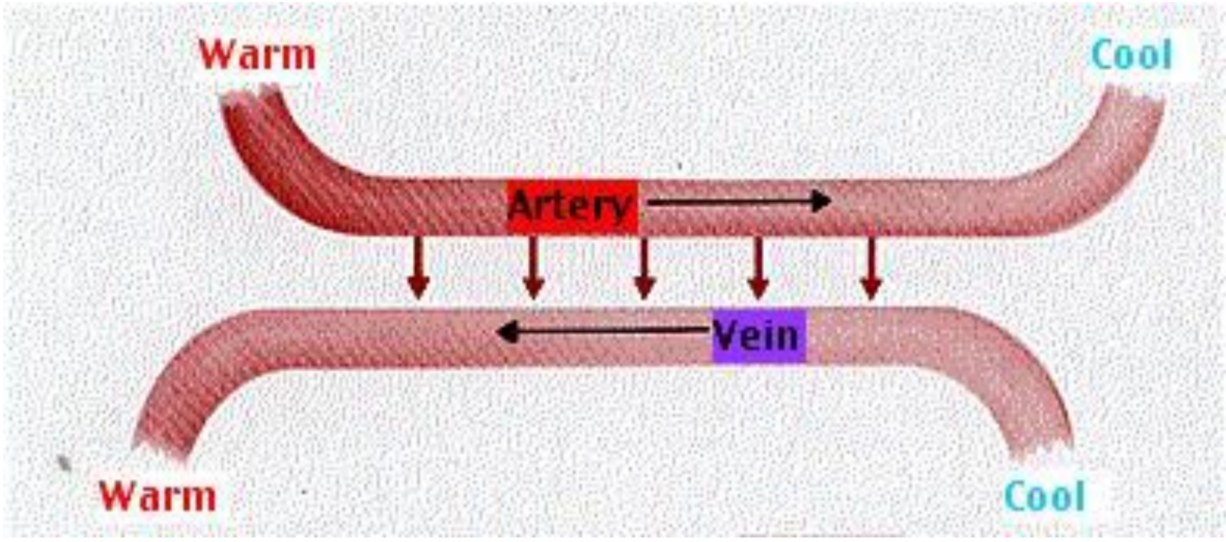


Figure 5: Heat transfer in an artery and vein.

When heat conducts into a static fluid, it leads to a local volumetric expansion , as a result of gravity induced pressure-gradients, the expanded fluid parcel becomes buoyant and displaces, thereby transporting heat by fluid motion which is convection(in addition to conduction) such heat-induced fluid motion in initially static fluids is known as free convection.

Convection, is the transport of thermal energy between a solid surface, in other words, at wall temperature T and a moving fluid, at a far-enough temperature T_∞, modelled by a thermal convection coefficient h named Newton's law; in this sense, heat convection is just heat conduction at the fluid interface in a solid, whereas in the more general sense used in Fluid Mechanics, thermal convection is the combined energy transport and heat diffusion flux at every point in the fluid. Notice, however, that what goes along a hot-water insulated pipe is not heat and there is no heat-transfer involved; it is thermal energy being converted, without thermal gradients. Related to fluid flow, but through porous media is percolation; a special case concerns heat transfer in biological tissue by blood perfusion, which is the flow of blood by permeation through tissues: skin, muscle, fat, bone, and organs, from arteries to capillaries and veins; the cardiovascular system is the key system by which heat is distributed throughout the body, from body core to limbs and head. The researcher has considered heat transfer by convection in trying to model the flow of blood through a stenosed artery.

2.2.2 Conduction

When heat is transferred by conduction, the substance itself does not flow, rather heat is transferred internally, by vibration of atoms and molecules, electrons can also carry heat which is the reason why metals are very good conductors of heat, metals have many free electrons, which move around randomly, these can transfer heat from one part of the metal to another.

2.2.3 Radiation

This is the third way to transfer heat, in which energy is transferred into electromagnetic waves. An electromagnetic wave is basically an oscillating electric and magnetic field travelling through space at the speed of light. The energy comes in the form of very high electromagnetic waves, as well as nuclear particles. The radiation associated with heat transfer is entirely electromagnetic waves, with a relatively low energy.

2.3 Fluid Dynamics

In physics, fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow which is the natural science of fluids (liquids and gases) in motion. Fluid dynamics offers a systematic structure which underlies practical disciplines that embrace empirical and semi-empirical laws derived from flow measurement and used to solve practical problems. The solution to a fluid dynamics problem typically involves calculating various properties of the fluid, such as velocity, pressure, density, and temperature, as functions of space and time [17].

The foundational axioms of fluid dynamics are the conservation laws, specifically, conservation of mass, conservation of linear momentum (also known as Newton's Second Law of Motion), and conservation of energy (also known as First Law of Thermodynamics). These are based on classical mechanics and are modified in quantum mechanics and general relativity. They are expressed using the Reynolds Transport Theorem.

In addition to the above, fluids are assumed to obey the continuum assumption. Fluids are composed of molecules that collide with one another and solid objects. However, the continuum assumption considers fluids to be continuous, rather than discrete. Consequently, properties such as density, pressure, temperature, and velocity are taken to be well-defined at infinitesimally small points, and are assumed to vary continuously from one point to another. The fact that the fluid is made up of discrete molecules is ignored [7].

Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipe lines, predicting weather patterns, understanding nebulae in interstellar space and reportedly modelling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid [17].

2.3.1 Continuum Assumption

Hydrodynamics meant something different than it is today. Before the twentieth century, hydrodynamics was synonymous with fluid dynamics [31]. The foundation axioms of fluid dynamics are the conservation laws, specifically conservation of mass, conservation of linear momentum also known as the Newton's Second Law of motion and the conservation of energy which is also known as the First Law of Thermodynamics. These are based on classical mechanics and are modified in quantum mechanics and general relativity. Overall, fluids are assumed to obey the continuum assumption [31].

Fluids are composed of molecules that collide with one another and solid objects [2]. However, the continuum assumption considers fluids to be continuous rather than discrete. Consequently, properties such as density, pressure, temperature and velocity are taken to be well defined infinitesimally small points and are assumed to vary continuously from one point to another. The fact that fluids are made up of discrete molecules is ignored [2].

For fluids which are sufficiently dense to be a continuum, do not contain ionized species and have velocities small in relation to the speed of light, the momentum equations for Newtonian fluids which is a set of nonlinear differential equations that describes the flow of a fluid whose stress depends linearly on velocity gradients and pressure. The unsimplified equations do not have a general-form solution, so they are primarily of use in Computational Fluid Dynamics. The equations can be simplified in a number of ways, all of which make them easier. Some of them allow appropriate fluid dynamics problems to be solved in closed form.

2.3.2 Viscous vs. Inviscid flow

Viscous problems are those in which fluid friction has significant effects on the fluid motion. The Reynolds number, which is a ratio between inertial and viscous forces, can be used to evaluate whether viscous or inviscid equations are appropriate to the problem [1]. Stokes flow is flow at very low Reynolds numbers, less than 1, such that inertial forces can be neglected compared to viscous forces. On the contrary, high Reynolds numbers indicate that the inertial forces are more significant than the viscous (friction) forces. Therefore, we may assume the flow to be an inviscid flow, an approximation in which we neglect viscosity completely, compared to inertial terms [1].

This idea can work fairly well when the Reynolds number is high. However, certain problems such as those involving solid boundaries may require that the viscosity be included. Viscosity often cannot be neglected near solid boundaries because the no-slip condition can generate a thin region of large strain rate (known as Boundary layer) which enhances the effect of even a small amount of viscosity, and thus generating vorticity. Therefore, to calculate net forces on bodies (such as wings) we should use viscous flow equations. As illustrated by d'Alembert's paradox, a body in an inviscid fluid will experience no drag force [40]. The standard equations of inviscid flow are the Euler equations. Another often used model, especially in computational fluid dynamics, is to use the Euler equations away from the body and the boundary layer equations, which incorporates viscosity, in a region close to the body [40].

The Euler equations can be integrated along a streamline to get Bernoulli's equation. When the flow is everywhere irrotational and inviscid, Bernoulli's equation can be used throughout the flow field. Such flows are called potential flows [7].

2.3.3 Compressible vs. Incompressible flow

All fluids are compressible to some extent, that is, changes in pressure or temperature will result in changes in density. However, in many situations the changes in pressure and temperature are sufficiently small that the changes in density are negligible. In this case the flow can be modelled as an incompressible flow [2].Otherwise the more general compressible fluid were the changes in temperature and pressure are sufficiently smaller that the changes in density are negligible.

Mathematically, incompressibility is expressed by saying that the density ρ of a fluid parcel does not change as it moves in the flow field. This additional constraint has a uniform density [17]. For liquids, whether the incompressible assumption is valid depends on the fluid properties (specifically the critical pressure and temperature of the fluid) and the flow conditions (how close to the critical pressure the actual flow pressure becomes). Acoustic problems always require allowing compressibility, since sound waves are compression waves involving changes in pressure and density of the medium through which they propagate [6].

2.3.4 Steady vs. unsteady flow

When all the time derivatives of a flow field vanish, the flow is considered to be a steady flow. Steady-state flow refers to the condition where the fluid properties at a point in the system do not change over time. Otherwise, flow is called unsteady. Whether a particular flow is steady or unsteady, can depend on the chosen frame of reference. For instance, laminar flow over a sphere is steady in the frame of reference that is stationary with respect to the sphere. In a frame of reference that is stationary with respect to a background flow, the flow is unsteady [7].

Turbulent flows are unsteady by definition. A turbulent flow can, however, be statistically stationary [3]. The random field $U(xt)$ is statistically stationary if all statistics are invariant under a shift in time. This roughly means that all statistical properties are constant in time. Often, the mean field is the object of interest, and this is constant too in a statistically stationary flow. Steady flows are often more tractable than similar unsteady flows. The governing equations of a steady problem have one dimension fewer (time) than the governing equations of the same problem without taking advantage of the steadiness of the flow field.

2.3.5 Newtonian vs. non-Newtonian fluids

Sir Isaac Newton showed how stress and the rate of strain are very close to linearly related for many familiar fluids, such as water and air. These Newtonian fluids are modelled by a coefficient called viscosity, which depends on the specific fluid. However, a fluid such as blood has more complicated non-Newtonian stress-strain behaviours.

2.3.6 Laminar vs. turbulent flow

2.3.6.1 Laminar Flow

The flow of blood in straight blood vessels, like the flow of liquids in narrow rigid tubes, is normally **laminar (streamline).** Within the blood vessels, an infinitely thin layer of blood in contact with the wall of the vessel does not move. The next layer within the vessel has a low velocity, the next a higher velocity, and so forth, velocity being greatest in the centre of the stream. Laminar flow occurs at velocities up to a certain **critical velocity.** At or above this velocity, flow is turbulent. Streamline flow is silent, but turbulent flow creates sounds. Laminar flow is illustrated in the diagram that follows.

