Analysis of Single Mode Step Index Fibres using Finite Element Method.

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Abstract

Single mode step index fibres are the most common waveguides employed in optical communication systems. To design and study such fibres, analytical methods are not sufficient and hence more advanced mathematical techniques such as finite element methods (FEM) are used. In this paper we employed the FEM software, COMSOL Multiphysics® (version 4.3) to study the effect of varying the core radius of a glass single mode step index fibre on the electric field intensities propagated from the core to the cladding region. The results show that with a cladding radius of 40 μm, it is possible for the single mode step index fibre to sufficiently support the propagation of the fundamental mode for core radii in the range 2.5 to 6 μm. The optimum core radius was found to be 4 μm.

Keywords – *single mode fibre, step index, core radius, FEM, field intensity*

1. Introduction

Optical fibres have become the transporting medium of choice for voice, video, and data, particularly for high speed communication systems. Compared to twisted pair and coaxial cables, optical fibres are compact, immune to electromagnetic interference and do not corrode. In addition, optical fibres also offer an almost unlimited bandwidth: the useful bandwidth per single fibre strand is one thousand times the total radio bandwidth worldwide [1,2]. The single mode fibre, having a step index profile with a higher refractive index in the centre core and a lower index in the outer cladding, is one of the most important fibre types in use today [3,4]. In addition, single mode step index fibres have a lower signal loss and a higher information capacity (bandwidth) than multimode fibres. Single mode fibres are capable of transferring higher amounts of data due to low intermodal dispersion. Numerical software plays an important role in the design of single-mode waveguides and fibres [5]. Numerical simulations provide a way of estimating the behaviour of an optical fibre by enabling the analysis of parameters such as magnetic and electric field intensities, effective refractive index among others [4,5]. Finite element methods (FEM) using commercial software packages have become the most efficient and attractive way of studying the behaviour of optical waveguides and devices [6,7]. In this paper, we demonstrate the use of the commercial FEM software, COMSOL, in studying the behaviour of single mode step index fibres. The RF Module (Optics and Photonics) in COMSOL was used to investigate the influence of core radius of the fibre on the effective refractive index, the electric and magnetic field intensities. This investigation was motivated by the need to optimize the geometrical parameters of the optical fibre.

2. Theory and Model

The basic structure of a single mode step index glass fibre is shown in Figure 1. The main parts are the core and the cladding. The core is a cylindrical rod made of dielectric material (pure silica glass). The core is described as having a diameter d_{core} and a refractive index n_{core}. The core is surrounded by a cladding layer of doped silica glass. The cladding has a refractive index of n_{clad} which is less than n_{core}; and has a diameter d_{clad}. A mode remains bound within the core if the propagation constant beta (β) meets the following boundary condition [1-4]:

$$n_{clad}\, k_0 < \beta < n_{core}\, k_0 \qquad (1)$$

where k_0 is the vacuum wave number.

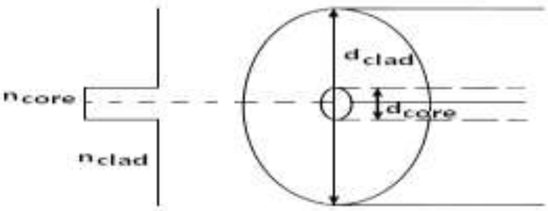

Figure 1. Single mode step index fibre

The boundary between truly guided modes and leaky modes is defined by the cut-off condition:

$\beta = n_{clad}\,k_0$. An important parameter connected to the cut-off condition is the normalized frequency (V) defined by Eq.(2):

$$V = \frac{2\pi r_{core}}{\lambda}\sqrt{n_{core}^2 - n_{clad}^2} \qquad (2)$$

where λ is the wavelength and r_{core} is the radius of the core. For a confined mode, the propagation constant (β) is related to the effective refractive index n_{eff} by the Eq.(3):

$$n_{eff} = \frac{\beta}{k_0} \qquad (3)$$

In this work, we have conducted a 2D mode analysis on a cross-section in the *xy*-plane of a single mode step index fibre. The electromagnetic fields in optical fibre waveguides are governed by the macroscopic Maxwell's equations in the absence of currents or external electric charges:

$$\nabla \times \mathbf{E}(x,y,z,t) = -\frac{\partial \mathbf{B}(x,y,z,t)}{\partial t} \qquad (3)$$

$$\nabla \times \mathbf{H}(x,y,z,t) = -\mathbf{J}(x,y,z,t) + \frac{\partial \mathbf{D}(x,y,z,t)}{\partial t} \qquad (4)$$