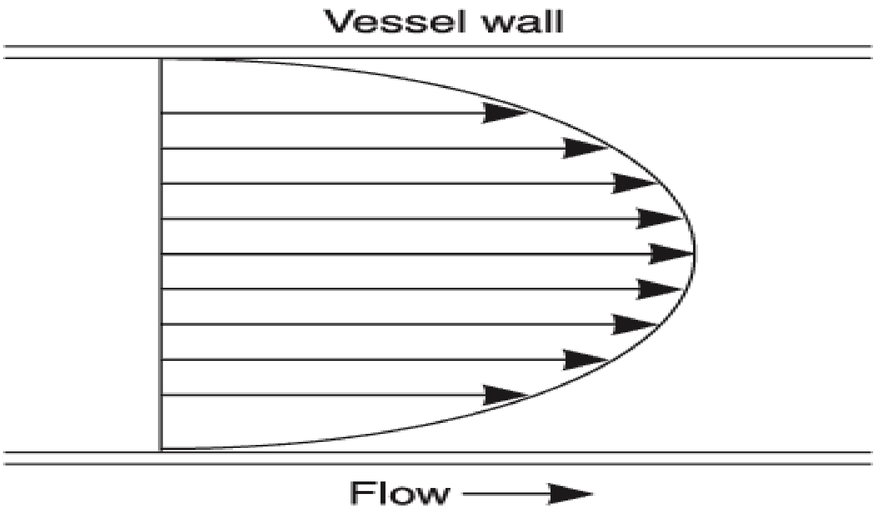


Figure 6: Viscous fluid flow in an artery.

2.3.6.2 Turbulence Flow

Flow considered turbulent is flow characterized by recirculation and apparent randomness [32]. Flow in which turbulence is not exhibited is called laminar. Mathematically, turbulent flow is often represented via a Reynolds decomposition, in which the flow is broken down into the sum of an average component and a perturbation component [4].

It is believed that turbulent flows can be described well through the use of the Navier–Stokes equations. Direct numerical simulation (DNS), based on the Navier–Stokes equations, makes it possible to simulate turbulent flows at moderate Reynolds numbers. Restrictions depend on the power of the computer used and the efficiency of the solution algorithm. The results of DNS have been found to agree well with experimental data for some flows [3].

The above characteristics of fluid flow are illustrated by Figure 7 which shows a section of a stenosed artery, the types of flow are highlighted.

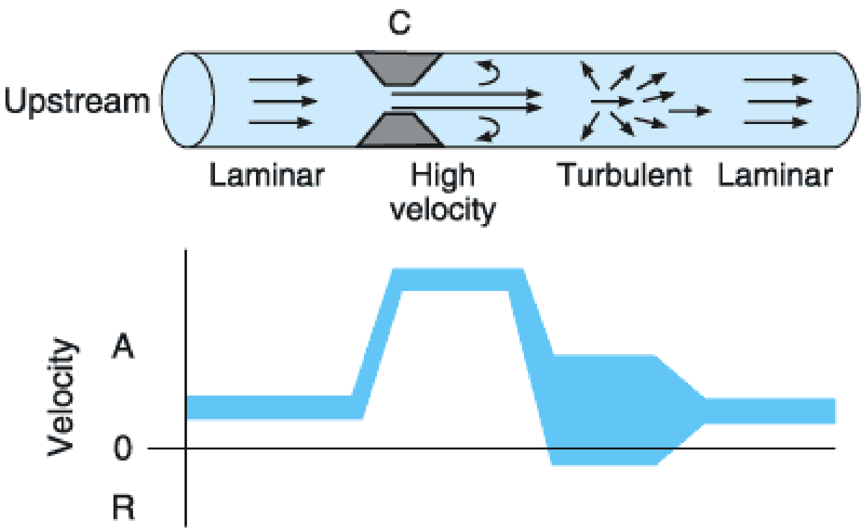


Figure 7: Blood flow in a stenosed artery

The top of the diagram shows the effect of a stenosis (C) on the profile of velocities in a blood vessel. The arrows indicate direction of velocity components, and their length is proportionate to their magnitude. The latter part of the diagram shows the range of velocities at each point along the vessel. In the area of turbulence, there are many different anterograde (A) and some retrograde (R) velocities.

2.3.6 Magneto hydrodynamics

Magneto hydrodynamics is the multi-disciplinary study of the flow of electrically conducting fluids in electromagnetic fields. Examples of such fluids include plasmas, liquid metals, and salt water. The fluid flow equations are solved simultaneously with Maxwell's equations of electromagnetism [4].

2.4 Exponential Stretching in Arteries.

Arteries serve as efficient conduits or tunnels for the movement of blood and also as pressure reservoir that keep blood moving during a diastole. A diastole is the widening of the heart chambers between two contractions when the chambers fill with blood [29]. Arteries have a large internal diameter made of the epithelial layer as shown in the diagram below. This layer is the one that stretches exponentially during a diastole. Arteries also contain an elastic layer in their walls. Elastin is a protein fibre that has elastic qualities which also helps in withstanding the pressure that comes with blood flow, this is shown in the Fig below.

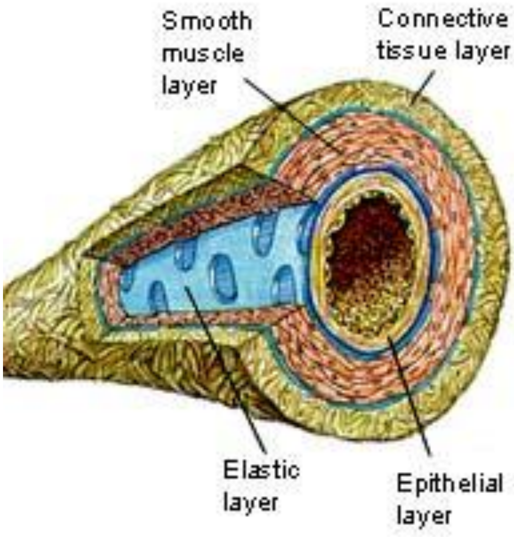


Figure 8: cross section of an artery

During systole, which is a condition whereby the heart chambers contract, large arteries distend with blood as their elastic walls stretch exponentially as blood will be moving from a region of a higher concentration to a region of lower concentration [31]. During diastole, the wall rebounds, thus pushing blood along. In this way the arteries act as a pressure reservoir that maintains the flow of blood through the capillaries despite pressure fluctuation during the cardiac cycle. Arteries also have a smooth muscular layer that functions to regulate the flow of blood through the artery. Contraction of the smooth muscle decreases the internal diameter of the vessel in the process called vasoconstriction. Relaxation of the smooth muscle increases the internal diameter in the process called vasodilation [31].

2.4 Boundary layer equations

The deduction of the boundary layer equations was one of the most important advances in fluid dynamics [10]. Using an order of magnitude analysis, the well-known governing Navier–Stokes equations of viscous fluid flow can be greatly simplified within the boundary layer.

2.4.1 Navier–Stokes equations

In physics, the Navier–Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of fluid substances [37]. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term. The equations are useful because they describe the physics of many things of academic and economic interest. They may be used to model the weather, ocean currents, water flow in a pipe and air flow around a wing. The Navier–Stokes equations in their full and simplified forms help with the design of aircraft and cars, the design of power stations, the analysis of pollution, the study of blood flow. The researcher is mostly interested in the latter.

2.4.1.1 Velocity field

The Navier–Stokes equations dictate not position but rather velocity. A solution of the Navier–Stokes equations is called a velocity field or flow field, which is a description of the velocity of the fluid at a given point in space and time. Once the velocity field is solved for, other quantities of interest (such as flow rate or drag force) may be found. This is different from what one normally sees in classical mechanics, where solutions are typically trajectories of position of a particle or deflection of a continuum. Studying velocity instead of position makes more sense for a fluid; however for visualization purposes one can compute various trajectories [37].

2.4.1.2 Nonlinearity

The Navier–Stokes equations are nonlinear partial differential equations in almost every real situation [39], [40]. In some cases, such as one-dimensional flow and Stokes flow (or creeping flow), the equations can be simplified to linear equations. The nonlinearity is due to convective acceleration, which is an acceleration associated with the change in velocity over position. Hence, any convective flow, whether turbulent or not, will involve nonlinearity. An example of convective but laminar (nonturbulent) flow would be the flow of blood through a stenosed artery [41].

2.4.1.3 Turbulence

Turbulence is the time dependent chaotic behaviour seen in many fluid flows. It is generally believed that it is due to the inertia of the fluid as a whole: the culmination of time dependent and convective acceleration; hence flows where inertial effects are small tend to be laminar (the Reynolds number quantifies how much the flow is affected by inertia). It is believed, though not known with certainty, that the Navier–Stokes equations describe turbulence properly [5].

The numerical solution of the Navier–Stokes equations for turbulent flow is extremely difficult, and due to the significantly different mixing-length scales that are involved in turbulent flow, the stable solution of this requires such a fine mesh resolution that the computational time becomes significantly infeasible for calculation. Attempts to solve turbulent flow using a laminar solver typically result in a time-unsteady solution, which fails to converge appropriately. To counter this, time-averaged equations such as the Reynolds-averaged Navier–Stokes equations (RANS), supplemented with turbulence models, are used in practical computational fluid dynamics (CFD) applications when modelling turbulent flows.

2.4.1.3 Applicability

Together with supplemental equations (for example, conservation of mass) and well formulated boundary conditions, the Navier–Stokes equations seem to model fluid motion accurately. The Navier–Stokes equations assume that the fluid being studied is a continuum (it is infinitely divisible and not composed of particles such as atoms or molecules), and is not moving at relativistic velocities. At very small scales or under extreme conditions, real fluids made out of discrete molecules will produce results different from the continuous fluids modelled by the Navier–Stokes equations [1].

Another limitation is simply the complicated nature of the equations. Time tested formulations exist for common fluid families, but the application of the Navier–Stokes equations to less common families tends to result in very complicated formulations which are an area of current research.

2.4.2 Derivation and Description

2.4.2.1 Derivation of the Navier–Stokes equations

The derivation of the Navier–Stokes equations begins with an application of Newton's second law: conservation of momentum (often alongside mass and energy conservation) being written for an arbitrary portion of the fluid. In an inertial frame of reference, the general form of the equations of fluid motion is: [7]

Navier-stokes equations (general)

$$\rho\left(\frac{\partial y}{\partial x}+v.\nabla v\right)=-\nabla p+\nabla.T+f \qquad (1)$$

where v is the flow velocity, ρ is the fluid density, p is the pressure, *T* is the (deviatoric) stress tensor, and f represents body forces (per unit volume) acting on the fluid and ∇ is the del operator. This is a statement of the conservation of momentum in a fluid and it is an application of Newton's second law to a continuum; in fact this equation is applicable to any non-relativistic continuum and is known as the Cauchy momentum equation.

Though the flow may be steady (time independent), the fluid decelerates as it moves down the diverging tube (assuming incompressible or subsonic compressible flow), hence there is an acceleration happening over position [40].

A very significant feature of the Navier–Stokes equations is the presence of convective acceleration: the effect of time independent acceleration of a fluid with respect to space.

The Navier–Stokes equations are strictly a statement of the conservation of momentum. In order to fully describe fluid flow, more information is needed (how much depends on the assumptions made). This additional information may include boundary data (no-slip, arterial surface, etc.), the conservation of mass, the conservation of energy, and/or an equation of state [40].

The Navier-Stokes equations are commonly used in 3 coordinates systems: Cartesian, cylindrical, and spherical. The Cartesian equations seem to follow directly which means that writing The Navier-Stokes equations in this form could be the most convenient compared to other coordinate systems. This explains why the researcher has used the Cartesian coordinate system of equations.

The continuity equation reads:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \qquad (2)$$

When the flow is incompressible, ρ does not change for any fluid parcel, and its material derivative vanishes: $\frac{D\rho}{Dt}=0$.The continuity equation is reduced to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (3)$$

The velocity components (the dependent variables to be solved for) are typically named *u, v, w*.

2.4.2.1.1 Cylindrical coordinates

A change of variables on the Cartesian equations will yield momentum equations for *r* ,ϕ, and *z*. The gravity components will generally not be constants, however for most applications either the coordinates are chosen so that the gravity components are constant or else it is assumed that gravity is counteracted by a pressure field (for example, flow in horizontal pipe is treated normally without gravity and without a vertical pressure gradient). The continuity equation is:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial(\rho u_\phi)}{\partial\phi}+\frac{\partial(\rho u_z)}{\partial z}=0 \qquad (4)$$

This cylindrical representation of the incompressible Navier–Stokes equations is the second most commonly seen (the first being Cartesian above). Cylindrical coordinates are chosen to take advantage of symmetry, so that a velocity component can disappear.

2.4.2.1.2 Spherical coordinates

In spherical coordinates, the *r*, ϕ, and θ momentum equations are as follows;

Mass continuity will read [31]:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin(\theta)}\frac{\partial(\rho u_\phi)}{\partial\phi}+\frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}(\sin(\theta)\,\rho u_\theta)=0 \qquad (5)$$

These equations could be slightly compacted by, for example, factoring $1/r^2$ from the viscous terms. However, doing so would undesirably alter the structure of the Laplacian and other quantities. Hence the researcher found the Cartesian Co-ordinate system to be more convenient.

The continuity and Navier–Stokes equations for a two-dimensional steady incompressible flow in Cartesian coordinates are given by the following equations;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (6)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mathrm{V}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\mathrm{V}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (8)$$

where u and v are the velocity components, ρ is the density, p is the pressure, and v is the kinematic viscosity of the fluid at a point. The approximation states that, for a sufficiently high Reynolds number the flow over a surface can be divided into an outer region of inviscid flow unaffected by viscosity (the majority of the flow), and a region close to the surface where viscosity is important [8].

Using scale analysis, it can be shown that the above equations of motion reduce within the boundary layer to become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (9)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mathrm{V}\frac{\partial^2 u}{\partial y^2} \qquad (10)$$

And if the fluid is incompressible (as liquids are under standard conditions):

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \qquad (11)$$

Since the static pressure p is independent of y, then pressure at the edge of the boundary layer is the pressure throughout the boundary layer at a given stream wise position. The external pressure may be obtained through an application of Bernoulli's equation. Let u_0 be the fluid velocity outside the boundary layer, where u and u_0 are both parallel. This gives upon substituting for p the following result to;

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_0\frac{\partial u_0}{\partial x} + \mathrm{V}\frac{\partial^2 u}{\partial y^2} \tag{12}$$

with the boundary condition

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{13}$$

For a flow in which the static pressure p also does not change in the direction of the flow then

$$\frac{\partial p}{\partial x} = 0$$

so u_0 remains constant.

Therefore, the equation of motion simplifies to become

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathrm{V}\frac{\partial^2 u}{\partial y^2} \tag{14}$$

These approximations are used in a variety of practical flow problems of scientific and engineering interest. The above analysis is for any instantaneous laminar or turbulent boundary layer, but is used mainly in laminar flow studies since the mean flow is also the instantaneous flow because there are no velocity fluctuations present.

2.5 Effects of a chemical reaction on the flow of blood.

Causes of chemical reactions in the body can either be external or internal. When they are external it means foreign substances to the body are taken in and these may result in chemical reactions in the bloodstream. Substances that are naturally found in the body can also cause chemical reactions. Effects of chemical reactions include decreasing blood flow.

Platelets are tiny components in blood that initiate blood clots. Platelets become stimulated when they encounter a damaged blood vessel, and flock to the site. The platelets clump together and form a plug, which reduces bleeding. Platelets also release substances that start the chemical reaction of blood clot formation [43]. Chemical reactions grow the blood clot. Blood contains dissolved proteins, also called clotting factors, which promote blood clots. (Most of the proteins have Roman numerals for names, including factors V, VII, VIII, IX, X, and XI). These signal to and amplify each other's activity in massive numbers at the site of bleeding. This results in a rapid chemical chain reaction whose end product is fibrin, the main protein forming blood clots. This results in the thickening of blood, which is what is referred to as viscosity. Hence blood flow is decreased by this chemical reaction [43].

During exercise, the muscles use up oxygen as they convert chemical energy in glucose to mechanical energy. This O_2 comes from hemoglobin in the blood. CO_2 and H^+ are produced during the breakdown of glucose, and are removed from the muscle via the blood. The production and removal of CO_2 and H^+, together with the use and transport of O_2, cause chemical changes in the blood. These chemical changes, unless offset by other physiological functions, cause the pH of the blood to drop. If the pH of the body gets too low (below 7.4), a condition known as acidosis results [43]. This can be very serious, because many of the chemical reactions that occur in the body, especially those involving proteins, are pH-dependent. Ideally, the pH of the blood should be maintained at 7.4. If the pH drops below 6.8 or rises above 7.8, death may occur. Fortunately, we have buffers in the blood to protect against large changes in pH. But as acidosis occurs in the blood, it comes with thickening of the blood which also slows down blood flow [44].

Sugar diabetes also slows down blood flow, the increase in glucose causes the blood to be viscous or thicker hence causing a decrease in blood flow [42].

Another effect of chemical reactions in the body is dilation of arteries. This can be caused by high levels of carbon dioxide in the blood hence causing acidosis as mentioned earlier [42].

Each artery in the human body has a baro-receptor. These baro-receptors, when innervated, cause dilation and constriction of the artery. For instance, if someone is losing a vast amount of blood, the baro-receptor notices a decrease in volume and sends an impulse to the brain which changes the amount of hormone released from the pituitary gland. The hormones are known as prostaglandins and have been pharmacologically refined to assist with the dilation in the pulmonary vessels [43]. This is also a chemical reaction that results in the decrease in blood flow.

External factors like marijuana and alcohol also decrease blood flow to some extent. This explains why it is not advisable to take in drugs. Also, concentration of blood solutes lowers blood flow [44].

2.6 Models for blood flow

The model is formulated under the following assumptions. Blood flows through the artery with multi-stenosis. The artery is of constant radius. The density and viscosity of the blood are assumed to be constant. The blood flowing through the artery is assumed to be Newtonian and homogenous. It is assumed that the arterial segment is a rigid cylindrical tube with stenosis [8].

[33] consider heat transfer and blood flow in a multi-stenosed artery with the effect of a magnetic field. They assumed the artery to be a rigid cylindrical tube with multi-stenosis and the Navier-stokes equations in cylindrical coordinates were solved using the vorticity stream function approach. The idea of exponential stretching during the flow of blood was left out.

[34] also considers heat transfer in a stenosed artery during blood flow where r, θ, z are taken to be the coordinates of a material point in the cylindrical polar coordinates system. The stenotic blood flow in the artery is taken to be two dimensional, unsteady, axi-symmetric and fully developed, where the flow of blood is treated to be non-Newtonic characterised by the generalised Polar-law model. Heat transfer, pressure, velocity and the density of blood are the factors taken into consideration. Again exponential stretching was not considered.

2.6.1 Model 1

A stenosed artery is constricted at a specified position as illustrated by the diagram below [35].

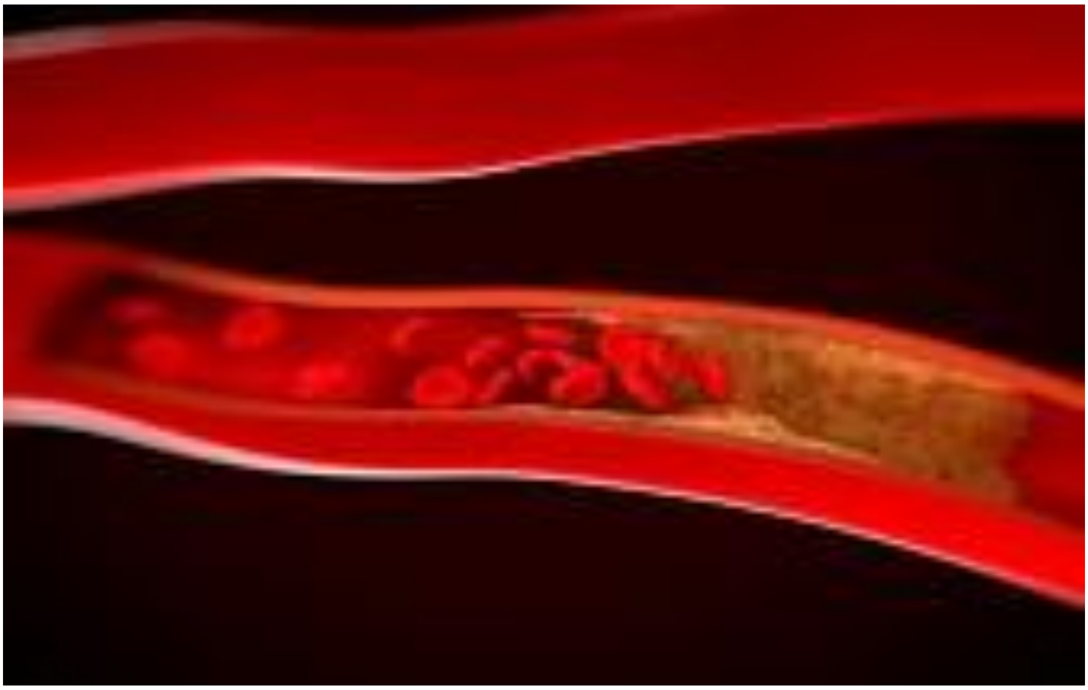


Figure 9: stenotic artery

The blood at the axi-symmetric stenosis can be simulated in two dimensions by making use of the of the cylindrical co-ordinate system. Let (r^*, θ^*, z^*) be the cylindrical polar coordinates with z^*-axis along the axis of symmetry of the artery. The region of interest is $0 \leq r^* \leq r_0(z^*)$, $0 \leq z^* \leq L^*$ (L^* being the finite length of the artery.) The incompressible two dimensional Navier-stokes equations can be taken to model the Newtonian flow past stenosis with exponential stretching of the artery and other constrictions namely pressure, density, velocity etc [1]. Let u^* and v^* be axial and radial velocity components respectively, p^* the fluid pressure, ρ the constant density and V denotes the kinematic viscosity of the fluid. Let U be the maximum inflow velocity specified in the luminal section of the artery [2].