$$\nabla \cdot \mathbf{D}(x,y,z,t) = -\rho \mathbf{B}(x,y,z,t) \qquad (5)$$

$$\nabla \cdot \mathbf{B}(x,y,z,t) = 0 \qquad (6)$$

where **E, H, D** and **B** are the electric, the magnetic, the dielectric and the magnetic induction fields, respectively. Also, the term **J** is the current density, ρ is the charge density, and t denotes time. For the model under study, the wave propagates in the z direction and has the form:

$$\mathbf{H}(x,y,z,t) = \mathbf{H}(x,y)e^{j(\omega t - \beta z)} \qquad (7)$$

where ω is the angular frequency and β is the propagation constant. An eigenvalue equation for the magnetic field **H**, which is solved for the eigenvalue $\lambda = -j\beta$, is derived from Helmholtz equation given by:

$$\nabla \times (n^{-2}\nabla \times \mathbf{H}) - k_0^2\mathbf{H} = \mathbf{0} \qquad (8)$$

3. Modelling and Simulation Procedure

The FEM in COMSOL study employed the RF Module which combines the optics and photonics interfaces. The material properties used in the study are shown in Table 1. The material properties in Table 1 are valid for the free space wavelength of 1.55 µm, which is in the infrared region where most of the fibre optic communications operate [1-3,7].

Table 1. Material properties used in the study

	Core	Cadding
Material	Silica Glass	Doped Silica Glass
Refractive Index	n_{core} = 1.4457	n_{clad} = 1.4378

A modal analysis was performed and the associated parametric sweeps where done in order to investigate the dependence of the fibre performance on the core radius. The core radius was varied from 2.5 to 6.0 µm in steps of 0.5 µm. This is the recommended range of core radii for the transmission wavelength of 1.55 µm [2,8,9]. The corresponding electric field intensities and effective refractive indices where determined for various values of core radius. The standard meshing tool was used with the mesh setting at physics – controlled mesh and element size set to "finer". Figure 2 shows the meshed geometry of the fibre cross section in 2D.

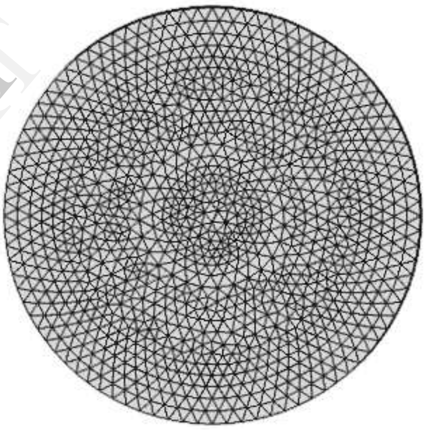

Figure 2. Structure of the finite elements in COMSOL for a single mode step-index fibre

4. Results and Discussion

3.1 Distribution of electric-field intensities

The first part of the modal study was a parametric analysis of the effect of core radius on the electric field intensities that propagate from the fibre core into the cladding. The cladding radius was fixed at 40 µm since this order of thickness avoids prohibitive losses. The core radius was varied from 2.5 to 6 µm in steps of 0.5 µm. The variation of electric field intensities propagating from the core into the cladding was determined, as shown in Figure 3.

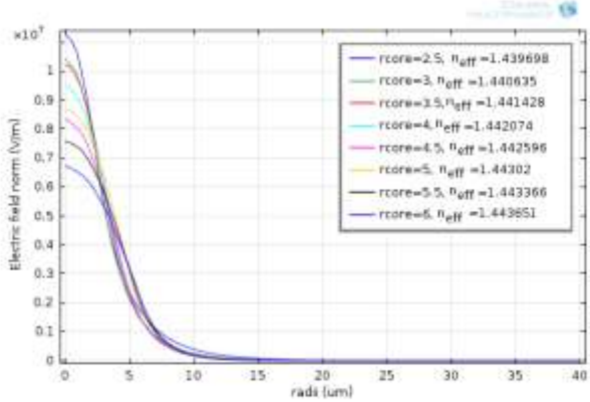

Figure 3. E-field intensities as a function of cladding radius for various core radii

Figure 3 shows the variation of electric field intensities from the centre of the fibre core into the cladding region. From Figure 3, it is evident that the electric field intensities for all the core radii studied fall to zero beyond a cladding radius of around 15 µm. Figure 4 shows the symmetrical distribution of the electric field intensities for cladding radii of up to 20 µm.