We introduce the non dimensional variables

$$t = t^* \frac{U}{D_0}, r = r^* \frac{U}{D_{0,}}, \quad z = z^* \frac{U}{D_{0,}}$$

$$u = \frac{u^*}{U}, v = \frac{v^*}{U}, \quad p = \frac{p^*}{U^2}$$

Where D_0 is the diameter of the stenosed portion [35].

2.6.2 Model 2

The Navier-stokes equations for incompressible fluid flow can be expressed cylindrical coordinates in dimensionless variables as

$$r\frac{\partial u}{\partial z} + \frac{\partial vr}{\partial r} = 0 \qquad (15)$$

$$\frac{\partial u}{\partial t} + \frac{\partial uv}{\partial r} + \frac{\partial u^2}{\partial z} + \frac{uv}{r} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2}\right) + \left(\frac{1}{r}\right)\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \qquad (16)$$

$$\frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial r} + \frac{\partial uv}{\partial z} + \frac{u^2}{r} = -\frac{\partial p}{\partial r} + \left(\frac{1}{Re}\right)\left(\frac{\partial^2 v}{\partial r^2}\right) + \left(\frac{1}{r}\right)\left(\frac{\partial v}{\partial r}\right) + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}) \qquad (17)$$

Where $Re = \frac{UD0}{V}$ is the Reynolds' number [35].

Boundary conditions

Along the axis of symmetry, the normal component of velocity and shear wall vanish so that

$\partial u\ (z, r, t)/\partial r = 0\ ,\ v\ (z, r, t) = 0\ on\ r = 0$

The boundary conditions on arterial wall when treated to be rigid are given by

$u\ (z\ , r, t\ ) = (\ z\ , r, t\ ) = 0\ at\ r = r_0(\ z\ ),$

When arterial wall is treated as flexible, the boundary conditions are given by

$u\ (z\ , r\ ,t\ ) = 0\ ,\ v\ (z\ , r,\ t\ ) = \partial r_0\ (z\ , t\ ) / \partial t\ on\ r_0 (z, t)$ [35].

In this case only shear stress, pressure and velocity have been analysed.

However we could consider a two dimensional flow of blood in an impermeable artery stretching exponentially with velocity U_W and a given temperature distribution T_W , the *x*-axis is taken along the arterial wall in the upward direction and the *y*-axis perpendicular to it into the blood. A uniform magnetic field B_0 is assumed to be applied in the *y* direction. We assume that the induced magnetic field of blood flow is negligible in comparison with the applied one which corresponds to a very small magnetic Reynolds number [14].

When the combined effects of the magnetic field forces are incorporated into the governing equations of an exponentially stretching stenosed artery with an exponential temperature distribution, the analytical solutions as well as the similarity solutions become intractable. The aim is to introduce a local similarity solution of an exponentially stretching surface with an exponential dependence of the temperature distribution in the presence of a magnetic field effect. Numerical solutions are obtained to study the characteristics of the thermal boundary layer in terms of different governing parameters.

2.6.3 Model 3

Under boundary layer approximation, the continuity, momentum and energy equations can be written as [15]:

$$\frac{\partial u}{\partial v} + \frac{\partial v}{\partial y} = 0 \qquad (18)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathrm{V}\frac{\partial^2 u}{\partial y^2} - (\frac{\sigma {B_0}^2}{\rho})\, u \qquad (19)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + (\frac{\sigma {B_0}^2}{\rho cp})u^2 \qquad (20)$$

These equations are subject to the following boundary conditions

$u = U_w,\ v = 0,\ T = T_w$ at $y = 0$

$u = 0,\ T = T_\infty,$ at $y = \infty$

u and v are the x and y components of the velocity field, respectively.

υ denotes the kinematic viscosity,

α is the thermal diffusivity,

σ is the electrical conductivity,

B_0 is the magnetic field flux density.

U_W is the stretching velocity defined as $U_W(x) = U_0 e^{x/D}{}_0$,

T_W is the exponential temperature defined as $T_W(x) = T_\infty + (T_0 - T_\infty)\, e^{ax/2D}{}_0$ [15].

2.7 Similarity theory

This method reduces PDEs to ODEs. Similarity solutions are defined mathematically as a solution where a change of variables allows for a reduction in the number of independent variables. In fluid dynamics , a similarity solution can be interpreted physically as a case that, when appropriately non-dimensionalised, causes the data taken at different locations or times to collapse for example in stagnation point flow [15].

The approach to similarity solutions involves the following steps:

- Simplify the equations taking advantage of geometry and any order of magnitude simplifications possible.
- Identify appropriate boundary conditions.
- Using physical reasoning, propose a functional form for the similarity variables which are normally non-dimensional.
- Substitute similarity variables in to equations and simplify.
- Use the resulting equation to determine the similarity variables' specific form. If there are possible forms, then a similarity solution is possible.
- Convert boundary conditions identified in physical variables for example velocity, to boundary conditions in similarity variables.

The result will be a set of equations that is simpler than the initial non-linear partial differential equations that the analysis started with. In some cases, these equations and the boundary conditions may be directly solved to get an analytical solution or numerically solved.

2.8 Numerical methods to solve Ordinary Differential Equations (ODEs)

The most popular, as well as accurate numerical Procedure used in obtaining appropriate solutions of an ordinary differential equation is the Runge-Kutta method [7]. The Runge-Kutta, also called the improved Euler method was named after the Germans mathematicians who developed it Carl Runge and Wilhelman Kutta [18]. These men improved the Euler method to higher orders, instead of approximating $f(t,x)$ by the value at the left end point of the interval, as the Euler procedure does, the Runge-Kutta method, specifically the second order Runge-Kutta, takes the average of the approximate values of $f(t,x)$ at both ends of the interval [16].

Apart from the above mentioned reasons, the researcher has resorted to Runge-Kutta method mostly because it has several solvers in Matlab namely ODE15, ODE23, ODE23s, ODE45, ODE45s and ODE113. This gives us an opportunity to compare results obtained from one solver to the next hence at the end of the day we are bound to get a perfect solution!

Shooting method was also the other method used to solve differential equations in this research. This was because it allows the equations to be solved simultaneous at once hence giving results on one graph. This is convenient especially to the reader. It gives a clear understanding and insight because comparisons can be made easily. In this case the researcher has translated the higher order differential equations to first order. This means increasing the number of equations and shooting method caters for all of them. Other numerical methods solve equations one by one.

There are other numerical methods for solving Ordinary Differential Equations. These include the Finite Volume Method, Finite Difference Method and Finite Element Methods. The procedures for these methods are more or less the same. These methods solve PDEs numerically.

In this case we initially had the common Navier-Stokes equations that were modified to suit our problem. These are the following PDEs;

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (18)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \qquad (21)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad (22)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 y}{\partial y^2} \quad (23)$$

with associated boundary conditions

$u = U_w \, , v = 0 \; at \; y = 0$

$u = 0 \, , T = T_{\infty \, ,} at \; y = \infty$

$u = 0 \, , C = C_{\infty \, ,} at \; y = \infty$

These were converted into a set of first order differential equations that were solved in Matlab software package.

2.9 Mathematics Software

The reason main reason why the researcher chose to use Matlab for solving the ordinary differential equations is that MATLAB has several advantages over other methods or languages:

Its basic data element is the matrix. A simple integer is considered a matrix of one row and one column. Several mathematical operations that work on arrays or matrices are built-in to the Matlab environment [18]. For example, cross-products, dot-products, determinants, inverse matrices. Vectorized operations. Adding two arrays together needs only one command, instead of a *for* or *while loop*. The graphical output is optimized for interaction. You can plot your data very easily, and then change colours, sizes, scales, etc, by using the graphical interactive tools. Matlab's functionality can be greatly expanded by the addition of toolboxes. These are sets of specific functions that provide more specialized functionality. Ex: Excel link allows data to be written in a format recognized by Excel, Statistics Toolbox allows more specialized statistical manipulation of data for example the ANOVA and Basic Fits [18].

Matlab is not only a programming language, but a programming environment as well. You can perform operations from the command line, as a sophisticated calculator. Or you can create programs and functions that perform repetitive tasks, just as any other computer language. One of the most important features of the MATLAB interface is the help. It is very thorough and you can learn almost anything you need from it. You can enter matrices into MATLAB in several different ways [41]:

- Enter an explicit list of elements.
- Load matrices from external data files.
- Generate matrices using built-in functions.
- Create matrices with your own functions in M-files.

The steps in which you can enter matrices in Matlab are very crucial as this is the very technique that the user used in solving the dimensionless ordinary differential equations [18]. It gives a clearer picture of the results because here the differential equations are further broken down to several first order differential.

CHAPTER THREE

The main purpose of this chapter is to describe the steps that the researcher took to ensure validity of the research. The chapter also gives an outline of the procedures and methods employed in the study by the researcher as well as numerical techniques and computer packages that were used. The researcher will give a layout of the research design and research instruments. These are illustrated by the diagram below.

3.0 Model development

The flow chart below summarises the steps in which the researcher has taken to formulate an appropriate model for heat transfer with exponential stretching of a stenosed artery during blood flow. The researcher has also inco-operated the effect of a chemical reaction to blood flow.

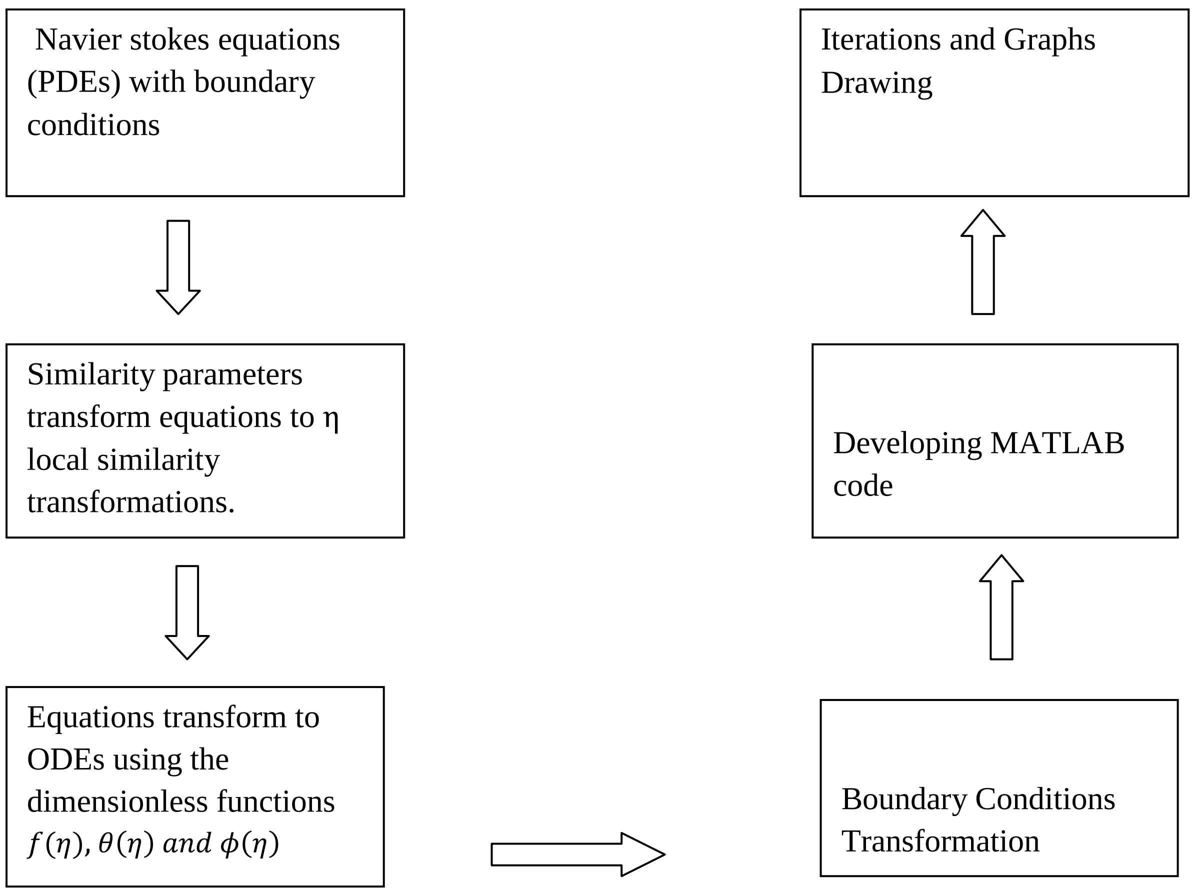


Figure 10: Procedure of model development.