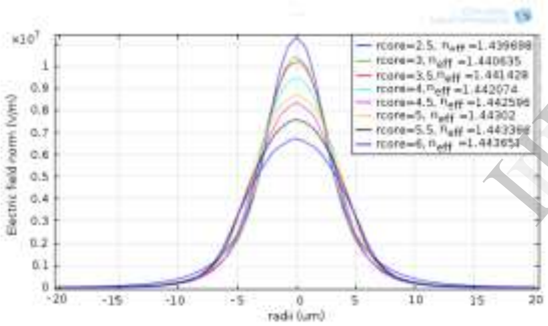

Figure 4. Electric field intensities as a function of cladding radius for various core radii

From Figure 4, a core radius of 2.5 µm has the highest electric field at the centre (1.13×10^7 V/m) while a core radius of 6.0 µm has the lowest electric field intensity at the centre of the core (7.61×10^6 V/m). Between 2.5 and 6 µm, the electric field intensity at the centre of the fibre decreases. As the radius of the core is increased, the electric potential gradient is reduced. Figure 5 shows the electric field intensity using the Mode Analyses study from COMSOL for the meshed structure in Figure 2. The results in Figure 5 confirm the single mode propagation in the optical fibre. From Figure 5(a), a single mode fibre with a core radius of 2.5 µm does not confine all the energy within the core. This penetration of low-order and high-order modes into the cladding region indicates that some are refracted out of the core.

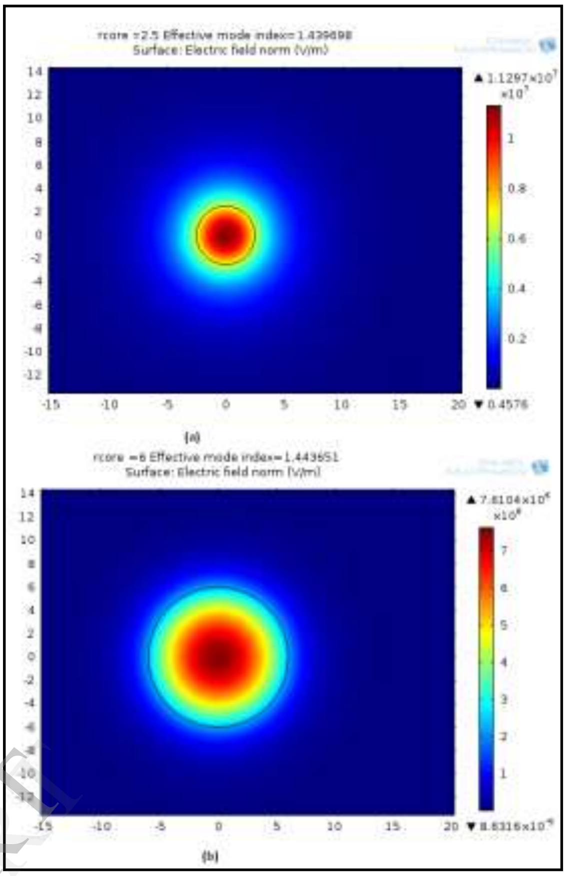

Figure 5. 2D Electric field distribution for (a) r_{core} = 2.5 µm (b) r_{core} = 6 µm

The refracted modes may become trapped in the cladding due to the thickness dimension of the cladding region. The modes trapped in the cladding region are called cladding modes. As the core and the cladding modes travel along the fibre, mode coupling occurs. Mode coupling may occur when any two modes exchange power and this result in the loss of power from the core mode(s). As a result, energy may leak deep into the cladding, leading to substantial power losses especially in short fibres [8-10]. On the other hand, a fibre with core radius of 6 µm, while it confines all the energy in the core, it also confines a substantial amount of weak fields within the core and hence reducing the peak intensity at the centre of the fibre core (see Figure 5(b)). Figure 6 shows the electric and magnetic fields for the fundamental mode for core radii of 2 and 6 µm. Figure 6(a) confirms the leakage of energy from the core into the cladding region.