As highlighted in Fig 10, initially we have the Navier stokes equations. These have been modified so as to include every aspect of the problem at hand. The partial differential equations are transformed into ordinary differential equations by using similarity theory. It is in this theory that stenosis, exponential stretching and the chemical reaction components are factored in. This is by means of similarity transformations, thus, the governing equations using dimensionless functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ become ordinary differential equations. The boundary conditions are also transformed similarly. The resulting ODEs are further transformed into first order differential equations that are finally solved by Matlab.

3.1 The governing equations

Consider a two-dimensional steady flow of incompressible viscous blood near the impermeable stenosed artery stretching exponentially with velocity U_w ,a chemical component C_w a given temperature distribution T_w. Under boundary layer approximation, the continuity, momentum, energy and chemical equations can be written as [25], [29], [31]:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{18}$$

$$\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \tag{21}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{22}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{23}$$

where u and v are the x and y components of the velocity field, respectively.υ denotes the kinematic viscosity,α is the thermal diffusivity and D is a constant [25].

with associated boundary conditions

$$u = U_w\ , v = 0 \ \ at \ \ y = 0$$

$$u = 0\ ,\ \ T = T_{\infty}\ , at\ y = \infty$$

$$u = 0\ ,\ \ C = C_{\infty}\ , at\ y = \infty \tag{24}$$

The stretching velocity U_w and the exponential temperature distribution T_w and the chemical reaction component C_w are defined as:

$$U_w(x) = U_0 e^{x/D_0} \quad (25)$$

$$T_w(x) = T_\infty + (T_0 - T_\infty)e^{ax/2D_0} \quad (26)$$

$$C_w(x) = C_\infty + (T_0 - T_\infty)e^{\gamma x/2D_0} \quad (27)$$

where D_0 is the length of the stenosed portion in the artery [25].

The continuity equation (equation 18) is satisfied by the stream function φ

where $u = \frac{\partial \varphi}{\partial y}$ and $v = -\frac{\partial \varphi}{\partial x}$

This eliminates it from the rest of the equations leaving us with equations (21)-(23). The following parameters are substituted into equations (21)-(23) to obtain similarity solutions

$$u(x,y) = U_0 e^{\frac{x}{D_0}} f'(\eta) \quad (25)$$

$$T(x,y) = T_\infty + (T_0 - T_\infty)e^{\frac{ax}{2D_0}}\theta(\eta) \quad (26)$$

$$\eta = \sqrt{Re}\, y/D_0 e^{x/2D_0.} \quad (27)$$

$$C(x,y) = C_\infty + (C_w - C_\infty)e^{\gamma x/2D_0}\, \phi(\eta) \quad (28)$$

The similarity parameters above are used to transform equations (21)-(23) to η local transformations.

3.2 Transforming the PDEs to ODEs.

The following working shows how equation (21) is transformed into an Ordinary Differential Equation.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathrm{V}\frac{\partial^2 u}{\partial y^2} \qquad (21)$$

let $u = U_0 e^{\frac{x}{D_0}} f'(\eta)$,

$v = 0$ (Parallel flow, [24])

$$\eta = \sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}$$

where $Re = \frac{U_0 D_0}{\mathrm{V}}$

therefore $\frac{\partial u}{\partial x} = \frac{1}{D_0}U_0 e^{\frac{x}{D_0}} f'(\eta) + U_0 e^{\frac{x}{D_0}} f''(\eta)\frac{\partial \eta}{\partial x}$

but $\frac{\partial \eta}{\partial x} = \frac{1}{2D_0}\sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}$

$$= \frac{\eta}{2D_0}$$

therefore $\frac{\partial u}{\partial x} = \frac{1}{D_0}U_0 e^{\frac{x}{D_0}}\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right)$

and so $u\frac{\partial u}{\partial x} = U_0 e^{\frac{x}{D_0}} f'(\eta).\frac{1}{D_0}U_0 e^{\frac{x}{D_0}}\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right)$

$$= \frac{U_0^2}{D_0}e^{\frac{2x}{D_0}}(f'^2 + \frac{\eta}{2}f'f''(\eta))$$

$$\frac{\partial u}{\partial y} = U_0 e^{\frac{x}{D_0}} f''(\eta)\frac{\partial \eta}{\partial y}$$

$$\frac{\partial^2 u}{\partial y^2} = U_0 e^{\frac{x}{D_0}} f'''(\eta)\frac{\partial \eta}{\partial y}\frac{\partial \eta}{\partial y} + U_0 e^{\frac{x}{D_0}} f''(\eta)\frac{\partial^2 \eta}{\partial y^2}$$

but $\frac{\partial \eta}{\partial y} = \sqrt{Re}\frac{1}{D_0}e^{\frac{x}{2D_0}}$

so $$\frac{\partial^2 \eta}{\partial y^2} = 0$$

therefore $$\frac{\partial^2 u}{\partial y^2} = U_0 e^{\frac{x}{D_0}} f'''(\eta). \left(\sqrt{Re} \frac{1}{D_0} e^{\frac{x}{2D_0}} \right) \left(\sqrt{Re} \frac{1}{D_0} e^{\frac{x}{2D_0}} \right)$$

$$= U_0 e^{\frac{x}{D_0}} f'''(\eta). \frac{Re}{D_0^2} e^{\frac{x}{D_0}}$$

$$= \frac{ReU_0}{D_0^2} e^{\frac{2x}{D_0}} f'''(\eta)$$

Therefore equation (21) becomes

$$\frac{U_0^2}{D_0} e^{\frac{2x}{D_0}} \left(f'^2 + \frac{\eta}{2} f' f''(\eta) \right) = \mathrm{V} \frac{ReU_0}{D_0^2} e^{\frac{2x}{D_0}} f'''(\eta)$$

$$f'^2 + \frac{\eta}{2} f' f'' = \frac{Re\mathrm{V}}{U_0 D_0} f'''$$

$$Re = \frac{U_0 D_0}{\mathrm{V}}$$

Hence the constants cancel out to yield the ODE

$$f'^2 + \frac{\eta}{2} f' f'' = f''' \qquad (29)$$

The following working shows how equation (21) is transformed into an Ordinary Differential Equation.

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{22}$$

taking $u=U_0e^{\frac{x}{D_0}}f'(\eta),$

$v=0$ (parallel flow ,[24])

$$\eta=\sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}$$

and $T(x,y)=T_\infty+(T_0-T_\infty)e^{\frac{ax}{2D_0}}\theta(\eta)$

we have

$$\frac{\partial T}{\partial x}=\frac{a}{2D_0}(T_0-T_\infty)e^{\frac{ax}{2D_0}}\theta(\eta)+(T_0-T_\infty)e^{\frac{ax}{2D_0}}\theta'(\eta)\frac{\partial\eta}{\partial x}$$

$$=(T_0-T_\infty)e^{\frac{ax}{2D_0}}\left(\frac{a}{2D_0}\theta(\eta)+\theta'(\eta)\frac{\partial\eta}{\partial x}\right)$$

but

$$\frac{\partial\eta}{\partial x}=\frac{1}{2D_0}\sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}$$

$$=\frac{\eta}{2D_0}$$

therefore

$$\frac{\partial T}{\partial x}=\frac{1}{2D_0}(T_0-T_\infty)e^{\frac{ax}{2D_0}}\left(a\,\theta(\eta)+\theta'(\eta)\sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}\right)$$

$\Rightarrow$

$$u\frac{\partial T}{\partial x}=U_0e^{\frac{x}{D_0}}f'(\eta).\frac{1}{2D_0}(T_0-T_\infty)e^{\frac{ax}{2D_0}}\left(a\theta(\eta)+\theta'(\eta)\sqrt{Re}\frac{y}{D_0}e^{\frac{x}{2D_0}}\right)$$

$$=\frac{U_0}{2D_0}(T_0-T_\infty)e^{\frac{2x+ax}{2D_0}}(\,af'\theta+\eta f'\theta')$$

$$\frac{\partial T}{\partial y}=(T_0-T_\infty)e^{\frac{ax}{2D_0}}\theta'(\eta)\frac{\partial\eta}{\partial y}$$

$$\frac{\partial^2 T}{\partial y^2}=(T_0-T_\infty)e^{\frac{ax}{2D_0}}\,\theta''(\eta).\frac{\partial\eta}{\partial y}\frac{\partial\eta}{\partial y}+(T_0-T_\infty)e^{\frac{ax}{2D_0}}\theta'(\eta)\frac{\partial^2\eta}{\partial y^2}$$

but $$\frac{\partial \eta}{\partial y} = \sqrt{Re}\,\frac{1}{D_0}\,e^{\frac{x}{2D_0}}$$

so $$\frac{\partial^2 \eta}{\partial y^2} = 0$$

therefore $$\frac{\partial^2 T}{\partial y^2} = (T_0 - T_\infty)e^{\frac{ax}{2D_0}}\,\theta''(\eta)\left(\sqrt{Re}\,\frac{1}{D_0}\,e^{\frac{x}{2D_0}}\right)\left(\sqrt{Re}\,\frac{1}{D_0}\,e^{\frac{x}{2D_0}}\right)$$

$$= \frac{Re}{D_0^2}(T_0 - T_\infty)e^{\frac{2x+ax}{2D_0}}\,\theta''(\eta)$$

therefore equation (22) becomes

$$\frac{U_0}{2D_0}(T_0 - T_\infty)e^{\frac{2x+ax}{2D_0}}(\,af'\theta + \eta f'\theta') = \alpha\,\frac{Re}{D_0^2}(T_0 - T_\infty)e^{\frac{2x+ax}{2D_0}}\,\theta''(\eta)\,,$$

that is $$f'\theta + \eta f'\theta' = \frac{2\alpha Re}{U_0 D_0}\,\theta''(\eta)\,. \qquad (30\,)$$

The following working shows how equation (23) is transformed into an Ordinary Differential Equation.

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 y}{\partial y^2} \qquad (23)$$

Taking $u = U_0 e^{\frac{x}{D_0}} f'(\eta)$,

$v = 0$ (parallel flow ,[24])

$$\eta = \sqrt{Re}\frac{y}{D_0} e^{\frac{x}{2D_0}}$$

and $C = C_\infty + (C_w - C_\infty)e^{\gamma x/2D_0}\,\phi(\eta)$

we have $\frac{\partial C}{\partial x} = \frac{\gamma}{2D_0}(C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\phi(\eta) + (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\phi'(\eta)\frac{\partial \eta}{\partial x}$

$$= (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\left(\frac{\gamma}{2D_0}\,\phi(\eta) + \phi'(\eta)\frac{\partial \eta}{\partial x}\right)$$

but $\frac{\partial \eta}{\partial x} = \frac{1}{2D_0}\sqrt{Re}\frac{y}{D_0}e^{x/2D_0}$

$$= \frac{\eta}{2D_0}$$

therefore $\frac{\partial C}{\partial x} = \frac{(C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}}{2D_0}(\gamma\phi(\eta) + \eta\,\phi'(\eta))$

so $u\frac{\partial C}{\partial x} = U_0 e^{\frac{x}{D_0}} f'(\eta).\frac{(C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}}{2D_0}(\gamma\phi(\eta) + \eta\,\phi'(\eta))$

$$= U_0.\frac{(C_w - C_\infty)e^{\frac{2x+\gamma x}{2D_0}}}{2D_0}(\gamma f'(\eta)\phi(\eta) + \eta\, f'(\eta)\phi'(\eta))$$

$$\frac{\partial C}{\partial y} = (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi'(\eta)\frac{\partial \eta}{\partial y}$$

$$\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi''(\eta)\frac{\partial \eta}{\partial y}\frac{\partial \eta}{\partial y} + (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi'(\eta)\frac{\partial^2 \eta}{\partial y^2}$$

but $\frac{\partial \eta}{\partial y} = \frac{\sqrt{Re}}{D_0}e^{x/2D_0}$

$$\frac{\partial^2 \eta}{\partial y^2} = 0$$

so $\frac{\partial^2 C}{\partial y^2} = (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi''(\eta)(\frac{\sqrt{Re}}{D_0}e^{x/2D_0})^2$

$$= (C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi''(\eta)\,\frac{Re}{D_0^2}e^{\frac{x}{D_0}}$$

therefore equation (23) becomes

$$U_0 e^{\frac{x}{D_0}}.\frac{(C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}}{2D_0}\left(\gamma f'(\eta)\phi(\eta) + \eta\, f'(\eta)\phi'(\eta)\right) = \mathrm{D}(C_w - C_\infty)e^{\frac{\gamma x}{2D_0}}\,\phi''(\eta)\,\frac{Re}{D_0^2}e^{\frac{x}{D_0}}$$

$$\Rightarrow \qquad \left(\gamma f'(\eta)\phi(\eta) + \eta\, f'(\eta)\phi'(\eta)\right) = \phi''(\eta)\,\frac{2DRe}{U_0 D_0}$$

$$\phi'' = \frac{U_0 D_0}{2DRe}\,(\gamma f'\phi + \eta f'\phi' \qquad (31)$$

3.3 Resultant ODEs

After using the similarity theory, the governing equations using the dimensionless functions $f(\eta), \theta(\eta) and\ \phi(\eta)$ become;

$$f''' = f'^2 + \frac{\eta}{2} f' f'' \quad (29)$$

$$\theta'' = \frac{U_0 D_0}{2\alpha Re}(af'\theta + \eta f'\theta') \quad (30)$$

$$\phi'' = \frac{U_0 D_0}{2DRe}(\gamma f'\phi + \eta f'\phi') \quad (31)$$

where $Re = \frac{U_0 D_0}{\upsilon}$ is the Reynolds' number.

And these are subject to boundary conditions that follow. These have also been transformed into dimensionless functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$.

$$f(0) = 0\,, f'(0) = 1, f'(\infty) = 0$$

$$\theta(0) = 1, \theta(\infty) = 0$$

$$\phi(0) = 1, \phi(\infty) = 0 \quad (32)$$

These are clearly ODEs which can now be solved by the MATLAB software package.

CHAPTER FOUR

The dimensionless ordinary differential equations (29)-(31) subject to the boundary conditions (32) were solved numerically using the Shooting and Runge-Kutta methods. The Matlab software package has been successfully used to solve the boundary layer problem. The results are given to carry out a parametric study showing influences of several non-dimensional parameters, namely the thermal viscosity α, the temperature exponent η and the Reynolds' number, Re.

Having explored the general theory of the boundary layer problems in the preceding chapter, applied the methodology on the two ordinary differential equations, translating the equations into a language that Matlab understands, formulated codes and finally using the software to acquire results, this chapter is generally about presenting the numerical results using different constants of fluid physics and discussing their different behaviours during blood flow. The researcher has divided the results into four different scenarios so that the reader may appreciate the effects of the constants in the ordinary differential equations on the stenosed artery during blood flow.

4.1 Scenario 1: Effect of differently shaped stenosis on the rate of blood flow.

The first scenario gives the result of considering both symmetric and asymmetric cases of stenosis, η=2 and η=4 respectively for $f'(\eta)$ (velocity component) against η.

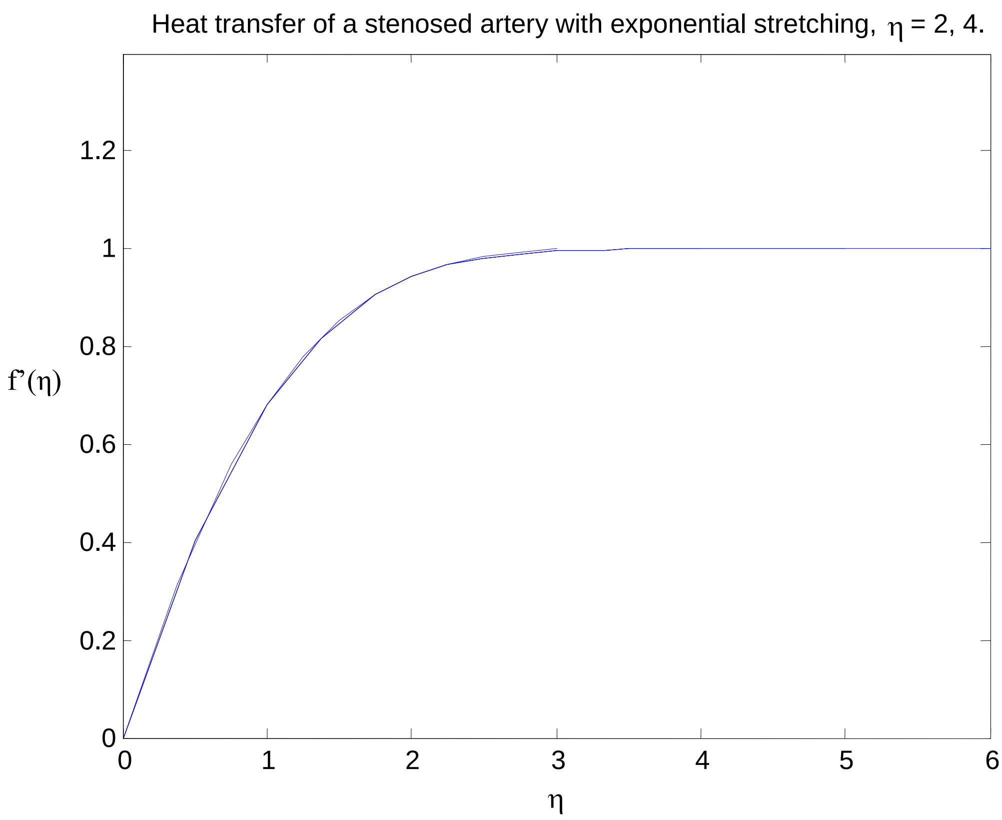


Figure 11: Symmetric and asymmetric stenotic conditions.

From the graph above we clearly can appreciate how the walls of the stenosed artery exponentially stretch in a condition of symmetry and increase the rate at which heat is being transferred in the artery. After η=2 the stretch levels off as blood velocity is now uniform in the artery. This is now a case where the stenosed portion is asymmetric.

4.2 Scenario 2: Effect of differently shaped stenoses on heat mass transfer.

In the second scenario the temperature exponent η is plotted against θ'(η) again in both conditions, symmetric and asymmetric stenosis.

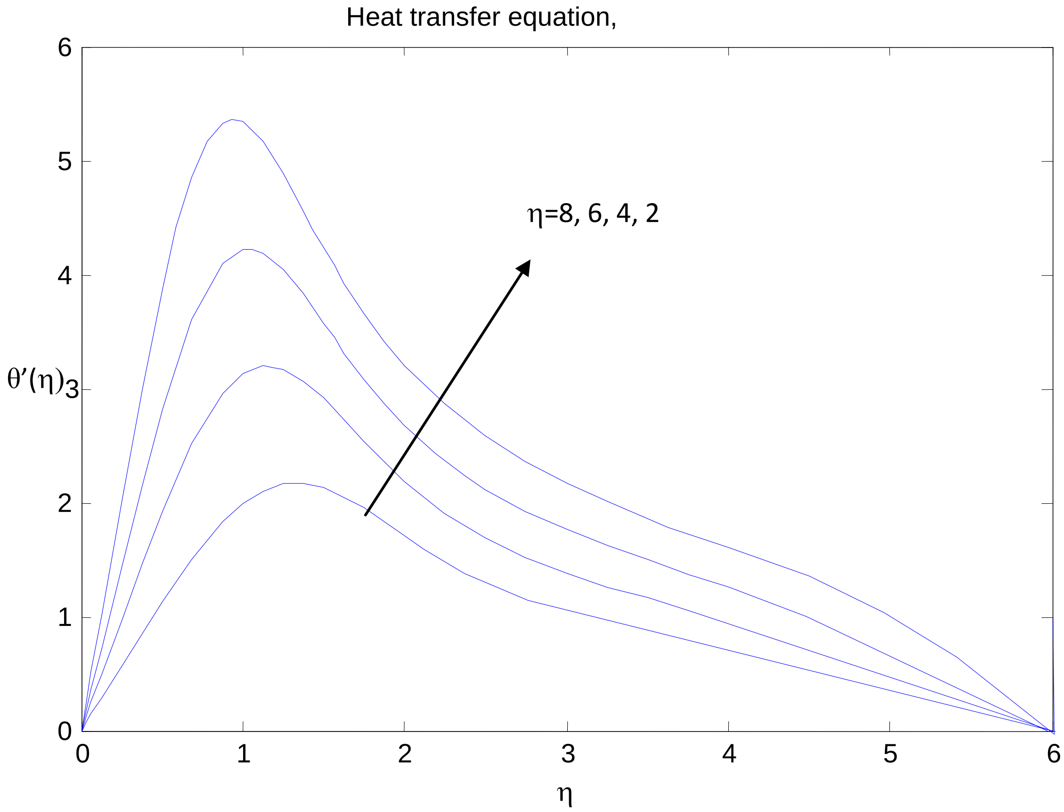


Figure 12: Symmetric and asymmetric stenotic conditions.