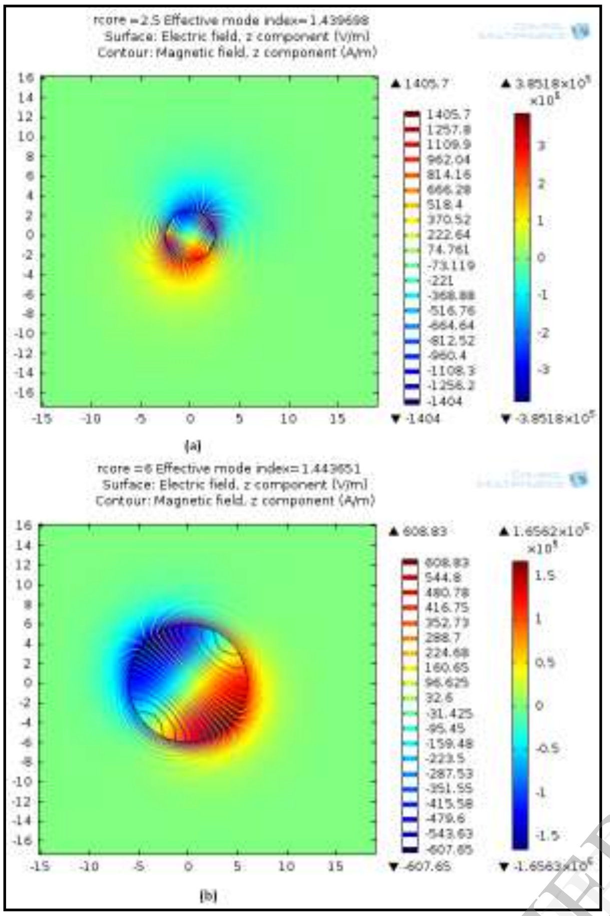

Figure 6. Electric an Magnetic fields (a) r_{core} = 2.5 µm (b) r_{core} = 6 µm

The maximum radius of the core that gives optimum performance of the single mode fibre can be determined by using the maximum value of the normalised frequency (V). For single mode operation, the maximum value of V is 2.4 [2,11]. This translates to a core radius of 3.92 µm ~ 4.0 µm. This optimum core radius value is in agreement with values reported in literature [1-3, 11-13]. Figure 7 shows the electric field profiles for the optimum core radius of 4 µm.

Small core radii below 4 µm pose problems with launching light into the fibre. Small core diameters require relatively expensive connectors to minimize connection losses. In addition, reduced relative refractive index difference between the core and cladding presents difficulties in fabrication of fibres with small core diameters [11-13]. On the other hand, single mode fibres with core radii greater than 4 µm have a marked reduction in the peak electric field intensity at the centre of the core. Single mode fibres with large core radii are easy to connect with other fibres. However, they suffer from losses due to intermodal dispersion since they allow other modes to propagate.

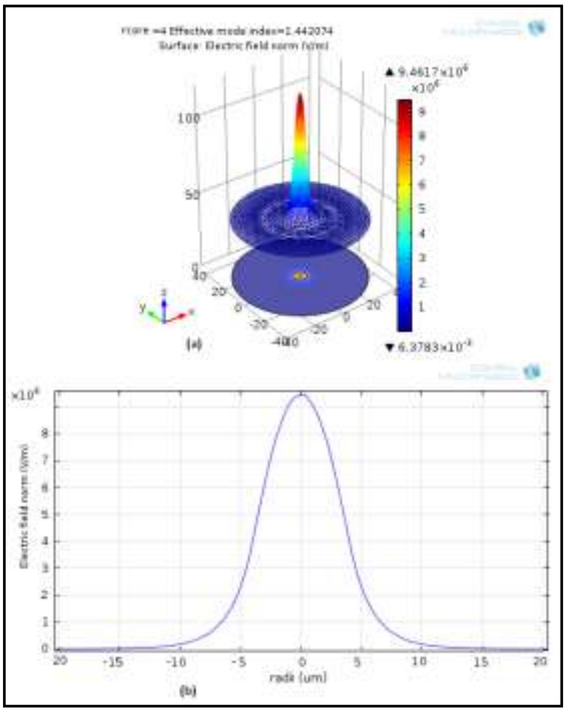

Figure 7. Electric field profiles for the optimum core radius of 4 µm

4. Conclusions

A single mode step index glass fibre was successfully analysed using the FEM in COMSOL Multiphysics®. The results show that for core radii in the range 2.5 to 6 µm, with a cladding radius of 40 µm, it is possible is possible for the single-mode step index glass fibres to sufficiently support the propagation of the fundamental mode. The optimum core radius was found to be 3.92 µm ~ 4.0 µm which agrees well with values reported in literature. It was also demonstrated that the use of FEM computational package, COMSOL Multiphysics® improves the way of estimating the behaviour of optical fibres by enabling the analysis of critical parameters such as effective mode index, magnetic and electric field intensities.

5. References

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