Fig 12 exhibits the variation of centre line velocity in axial direction at Re=600 for differently shaped stenoses, (η=2, 4, 6, 8). It is very clear from the Fig 12 that the maximum centre line velocity occurs slightly in the downstream of the constriction due a formation of recirculation zone near the wall as a result of flow separation. The symmetry geometry (about the centre of the arterial lumen) of the stenosis is generally lower at the converging part of the constriction. So it induces an excess flow acceleration, which explains the exponential stretching, as compared to asymmetric stenosis (η=4). The centre line velocity is seen to take a larger distance to recover its initial value as Reynolds number increases.

4.3 Scenario 3: Effects of thermal diffusivity.

Scenario 3 illustrates the effects of the different values of α in equation (29). $f'(\eta)$ is plotted against η on different values of alpha ranging from -5 to 10.

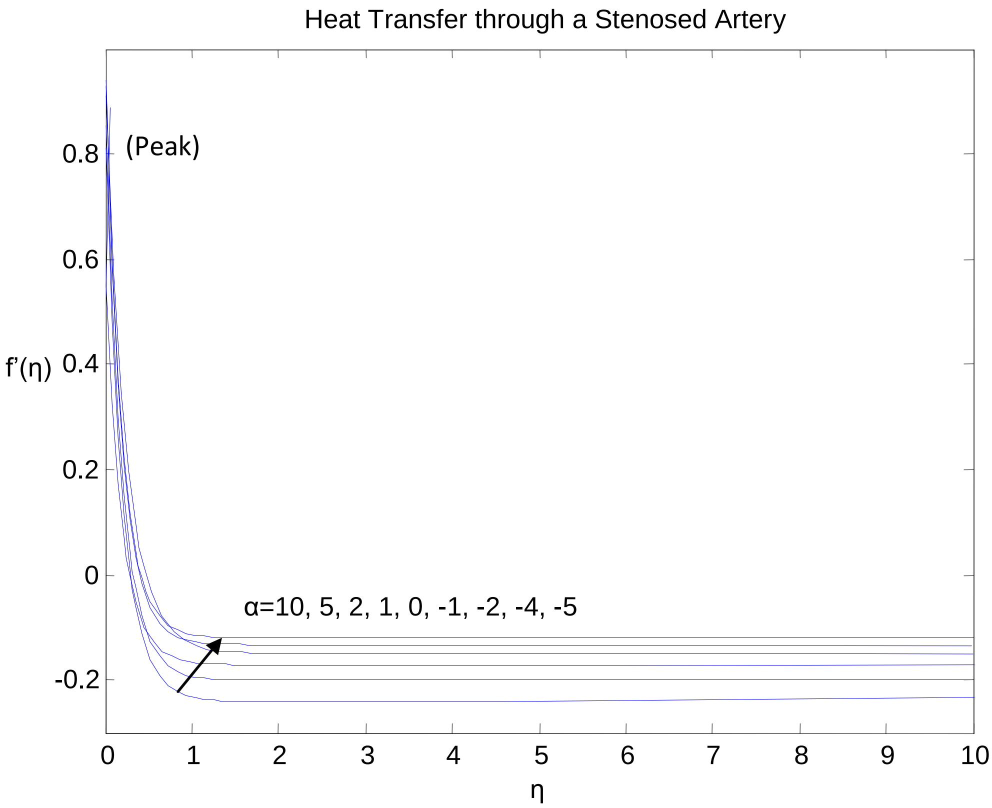


Figure 13: Temperature profiles that are dimensionless for different values of α.

The above Fig reveals that heat transfer is increased by decreasing the value of α. The presence of the peak indicates that the maximum value of temperature occurs in the body of the fluid close to the surface and not on the surface. It is interesting to note that the increase in temperature accompanied by a greater increase in the temperature peak value which increases the temperature difference between the stretched wall and the adjacent fluid is the reason for triggering the heat transfer process form the ambient fluid to the surface. On the other hand increasing the value of *a* above the adiabatic value leads also to increasing the heat transferred from the wall to the ambient fluid. This behaviour can be seen in Fig 14 (next page). The increase in the Reynolds' number in Fig 14 is clearly noticed by moving away from the edge of the stretching surface. Also temperature distribution through the thermal boundary layer for a=-3 and -6 below which is below the adiabatic value tells us that heat transfer is enhanced. This result is concluded from Fig 14.

4.4 Scenario 4: Effects of the Reynolds' number on heat flow.

This scenario deals with the effect of the Reynolds' number on heat flow. Again $f'(\eta)$ is plotted against η at symmetric and asymmetric stenosis. This is illustrated by the diagram below.

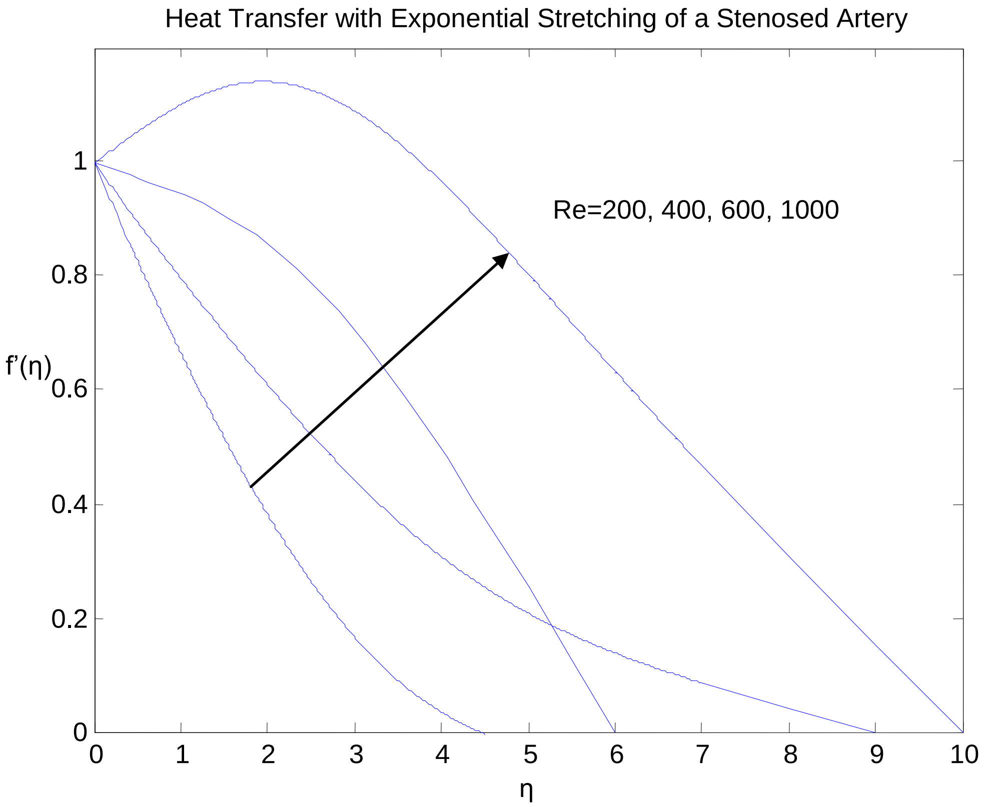


Figure 14: Effects of the Reynolds' number.

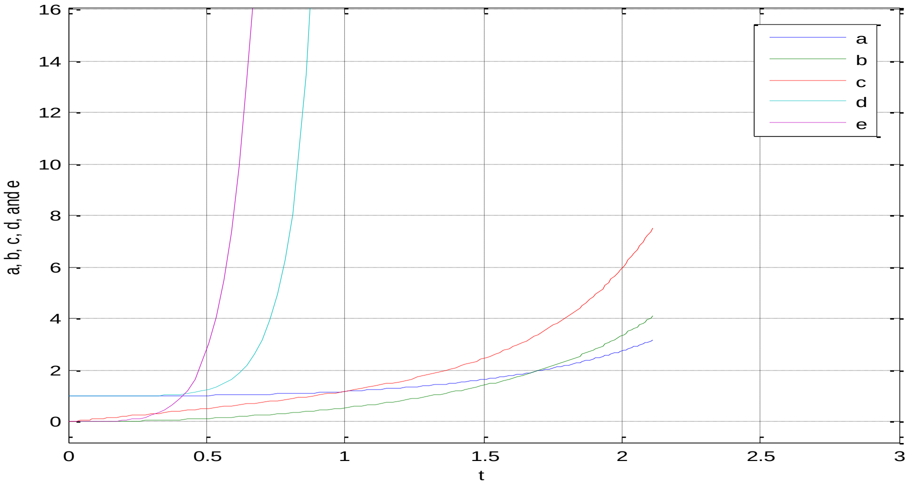
When the Reynolds number is low, as shown in Fig 14, some curves become concave up near the wall. It is interesting to note that at the downstream of the stenosis, back flow, occurs exponentially near the wall where the direction of the velocity changes from negative to positive, and that in turn causes separation in the flow filed.

When the Reynolds number is relatively higher the graphs tend to be concave down, the flow is upstream for a while in a symmetric case then downstream in symmetric case. The fluctuation levels in a case of the stretching artery are of considerable interest. This can cause the prevention of endothelial cells from aligning in the direction of the flow, thereby causing the intima more permeable of the lipoproteins and monocytes. This also results in the predilection towards Athesclerosis.

4.5 Scenario 5: Heat mass transfer in a stenosed artery.

Scenario 5 basically intends to highlight the effects of a chemical reaction in blood flow when all the factors have been held constant. The researcher dis-integrated the dimensionless ordinary differential equations into first order differential equations and solved them simulteneously using the Runge-Kutta method in Matlab software package. The results are shown in the table below.

a)



b)

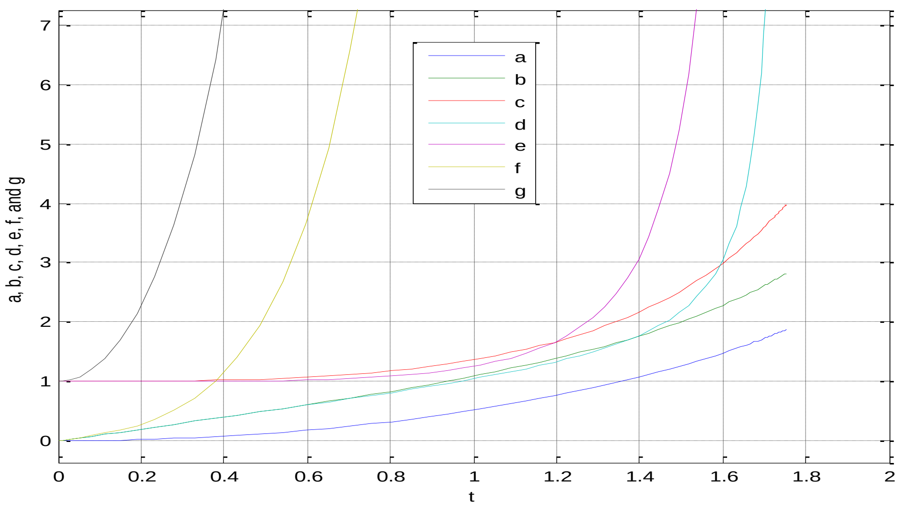


Figure 15: Heat mass transfer. a) Without chemical reaction. b) With chemical reaction.

Fig 15a shows that blood flows exponentially with time in the artery. The five graphs represent the differential equations which resulted from transforming equations (29-30) to first order. These were plotted against time. Stenosis generally induces an excess flow acceleration, which explains the exponential stretching of the artery in trying to withstand the pressure that comes with the increase in blood flow. The flow is also exponential due to the fact that blood flows from a point of a higher concentration to a lower concentration as it is pumped from the heart to the rest of the body through these arteries. The heart acts as a reservoir and so the flow follows a gradient as it is distributed around the entire parts of the body. Fig 15a also acts as a control in trying to reveal the effects of a chemical reaction to the flow of blood. Comparing Fig 15a to Fig 15b gives the reader some appreciation of the effects of chemical reactions to the flow of blood.

Fig 15b involves the effect of a chemical reaction to the flow of blood in arteries. Equations (29)-(31) were transformed into seven first order differential equations and these were plotted against time as shown in Figure 15b.

In the presence of a chemical reaction, the steepness of graphs *a-e* is lower as compared to the corresponding graphs in Figure 15a this is due to the fact that chemicals cause the blood to be more viscous or thicker. Blood tends to be more resistant to flow which ultimately slows down the process of heat mass transfer. Blood flows through the vessels in what is described as laminar flow. That is, the blood forms layers (lamina) that slide easily over each other. If we were to look at the blood vessel from the side, we would see the fastest flowing blood in the center layers, with slower moving blood in the outer layers near the wall of the vessel. Highly viscous blood does not slide as smoothly as less viscous blood. This result requires the heart to work harder, pushing the viscous blood out at even higher pressures, further damaging the intimal layer (inner lining of the artery) [15]. We also see the consequences of hyper viscous blood primarily in damaged blood vessels and in decreased delivery of oxygen to the tissues.

Viscous Blood can cause painful leg cramps or leg pain caused by poor circulation, a condition called *intermittent claudication* [23]. Physicians also may prescribe medicine for these conditions, including stroke, impotence, male infertility and Reynaud’s disease, and nerve and circulation problems caused by diabetes. Taking aspirin, (among other treatments).would decrease viscosity and increase blood flow hence increasing heat mass transfer [23]

CHAPTER FIVE

5.1 Conclusions and Recommendations

A mathematical model of heat transfer through an exponentially stretching stenosed artery is presented here. Potential improvements to previous models by [21], [25], [29] and [31] have been made by the inco-operation of the effect of exponential stretching of the stenosed artery as blood flows and also by including the effect of a chemical reaction to blood flow.

From scenarios 1-3, the dimensionless temperature field depends on thermal diffusivity (α), heat component (η) and the dimensionless $U_0.D_0/Re$. There are three cases defining the flow of heat which depends upon the above parameters. The first case corresponds to a value of α above the adiabatic one, this is when heat transfer is directed from the wall to ambient fluid (direct flow).The second case corresponds to a value of α that is below the adiabatic value, here the flow of heat is directed form the ambient flow towards the wall. Lastly we have a case where there is no heat transfer (adiabatic case), in this case $\alpha = 1$. Decreasing α (below the adiabatic value), increases the thickness of the stenosed artery, this is when the stenosed artery stretches exponentially and this enhances the heat transfer process.

We also conclude that Increasing Re reduces $U_{0.}D_0$/Re and this exhibits the variation of centre line velocity in axial direction for differently shaped stenoses ($\eta = 2, 4, 6, 8$). In the downstream diverging part, the deceleration of the fluid is more modest than at low Reynolds number. The development of separation zones towards the diverging section of the constriction is believed to be the prime areas for further deposition of atherosclerotic plaques. This is found in scenario 4. The role of asymmetric stenosis ($\eta > 2$) in a realm of the arterial plaque may be useful for early detection of cardiovascular diseases. Hence the researcher recommends that people take low fat and cholesterol diets as these highly lead to stenotic conditions.

We have seen that chemical reactions generally lower the rate at which blood flows by increasing blood viscosity. This ultimately reduces heat mass transfer. In view of the articles in literature about the treatment of increased viscosity, it is possible to summarize that the treatment is based on blood dilution which is scientifically known as hemodilution, decreasing erythrocyte mass, and filtration of blood components for example fibrinogen and treatment of underlying disease.

In conclusion, a lowered blood viscosity level promoted by minimal chemical reaction may constitute a lesser blood pressure, friction and damage on the vessel lumen. It may also slow down the atherosclerotic process. It also enhances blood flow which in turn increases heat mass transfer. Therefore, additional studies probably will lead to further biologic and medical advantages.

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Appendix 1

```
function stenosis1

p=1.5;('p = (uo*do)/(alpha*re)');
n= 5;
a1=8;
parameters = [n,p];

infinity = 5;
maxinfinity = 10;

solinit = bvpinit(linspace(0,infinity,5),[0 0 0 0 0]);

solution = bvp4c(@matode,@matbc,solinit);

a = solution.x;
b = solution.y;

figure
plot(a, b(1,:), a(end),b(1,end),'o');
axis([0 maxinfinity  0 1.2]);
title('Heat Transfer through a Stenosed Artery')

xlabel('x')
ylabel('dy/dx')
hold on
drawnow
shg

for Bnew = infinity+1:maxinfinity

  solinit = bvpinit(solution,[0 Bnew]); % Extend solution to Bnew.
  solution = bvp4c(@matode,@matbc,solinit);
```

```
  a = solution.x;
  b = solution.y;
  c = solution.yp;

  fprintf('Value computed using infinity = %g is %7.5f.\n= 2,4', ...
          Bnew,b(5,1))
  plot(a,b(1,:),a(end),b(1,end),'o');
  drawnow

end
hold off



function dydx = matode(parameters,y);
dydx = [ y(2)
         y(3)
         (0.5*p)*(a1*y(2)*y(4) + n*y(2)*y(5))
         y(5)
         (n/2)*y(2)*y(3) + y(2)^2];
end

function bc = matbc(ya, yinf);

bc = [ ya(1)
       ya(2) - 1
       yinf(2)
       ya(4) - 1
       yinf(4)];
end



end
```

Appendix 2

```
function stenosis2

p=1.5;('p = (uo*do)/(alpha*re)');
n= 5;
a1=8;
parameters = [n,p];

infinity = 5;
maxinfinity = 10;

solinit = bvpinit(linspace(0,infinity,5),[0 0 0 0 0]);

solution = bvp4c(@matode,@matbc,solinit);

a = solution.x;
b = solution.y;

figure
plot(a, b(2,:), a(end),b(2,end),'o');
axis([0 maxinfinity  0 6]);
title('Heat Transfer through a Stenosed Artery')

xlabel('x')
ylabel('dy/dx')
hold on
drawnow
shg

for Bnew = infinity+1:maxinfinity

  solinit = bvpinit(solution,[0 Bnew]); % Extend solution to Bnew.
  solution = bvp4c(@matode,@matbc,solinit);
```

```
  a = solution.x;
  b = solution.y;
  c = solution.yp;

  fprintf('Value computed using infinity = %g is %7.5f.\n=2,4,6,8', ...
          Bnew,b(5,1))
  plot(a,b(2,:),a(end),b(2,end),'o');
  drawnow

end
hold off



function dydx = matode(parameters,y);
dydx = [ y(2)
         y(3)
         (0.5*p)*(a1*y(2)*y(4) + n*y(2)*y(5))
         y(5)
         (n/2)*y(2)*y(3) + y(2)^2];
end

function bc = matbc(ya, yinf);

bc = [ ya(1)
       ya(2) - 1
       yinf(2)
       ya(4) - 1
       yinf(4)];
end



end
```

Appendix 3

```
function stenosis3

p=1.5;('p = (uo*do)/(alpha*re)');
n= 5;
a1=8;
parameters = [n,p];

infinity = 5;
maxinfinity = 10;

solinit = bvpinit(linspace(0,infinity,5),[0 0 0 0 0]);

solution = bvp4c(@matode,@matbc,solinit);

a = solution.x;
b = solution.y;

figure
plot(a, b(3,:), a(end),b(3,end),'o');
axis([0 maxinfinity  -0.3 1]);
title('Heat Transfer through a Stenosed Artery')

xlabel('x')
ylabel('dy/dx')
hold on
drawnow
shg

for Bnew = infinity+1:maxinfinity

  solinit = bvpinit(solution,[0 Bnew]); % Extend solution to Bnew.
  solution = bvp4c(@matode,@matbc,solinit);
```

```
  a = solution.x;
  b = solution.y;
  c = solution.yp;

  fprintf('Value computed using infinity = %g is %7.5f.\a=10, 5, 2, 1, 0, -1, -2, -4, -5

', ...
  Bnew,b(5,1))
  plot(a,b(3,:),a(end),b(3,end),'o');
  drawnow

end
hold off



function dydx = matode(parameters,y);
dydx = [ y(2)
         y(3)
         (0.5*p)*(a1*y(2)*y(4) + n*y(2)*y(5))
         y(5)
         (n/2)*y(2)*y(3) + y(2)^2];
end

function bc = matbc(ya, yinf);

bc = [ ya(1)
       ya(2) - 1
       yinf(2)
       ya(4) - 1
       yinf(4)];
end



end
```

Appendix 4

```
function stenosis4

p=1.5;('p = (uo*do)/(alpha*re)');
n= 5;
a1=8;
parameters = [n,p];

infinity = 5;
maxinfinity = 10;

solinit = bvpinit(linspace(0,infinity,5),[0 0 0 0 0]);

solution = bvp4c(@matode,@matbc,solinit);

a = solution.x;
b = solution.y;

figure
plot(a, b(4,:), a(end),b(4,end),'o');
axis([0 maxinfinity  0 1.2]);
title('Heat Transfer through a Stenosed Artery')

xlabel('x')
ylabel('dy/dx')
hold on
drawnow
shg

for Bnew = infinity+1:maxinfinity

  solinit = bvpinit(solution,[0 Bnew]); % Extend solution to Bnew.
  solution = bvp4c(@matode,@matbc,solinit);
```

```
  a = solution.x;
  b = solution.y;
  c = solution.yp;

  fprintf('Value computed using infinity = %g is %7.5f.\Re=200,400,600,1000', ...
          Bnew,b(5,1))
  plot(a,b(4,:),a(end),b(4,end),'o');
  drawnow

end
hold off



function dydx = matode(parameters,y);
dydx = [ y(2)
         y(3)
         (0.5*p)*(a1*y(2)*y(4) + n*y(2)*y(5))
         y(5)
         (n/2)*y(2)*y(3) + y(2)^2];
end

function bc = matbc(ya, yinf);

bc = [ ya(1)
       ya(2) - 1
       yinf(2)
       ya(4) - 1
       yinf(4)];
end



end
```